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THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

Volume XXIII, Number 1 October 1983

Mathematics Council, ATA 23rd Annual Conference

"Motivation in Mathematics"

October 28 and 29, 1983
University of Calgary

Mathematics Council, ATA
23rd Annual Conference
Friday evening, October 28
and Saturday, October 29, 1983
The University of Calgary

THEME

"MOTIVATION IN MATHEMATICS"

Keynote Speaker

GORDON ELHARD

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Editorial

Professor George Cathcart Retires as *delta-K* Editor

Professor George Cathcart, editor of *delta-K*, has decided to "retire" after two very successful years. MCATA wishes to acknowledge the very great contribution Professor Cathcart has made not only to mathematics education, but also to educational publishing in Alberta.

Under his able leadership, the last two years of *delta-K* have provided outstanding articles and reporting, useful ideas, and a treasury of current and new thinking which have assisted mathematics teachers in their constant pursuits of keeping abreast of recent developments, remaining "current" in the classroom, and developing teaching that is relevant to the everyday interests of their students.

He has also succeeded in enlarging our perpective of developments in North America, Canada, and Alberta, and posed the questions that focused our attention on some of the pressing problems of curriculum development and selection of text materials that face mathematics teachers in Alberta.

Along with these contributions, Professor Cathcart is held in high esteem for his practical comment, which is confidence-inspiring and timely. The future is bright, for we feel sure that his valuable influence will continue at the leading edge of mathematics education. Thank you, Professor Cathcart, for your valued contributions. And to all our readers, as he wrote a year ago, "best wishes for a professionally rewarding year."

--*Gordon Nicol*

Announcement

New Editor for *delta-K*

As President of MCATA, it is my privilege to introduce our new *delta-K* editor, Gordon Nicol. Gordon is a Grade 6 teacher in Mistassiniy School, Desmarais, and he volunteered his services after reading, in the second issue of the *MCATA Newsletter*, about our need of an editor for the journal. Gordon has edited a variety of materials since high school. We welcome him to the MCATA Executive and trust his efforts will be rewarding for him as well as for us.

--*Gary R. Hill*

An Advisory Exam in Mathematics

Z. M. Trollope
Department of Mathematics
University of Alberta, Edmonton

The mathematics department at the University of Alberta gave the following 50-minute examination to 2629 students in its introductory calculus courses at the beginning of the 1982 fall term. The mean score was 11.92. Those students whose background appeared weak were advised to participate in the mathematics remedial program.

Advisory Exam Statistics

Table I gives the percentage of students answering each of the 25 questions correctly.

Table II gives the relative frequency (R.F.) and cumulative frequency (C.F.) for each of the possible scores (that is, 0 to 25).

Table I

Question	Percentage
1	77
2	37
3	56
4	64
5	66
6	22
7	26
8	60
9	82
10	62
11	61
12	31
13	24
14	17
15	47
16	23
17	58
18	41
19	55
20	49
21	59
22	46
23	39
24	40
25	50

Table II

Score	R.F.	C.F.
0	0.3	0.3
1	0.6	0.9
2	0.6	1.6
3	2.1	3.6
4	3.0	6.7
5	4.0	10.6
6	5.5	16.1
7	6.2	22.3
8	7.0	29.3
9	7.6	36.9
10	6.9	43.8
11	7.1	50.9
12	6.7	57.6
13	5.2	62.8
14	5.6	68.4
15	5.6	74.1
16	5.2	79.2
17	4.4	83.6
18	3.5	87.1
19	3.1	90.1
20	2.7	92.8
21	2.0	94.8
22	2.3	97.1
23	1.3	98.4
24	1.0	99.4
25	0.6	100.0

1. If $\frac{1}{x-5} + 3 = \frac{x}{x-5}$, then $x =$

(a) -2 (b) 1 (c) 2 (d) 4 (e) 7

---

2. $\frac{2x}{x^2-1} - \frac{1}{x-1} =$

(a) $\frac{1}{x-1}$ (b) $\frac{1}{x+1}$ (c) $\frac{2x-1}{x^2-1}$ (d) $x-1$ (e) $x+1$

---

3. Suppose the sides of a rectangle with length x and width y are each increased by h. The increase in the area of the rectangle is

(a) h^2 (b) $xh+yh+h^2$ (c) $xh+yh$ (d) xy (e) xyh

---

4. If the equation $(x-k)^2 = k^2+2x+x^2$ is to be true for all x, then $k =$

(a) -2 (b) -1 (c) 0 (d) 1 (e) 2

---

5. The solutions of $(5y-1)(y+1) = 8y$ are

(a) $1, 0$ (b) $-1, -\frac{1}{5}$ (c) $-\frac{1}{5}, 1$ (d) $\frac{1}{2}, -\frac{1}{2}$ (e) $\frac{1}{5}, -1$

---

6. The polynomial equation $x^5-16x = 0$ has how many real roots?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

---

7. The graph of the parabola $y = x^2-16x$ is symmetric with respect to the line

(a) $x = 8$ (b) $x = 4$ (c) $x = -4$ (d) $y = 4$ (e) $y = -4$

---

8. The x-coordinate of the intersection of the graphs of $2x-y = 6$ and $x+y = -3$ is

(a) -3 (b) -2 (c) -1 (d) 0 (e) 1

---

9. A rectangle has vertices at $(2,3)$, $(8,3)$, $(2,-5)$ and $(8,-5)$. The length of a diagonal is

(a) $\sqrt{10}$ (b) 6 (c) 10 (d) 40 (e) 100

---

10. The graph of $x^2+y = 1$ is

(a) a circle with center $(0,0)$ and radius 1

(b) a line through $(0,1)$ with slope -1

(c) a line through $(0,1)$ with slope 1

(d) a parabola with vertex $(0,1)$ opening downward

(e) a parabola with vertex $(0,1)$ opening to the left

---

11. $-3 < 5 - 2x < 3$ is equivalent to

(a) $x > 1$ (b) $x < 1$ (c) $1 < x < 4$ (d) $-1 < x < 1$

(e) $x < 1$ or $x > 4$

---

12. The equation $|x| = -x$ is an identity for

(a) all $x \geq 0$ (b) all $x \leq 0$ (c) only $x = 0$

(d) all real numbers (e) no real numbers

---

13. $\log_3 \frac{1}{9} =$

(a) -3 (b) -2 (c) $\frac{1}{2}$ (d) 2 (e) 3

---

14. If $\log_b x = \frac{7}{10}$ and $\log_b y = \frac{1}{5}$, then $\log_b (xy)^2 =$

(a) $\frac{7}{25}$ (b) $\frac{81}{25}$ (c) $\frac{49}{625}$ (d) $\frac{9}{5}$ (e) $\frac{81}{100}$

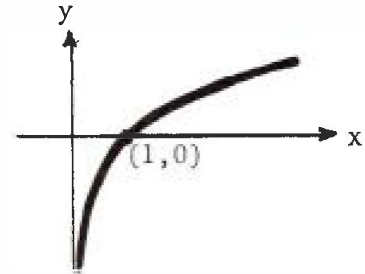
---

15. If $x > 0$ and $y > 0$, then $\sqrt{8\sqrt{4x^6y^4}} =$

(a) $8xy\sqrt{x}$ (b) $4xy\sqrt{2x}$ (c) $4xy\sqrt{x}$ (d) $4x^2y$ (e) $4x^3y^2\sqrt{2}$

---

16. The given sketch best represents the graph of



(a) $y = x^2-1$ (b) $y = \log_{10} x$

(c) $y = 10^{-x}$ (d) $y = 1-x$

(e) $y^2 = x-1$

---

17. The number of bacteria in a certain culture at time t is given by $N(t) = 2^{kt}$. If at time $t = 5$ the number of bacteria equals 32 units, then $k =$

(a) 5 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1 (e) $\frac{6}{5}$

---

18. If $f(x) = 2^x - x^2$, then $f(0)-f(-1) =$

(a) 5 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1 (e) $\frac{3}{2}$

---

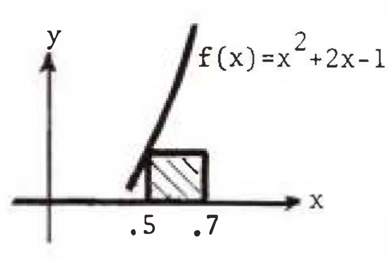
19. Let $f(x) = \sqrt{x^2-4}$. The domain of f is

(a) $|x| \geq 2$ (b) $|x| \leq 2$ (c) $x \geq -2$ (d) $x \leq 2$

(e) all real numbers

---

20. The area of the rectangle pictured on the right is



(a) 0.05 (b) 0.2

(c) 0.25 (d) 0.35

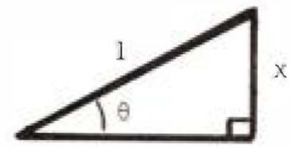
(e) 1.2

---

21. The radian measure of the angle 240° is

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{4\pi}{3}$ (d) $\frac{5\pi}{3}$ (e) $\frac{2\pi}{3}$

22. In the right triangle shown, $\cos\theta =$



(a) x (b) $x\sqrt{1-x^2}$

(c) x^2+1 (d) $\frac{1-x^2}{2}$

(e) $\sqrt{1-x^2}$

23. Which of the following numbers is largest?

(a) $\sin(-\pi)$ (b) $\sin(-\frac{\pi}{2})$ (c) $\sin 0$ (d) $\sin\frac{\pi}{2}$ (e) $\sin\pi$

24. $\cos(90^\circ - \theta) =$

(a) $\cos\theta$ (b) $-\cos\theta$ (c) $\sin\theta$ (d) $-\sin\theta$ (e) $1 + \cos\theta$

25. $\frac{(\cos^2\theta)(\tan\theta)}{\sin\theta} =$

(a) $\sin\theta$ (b) $\cos\theta$ (c) $\tan\theta$ (d) $\sin^2\theta\cos^2\theta$ (e) $\sec\theta$

Introducing Geometry and Algebra through Computer Programming

John Curda
Capitol Hill Elementary School, Calgary

While teaching a geometry unit to my Grade 5 and 6 class at Capitol Hill Elementary School, I decided to tie coordinate geometry to computer programming using high-resolution graphics on the Apple II+ microcomputer. I have a portion of my blackboard squared off with white lines so that I can easily number the axis for a grid for coordinate geometry and/or for the high-resolution graphics screen.

I began this particular introductory lesson on coordinate geometry by marking the grid in the same manner that is used for Apple high-resolution graphics. Students were told how to locate a point on the screen by starting at the upper left-hand corner, then going across, and then going down. A simple straight-line shape was then drawn on this grid (see Table 1).

Students were asked to identify the location of each point at the end of a straight line (see Table 2). Students had to locate the coordinates of the points from A to M in alphabetical order, and then end at B. Each point was written down on the blackboard. Where we had written the location of the coordinates, a space was left on the left-hand side so that we could add line numbers and key BASIC command words. After the location of the coordinate point was written on the blackboard, the word "TO" was written after it. This was followed by the coordinates of the point where the straight line ended. This procedure was continued until we returned to the starting point. To tie this into computer programming, the command "HPLOT" was placed in front of the line containing the coordinates of all the points. It was also necessary to add two BASIC commands to the program, "HGR" and "HCOLOR=3," before the location of the coordinate points. I then added line numbers to complete the required programming instructions. Following is the simple program that we ended up with in order to draw the rocket.

```
10 HGR
20 HCOLOR=3
30 HPLOT 10,0 TO 10,10 TO 15,15 TO 15,25 TO 20,30 TO 15,40 TO 15,35 TO 10,40 TO
         5,35 TO 5,40 TO 0,30 TO 5,25 TO 5,15 TO 10,10
40 END
```

To culminate this activity, the program was entered into the computer and run. The follow-up activity consisted of giving each student a sheet of graph paper (with the top and left-hand margin marked with numbers for plotting points in high-resolution graphics). Each student was asked to created a simple straight-line picture, and then write a computer program to draw this picture.

Those who completed the assignment were permitted to enter their program on the computer. To save time and reduce frustration, the "HELLO" program automatically loaded a "LINE EDITOR" program into the computer's memory. If corrections were required, the need to retype a complete line was eliminated. The students, once the program was entered into the computer, had instant feedback on their programs. If the computer drew what they had intended, they knew immediately that they were right. If the computer drew something else, they then had to go back to the graph paper and check the coordinates against what had been entered into the program. Students helped one another to do the checking. This cooperation resulted in students learning how to check the locations of the points and provided them with more time to enter their programs. Not only did students enter their programs during the math period, but they also asked for and received permission to enter their programs before school, at noon, or after school.

Having achieved this in a relatively short period of time, I thought that it would be interesting to teach students how to get their diagram to move either horizontally or vertically. This was done by going back to the grid on the blackboard (see Table 3). To get the rocket at the bottom of the screen, we assigned a variable to Y so that Y + 40 = 159. To get the rocket to move up, we decreased the value of Y.

In effect, what we were doing was placing a grid within a grid. This enabled students to plot various points using (X + the point on the second grid) as the distance over, and (Y + the point on the second grid) as the distance down. I felt that the students did not immediately grasp the reason why this was required, but they could follow the rules. Having the students watch the program with movement and asking them well-directed questions enabled the students to see what has happening.

This enabled us to use a subroutine alternately to draw, erase, move, redraw, erase, and so on, until the movement was complete. This movement was done through the use of a "FOR . . . NEXT . . . STEP" loop. At this time, the students were able to see what happened if we changed the value of either X or Y by a constant. The following is the program that we used to create movement:

```
10 HGR
20 X = 50
30 FOR Y = 1 TO 119 STEP -1
40 HCOLOR=3: GOSUB 20000
50 HCOLOR=0: GOSUB 20000
60 NEXT Y
70 HCOLOR=3: GOSUB 20000
80 GOTO 30000
20000 HPLOT X + 10, Y + 0 TO X + 10, Y +
          10 TO X + 15,Y + 15 TO X + 15,Y + 25 TO X + 20,Y + 30 TO X + 15,Y + 40
          TO X + 15,Y + 35 TO X + 10,Y + 40 TO X + 5,Y + 35 TO X + 5,Y + 40 TO X
          + 0,Y + 30 TO X + 5,Y + 25 TO X + 5,Y + 15 TO X + 10,Y + 10
20010 RETURN
30000 END
```

Coordinate geometry, algebra, number patterns, and computer programming had been introduced in a meaningful manner to students in the space of one school week. Students were highly motivated and delighted with the results.

Table 1

Grid showing a diagram of the rocket.

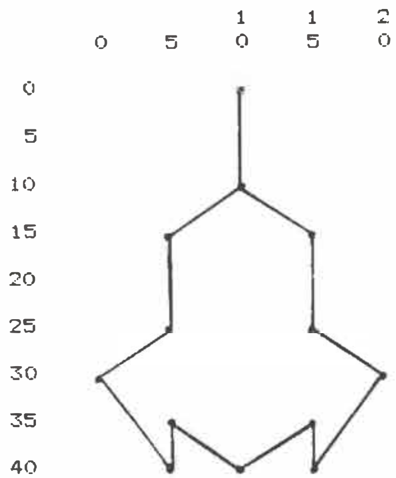


Table 2

Grid showing coordinate points of the rocket.

```
                    1    1    2
         0    5     0    5    0

 ●                  A

 5

10                  B

15            M          C

20

25            L          D

30       K                    E

35            I          G

40            J     H    F
```

Table 3

Grid showing coordinate points of a moveable rocket.

```
      0    5    0    .    .    .

0 0

5

10                        Y

                                              1    1    2
                     X    +        0    5     0    5    0

                          0                  A

                          5

                         10                  B

                         15            M          C

                         20

                         25            L          D

                         30       K                    E

                         35            I          G

                         40            J     H    F
```

Geometric Patterns for Combinatorial Results

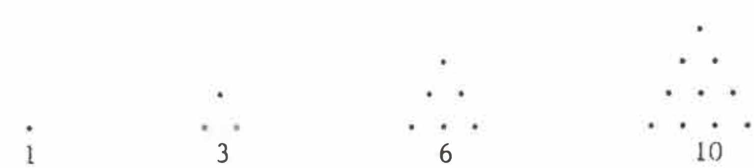
David R. Duncan and Bonnie H. Litwiller

Professors of Mathematics

University of Northern Iowa, Cedar Falls, Iowa

Teachers and their students frequently wish to use activities that provide for drill and practice and also encourage the review or discovery of number patterns. A rich source for such activities is found in sets of figurate numbers; that is, numbers denoting points that can be arranged in geometric shapes.

A familiar set of figurate numbers is the set of triangular numbers. The first four triangular numbers are shown below; they are so named because of the triangular shapes they form.



Verify, by drawing, the appropriate pictures that the first 10 triangular numbers are 1, 3, 6, 10, 15, 21, 28, 36, 45, and 55.

A three-dimensional extension of the notion of triangular numbers results in the tetrahedral numbers 1, 4, 10, 20, 35, 56, 84, 120, 165, 220, ··· These tetrahedral numbers can be depicted as pyramidal "stacks" of triangular numbers, similar to those formed by stacks of cannonballs in an American Civil War exhibit. Verify that the differences between consecutive numbers ($4 - 1 = 3$; $10 - 4 = 6$; $20 - 10 = 10$; $35 - 20 = 15$; ···) are consecutive triangular numbers. That this should be true is apparent if one recalls that tetrahedral numbers are formed three-dimensionally by stacking two-dimensional layers of triangular numbers. We will now generate these two sets of numbers in a different way.

Recall that $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

Activity 1

$$\binom{2}{2} = \frac{2!}{2!\ 0!} = 1$$

$$\binom{3}{2} = \frac{3!}{2!\ 1!} = 3$$

$$\binom{4}{2} = \frac{4!}{2!\ 2!} = \frac{4\cdot 3}{2\cdot 1} = 6$$

$$\binom{5}{2} = \frac{5!}{2!\ 3!} = \frac{5\cdot 4}{2\cdot 1} = 10$$

Verify that in general $\binom{n}{2}$ generates the $(n-1)$ st triangular number.

Activity 2

$$\binom{3}{3} = 1$$

$$\binom{4}{3} = 4$$

$$\binom{5}{3} = \frac{5\cdot 4}{2\cdot 1} = 10$$

$$\binom{6}{3} = \frac{6\cdot 5\cdot 4}{3\cdot 2\cdot 1} = 20$$

$$\binom{7}{3} = \frac{7\cdot 6\cdot 5}{3\cdot 2\cdot 1} = 35$$

Verify that in general $\binom{n}{3}$ generates the $(n-2)$ nd tetrahedral number.

Activity 3

$$\binom{4}{4} = 1$$

$$\binom{5}{4} = 5$$

$$\binom{6}{4} = \frac{6\cdot 5}{2\cdot 1} = 15$$

$$\binom{7}{4} = \frac{7\cdot 6\cdot 5}{3\cdot 2\cdot 1} = 35$$

$$\binom{8}{4} = \frac{8\cdot 7\cdot 6\cdot 5}{4\cdot 3\cdot 2\cdot 1} = 70$$

Continue this pattern. Note that the differences of consecutive numbers in this sequence form consecutive tetrahedral numbers ($5 - 1 = 4$; $15 - 5 = 10$; $35 - 15 = 20$; $70 - 35 = 35$; $\cdots$). To give a geometric explanation would require the notion of a four-dimensional "stack" of three-dimensional tetrahedral numbers. How might it be pictured?

Activity 4

Continue this pattern for $\binom{r}{r}$, $\binom{r+1}{r}$, $\binom{r+2}{r}$, $\binom{r+3}{r}$, $\cdots$ for $r = 5, 6, 7, \cdots$. How can the results be described and/or pictured?

Activity 5

Depict "rectangular" numbers as shown by the following $n \times (n + 1)$ rectangles.

$1 \times 2 = 2$	$2 \times 3 = 6$	$3 \times 4 = 12$	$4 \times 5 = 20$
(1st rectangular number)	(2nd rectangular number)	(3rd rectangular number)	(4th rectangular number)

Note that:
$$\begin{aligned} 2 &= 2 \\ 6 &= 2 + 4 \\ 12 &= 2 + 4 + 6 \\ 20 &= 2 + 4 + 6 + 8 \\ &\vdots \end{aligned}$$

Continue this pattern. Is it always the case that the n th rectangular number is the sum of the first n positive even integers?

Activity 6

Note that:
$$\begin{aligned} 2 &= 2 \times 1 \\ 6 &= 2 \times 3 \\ 12 &= 2 \times 6 \\ 20 &= 2 \times 10 \\ &\vdots \end{aligned}$$

Continue this pattern. The n th rectangular number appears to be equal to twice the n th triangular number. To verify the pattern, we can partition the triangular numbers as follows:

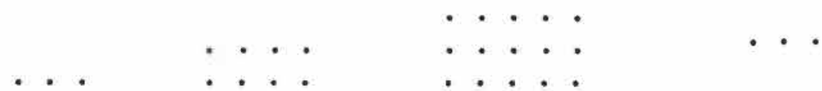
Activity 7

The following figure depicts the interior of a multiplication table. Note that the circled set represents the rectangular numbers of Activities 5 and 6.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57
0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114
0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133
0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152
0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171
0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190
0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209
0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228
0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247
0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266
0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285
0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304
0	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323
0	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342
0	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361

Activity 8

Replicate the steps of Activities 5, 6, and 7 using rectangular numbers in an n X (n + 2) shape:



1 X 3   2 X 4   3 X 5   . . .

What patterns can be found?

This article has only scratched the surface of pattern-finding involving figurate numbers. Can you and your students find more?

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