The Development of Problem-Solving Skills: Some Suggested Activities (Part II)

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Activity 3: Collect and Record Data

The spatial visualization activities may also be used to develop the skill of collecting and recording data. Choose a particular configuration and record data. The data from Activity 2 is presented in tabular form below.

Diagram	Number of Rows of 3 Objects	Number of Objects
000	1	3
0 000 0	2	5
0 0 0 0 0 0 0 0	3	7
0 0 0 0 0 0 0 0 0	___	___
	___	___

At the point where the partial table includes the first three sets of data, children may mark the rows of three and count the objects. Challenge the children to complete the next two rows. Have them predict the number of rows and the number of objects that would be required. The recognition of the patterns (counting numbers, and odd numbers starting with 3) is a skill. Extending the pattern to answer a question such as the following begins to develop into a problem-solving strategy: "If 21 objects were arranged in this pattern, how many rows each containing three objects would there be?" Children are challenged to find a short way of determining the number of objects used or number of rows required, in at least two ways. At this level, they are using a problem-solving strategy. For further applications, consider:

1. 5 objects per row

--5 objects
--9 objects
--13 objects
--___ objects
--___ objects
--___ objects

2.

Conditions?
What comes next?

3.

Conditions?
What comes next?

Use real materials, make diagrams, collect and record, search for patterns, extend, look for relations, and predict and generalize.

Activity 4: Recording and Listing

A second set of collecting and recording activities follows: activities relating to addition facts. Skills that are involved are listing, organizing data, and diagramming. The problem-solving strategy that may evolve is generalizing a rule or relationship.

Have children list all the sets of two addends that yield a sum of five. The listing would include: 1 + 4, 3 + 2, 0 + 5, 2 + 3, 5 + 0, and 4 + 1 (not necessarily in this order).

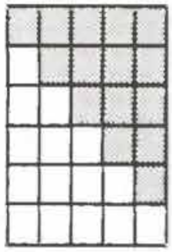
Challenge the students to organize the list. Give the cue to let one addend increase or decrease. The organized list is evident:

0 + 5 Commutative patterns
1 + _ (for example, 0 + 5 and 5 + 0)
2 + _
3 + _
4 + _
5 + _

Repeat the activity with the sum of 6, 7, 8, . . .

Encourage students to diagram the facts. A diagram of the addition facts for a sum of 5 is shown below.

$0 + 5 = 5$
$1 + 4 = 5$
$2 + 3 = 5$
$3 + 2 = 5$
$4 + 1 = 5$
$5 + 0 = 5$



Explore the following after the completion of listing and/or diagramming: How do odd and even numbers differ? If one addend is increased by two, what happens to the other addend? [The compensation principle may be expressed thus: If $a + b = c$, then $(a + k) + (b - k) = c$.] How does the number of 2 addend facts for a given sum compare to the sum $(S + 1)$? These questions will always produce a finite list with $S + 1$ members where "S" is the sum.

Activity 5: Listing – Subtraction

Ask students to list two numbers that produce a difference of two. Accept any and all correct responses, such as:

$$\begin{array}{r}4\\-2\\\hline 2\end{array}\qquad\begin{array}{r}15\\-13\\\hline 2\end{array}\qquad\begin{array}{r}2002\\-2000\\\hline 2\end{array}$$

Many students will realize intuitively that the list is infinite. The teacher may wish to develop an organized list, such as:

$$\begin{array}{r}2\\-0\\\hline 2\end{array}\qquad\begin{array}{r}3\\-1\\\hline 2\end{array}\qquad\begin{array}{r}4\\-2\\\hline 2\end{array}\qquad\begin{array}{r}5\\-3\\\hline 2\end{array}$$

Have students examine the list. Ask them if they can state a relationship. [The compensation principle in subraction may be expressed thus: If $a - b = c$, then $(a + k) - (b + k) = c$.] Have the students use the generalization of compensation in addition.

If $2 - 0 = 2$, then is $(2 + 10) - (0 + 10) = 2$?
If $2 - 0 = 2$, then is $(2 + 20) - (0 + 20) = 2$?

Further examples may be explored, such as:

If $32 - 30 = 2$, then is $42 - 40 = 2$?
Is the answer to $34 - 17$ the same as the answer to $37 - 20$? Which subtraction is easier to perform?

An activity such as the one above can be used to introduce the "equal addition algorithm" for subtraction.

Activity 6: Organized Lists in a Problem

Problem: I am thinking of two numbers. The sum of the two numbers is 13. The difference of the two numbers is 3. The numbers are ___ and ___. Obviously, the elementary student could guess and check. However, a student who has a more organized approach could use the skills developed above. Ask such questions as:

What are we to find? Answer: two numbers.
How are they related? (What is the condition?) Answer: They have a sum of 13.
Is there another condition? Answer: Yes, they have a difference of 3.
How many two-addend addition facts are there for a sum of 13? Answer: 14.
Can you develop an organized list? Answer: Hopefully, yes.
How do you want to organize? Answer: Answers may vary.

The following table may be developed with the students:

Sum of 13	Difference	
13 + 0	13	
12 + 1	11	Look for patterns in addends
11 + 2	9	and in difference columns.
10 + 3	7	
______		How does the difference vary?
______		(By 2). Why?
7 + 6	1	
6 + 7	1	
______		
______		
______	Do we need to do the rest of the table?	
______		
______		
0 + 13		

Assume absolute value only at this stage.

Encourage discussion on how the problem was solved. Answers that indicate students realize that one condition (sum of 13) was satisfied first and then checked in the second condition (difference of 3) show the beginning of a problem-solving strategy, because the problem has been divided into two or more problems.

Vary the conditions. For example,

Sum of 14	Difference of 6
Sum of 14	Difference of 3 (This is impossible. Why?)
Sum of __	Difference of __

Have students work in pairs to make up problems for their classmates to solve.

Activity 7: Counting Patterns

Use the hundred board. Have students count and color every second, third, and so on, square. After the first three rows are completed, encourage the students to look for patterns and to use the patterns to complete the coloring of appropriate squares. Counting by twos and fives is suggested as a starting point. The partial pattern for each is shown.

Counting by twos pattern:

1		3		5		7		9	
11									
21				25					
						47			
								69	
				95				99	

Counting by fives pattern:

1	2	3	4		6	7	8	9	
11	12	13	14		16	17	18	19	
21	22	23	24		26	27	28	29	
				65					
				75					
				85					
				95					

Explore the counting by threes pattern:

1	2		4	5		7	8		10
11					16				
					26				
31									40
41					46				50
					56				
61									70
71					76				80
					86				
91									100

The numerals 3, 6, 9, 12, . . . may be arranged in the format:

3	6	9
12	15	18
21	___	___
___	___	___

After students have extended the format, regular patterns may be explored. Each column increases by nine. The sum of the digits in each column is three, six, or nine. This may not be evident for a numeral such as 39, where the sum of digits is 12. However, 12 is found in the column that sums to three.

Further questions may include:

Is 47 a multiple of three (included in the counting by threes table)? Justify your answer.

Can you place 81, 42, and 96 in the appropriate column? Justify your answer.

Find the "counting by nines" on the hundred board and in the table.

The counting by eights pattern is given below:

								9	10
11	12	13	14	15					20
21									
31									
						97	98	99	100
101	102	103	104	105	106	107	108	109	110

Again, encourage students to explore alternate arrangements, such as the following:

8, 16, 24, 32, 40,
48, 56, 64, etc.

See the next issue for activities on organizing data--multiplication, and a sample problem and solution.