The Development of Problem-Solving Skills: Some Suggested Activities (Part I)

John B. Percevault
University of Lethbridge

John B. Percevault is an Associate Professor at the University of Lethbridge. During 1982-83, while on administrative leave, he worked with Grade 3-6 teachers in Lethbridge elementary schools. This article presents some of the problem-solving skills and strategies that were used in the schools.

A quick overview of mathematics teaching in Alberta schools would reveal that activity is centred on problem solving. Alberta Education has produced an excellent monograph, *Let Problem Solving Be the Focus in the 1980s.*[1] This service document presents practical ideas for implementing Polya's four-step procedure for problem solving, namely, understanding the problem, developing a plan, carrying out the plan, and looking back. Another excellent monograph has been produced by the Mathematics Council of The Alberta Teachers' Association.[2] At the international level, the National Council of Teachers of Mathematics has produced a yearbook, *Problem Solving in School Mathematics.*[3]

The first recommendation in *An Agenda for Action*, another publication of the National Council, states, "Problem solving must be the focus of school mathematics in the 1980s." To implement the recommendation, recommended action 1.2 states:

> The definition and language of problem solving in mathematics should be developed and expanded to include a broad range of strategies, processes, and modes of presentation that encompass the full potential of mathematic applications.
>
> > Computational activities in isolation from a context of application should not be labeled "problem solving."
> > The definition of problem solving should not be limited to the conventional "word problem" mode. As new technology makes it possible, problems should be presented in more natural settings or in simulations of realistic conditions.
> > Educators should give priority to the identification and analysis of specific problem-solving strategies.
> > Educators should develop and disseminate examples of "good problems" and strategies and suggest the scope or problem-solving activities for each school level.[4]

Is There a Need to Define?

Authors use such terms as problem-solving strategies and problem-solving skills. For the purpose of this article, problem-solving skills are defined as the information the students may have, which includes facts, processes, and algorithms. Problem-solving strategies are procedures for processing the information.[5] Skills become the building blocks for the learner to develop new or different knowledge and to solve problems. Further, problem-solving skills may be taught and/or developed independently of the problem-solving situation. Teachers of elementary school children may wish to identify particular problem-solving skills. Instruction could then incorporate the skill during the teaching/learning situation, not only in mathematics, but in all subject areas. This does not mean that challenging problems should be excluded from the elementary curriculum. Hopefully, attention to the acquisition of problem-solving skills will enhance students' problem-solving ability.

The listings that follow are not intended to be all-inclusive:

Problem-Solving Skills

Reading

Computation

Representations (Models) such as:
--real materials
--drawing a picture
--a diagram

Recognize and continue a pattern

Develop a pattern

Estimate

Computation

Use formula

Collect and record data in:
--unorganized lists
--organized lists
--tables
--graphs

Problem-Solving Strategies

Modeling; for example:
--Develop a formula
--Transform or develop an equation
--Develop a system of notation

Relate to:
--another problem that has been solved
--a simpler but related problem (records and interprets)

Changes point of view

Break problem into two or more simpler problems

Inferencing skills, including:
--guess and check
--trial-and-error
--prediction
--generalize
--hypothesize
--seek relationship

Looks for pattern and then infers

Logical analysis

Synthesize (works backwards)

Asks an alternate question

It should be noted that the problem solver often employs two or more skills and/or strategies in arriving at a solution.

Suggested Activities

Activities that may be used to enhance the development of problem-solving skills should be incorporated into the teaching procedures for introducing the basic processes and introducing other strands of the elementary mathematics curriculum. Some suggestions follow.

Activity 1: Reading in Mathematics

A prerequisite to becoming an independent problem solver is the ability to read. To understand the problem, certain techniques may be employed, techniques that may be used with textbook story problems (routine) or with nonroutine problems.

Every problem in the text does not need to be solved. Use many of the problems for a directed reading lesson. Questions that the teacher may ask include:

1. What is the question? What are we to find? Have students read the question, state the question in their own words, or tell the question to a friend.

2. What quantities are involved? What is the information given? It is suggested that students encounter problem situations that have:

   --only sufficient information; the student states, tells, or writes.
   --insufficient information; the student identifies what is needed and supplies missing information.
   --extraneous information; the student crosses out extraneous information and rereads, omitting extraneous information.
   --information that has to be recalled or inferred; the student supplies the formula and orally states what must be recalled or inferred.
   --information that has to be determined from a diagram, graph, or picture; the student interprets a diagram, graph, or picture.
   --information that needs to be researched; the student uses the library to find information.

3. What process(es) is (are) to be used? Have students support the choice of process(es). Cue words (more, less, in total, and so on) may be identified. Students may also be asked to develop related problems that are at an easier, similar, or more difficult level. Students may also diagram or use a more abstract representation such as a number line.

4. Can you solve the problem? Each step of the directed reading procedure may be used as the basis of a lesson.[6]

Students who are not proficient readers may still develop the skills noted. The teacher or classmate should read the problem to the students.

Activity 2: Developing Models

Children learn to represent problems through the use of real material and through interpreting and developing pictures or diagrams. Textual materials utilize this technique to present the basic processes. Spatial visualization, a geometric skill, may also be introduced with real materials and diagrams. Early experience may help female achievement in spatial visualization, which is reported as being lower than male achievement.[7] Give children a supply of objects. Ask them to arrange the objects so that there are three in a row. Typical arrangements include:

```
 a        b        b        c

xxx      xxx      xxx     x x x

         xxx      x       x x x
                   x
                     x    x x x
```

Some may note that arrangement "c" actually has eight rows each containing three objects. Vary the activity so that children are asked to represent the following problems:

```
1.  3 objects        - 3 in a row       - 1 row
2.  5 objects        - 3 in a row       - 2 rows
3.  7 objects        - 3 in a row       - 3 rows only
4.  9 objects        - 3 in a row       - 4 rows only
5. 11 objects        - 3 in a row       - 5 rows only
```

The configurations may include:

Problem:

```
1.  000           0                0            0
       (1.1)        0  (1.2)     0  (1.3)      0  (1.4)
                      0        0               0

2.  000           000            0            0
      0 (2.1)     0 (2.2)       000 (2.3)     000 (2.4)
      0           0              0            0

3.  0 0              0          0             000
   0 0 0             0           0              000
    0 0  (3.1)    000 (3.2)       000 (3.3)    0     (3.4)
                  0                0
                  0                0


   000              000            0
    0  (3.5)         0    (3.6)   000 (3.7)
   000              000           000
```

Problems number 4 and 5 are left for the reader to determine.

Examine the representations with the children. Questions that may be explored include:

Is 1.1 the same as 1.2 and 1.3? (Rotate a representation using an overhead.)

Is 2.1 the same as 2.2? (2.2 is a "flip" of 2.1.)

Does 2.3 satisfy conditions in problem 2?

Does 2.4 satisfy the given conditions?

Examine 3.2 and 3.6. Are they the same?

How many rows each containing 3 objects are there in 3.5?

Which configurations satisfy the conditions of problem 3?

Geometric concepts of intersection, flips, and turns can be explored. Representations (real objects and diagrams) may be analyzed to determine whether conditions of the problems were met.

See next issue for activities on collecting and recording data, recording and listing, listing-subtraction, and organized lists in a problem.

Notes

[1]*Let Problem Solving Be the Focus for the 1980s* (Edmonton, Alberta: Alberta Department of Education, 1983).

[2]*Math Monograph No. 7: Problem Solving in the Mathematics Classroom*, edited by Sid Rachlin (Edmonton: Mathematics Council of The Alberta Teachers' Association).

[3]*Problem Solving in School Mathematics*, 1980 Yearbook of the National Council of Teachers of Mathematics, edited by Stephen Krulik and Robert E. Reys (Reston, Virginia: National Council of Teachers of Mathematics, 1980).

[4]*An Agenda for Action: Recommendations for School Mathematics of the 1980s* (Reston, Virginia: National Council of Teachers of Mathematics, 1980), p. 2.

[5]Adapted from Sydelle D. Ehrenberg, "Concept Learning: How to Make It Happen in the Classroom," *Educational Leadership* 39, no. 1 (October 1981):36.

[6]For an alternate listing, use Raymond S. Nickerson, "Thoughts on Teaching Thinking," *Educational Leadership* 39, no. 1 (October 1981):23.

[7]Elizabeth Fennema, "The Sex Factor," *Mathematics Education Research: Implications for the 1980's*, edited by Elizabeth Fennema (Alexandria, Virginia: Association for Supervision and Curriculum Development, 1981), page 96.