Let's Solve Problems with LOGO

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Problem solving and computer applications are two areas receiving the focus of our attention at this time. In *An Agenda for Action*, published by the National Council of Teachers of Mathematics (1980), Recommendation 1 states that "problem solving [must] be the focus of school mathematics in the 1980s," and Recommendation 3 states that "mathematics programs [must] take full advantage of the power of calculators and computers at all grade levels." In an attempt to correlate problem solving and computer programming in LOGO, both were chosen as the core of a three-month enrichment project for Grade 6 students at Landsdowne School.

A total of 14-16 hours was spent on the project. Students usually had one hour per week of school time plus one-half to three-quarters of an hour of early morning time to devote to the project. Each of the five students in the group had access to an Apple computer, and one printer was available.

Major Objective

The major objective was to have upper elementary pupils provide computer (LOGO) solutions to the problems deemed appropriate, which appear in the elementary mathematics problem-solving curriculum.

Minor Objectives

The minor objectives were as follows:

1. To make a selection of problems from: "Problem-Solving Elementary Math Levels D-F" (Edmonton Public Schools), *Let Problem Solving Be the Focus* (Alberta Education), and other sources, if necessary.

2. To determine the necessary LOGO primitives, procedures, and understanding that are required by pupils prior to tackling the solution of these problems using LOGO.

3. To solve and store solutions to problems.

4. To relate solutions to similar problems or to extend them to more complex problems.

5. To record further LOGO knowledge required and acquired as problems are dealt with.

6. To record unplanned instances in which students learned something new or found new applications for LOGO.

7. To gain some idea of the time involvement necessary to accomplish the above.

Procedures

1. Grade 6 students with no previous LOGO experience were the participants.

2. Charts were prepared to provide students with information on LOGO primitives and their application.

3. Introductory sessions gave students time to explore and discover LOGO turtle graphics, as well as some experience in handling text and numbers.

4. Application of LOGO to the elementary problem-solving curriculum was made.

5. One hour per week of "hands on" time was allotted. This was increased after the third week, at the request of students.

Outcomes

1. A set of LOGO charts.

2. A set of mathematics problems suitable for computer solution.

3. A list of necessary primitives and procedural methods required by students.

4. Computer solutions to the problems.

5. A collection of other problems that relate to LOGO.

6. A guide to the time allotments involved in such a project.

Pupils Involved

A group of five pupils composed the Grade 6 enrichment group. These pupils were selected by the homeroom teacher on the basis of their general achievement and ability. There were two girls and three boys. One boy (I will call him "G") had no computer experience prior to our first session. The other four had all done some programming in BASIC as a part of their computer literacy program in Grade 5.

As was expected, "G" required extra help and some additional time to keep pace with the others. He was much more inclined to work in "IMMEDIATE" mode and had to be encouraged to write procedures so he would have something to save at the end of the session. He was a very bright boy and had a good mathematical understanding of the problems.

Selection of Problems

The following resource materials were used as the source of problems: *Problem Solving Elementary Math Level D* (Edmonton Public School Board), *Problem Solving Elementary Math Level E* (Edmonton Public School Board), *Problem*

Solving Elementary Math Level F (Edmonton Public School Board), and *Let Problem Solving Be the Focus* (Alberta Education).

From the four problem-solving documents, 31 problems were selected as being suitable for solution on the computer using LOGO. It was found that the majority of the problems appeared to be inappropriate for LOGO solutions.

The 31 problems fell into the following categories:

a) pyramid building--four problems
b) ball falls and bounces--three problems
c) mix and match--five problems
d) squares on a checkerboard--four problems
e) handshake--five problems
f) pigs and chickens--six problems
g) miscellaneous--four problems

The problems that were solved by the group are included in Appendix A.

Required LOGO Primitives and Procedural Methods

Before Division II pupils can begin to write LOGO solutions for the problems in our problem-solving manuals, an understanding of and experience with LOGO is required. Approximately seven hours were spent introducing the LOGO language and philosophy, allowing pupils time for discovery. Considerably more time could have been spent in discovery and exploration, but there were some time constraints to this problem solving project. The only session that resembled a formal lesson was the one on the LOGO Editor. Other ideas were presented by way of charts or by challenging pupils to try a primitive and see the results.

As is often done, one of the first shapes drawn in LOGO was the square. Throughout our learning of LOGO we came back to the familiar square to learn and apply our new ideas, primitives, procedures, and variables. The square was used to introduce "REPEAT"; to design our first procedure--"TO SQUARE"; to learn about variables--"TO SQUARE :SIZE"; and to help teach the lesson on the editor.

Material Covered Prior to the First Problem

Primitives	Activities
FD BK RT LT CS ST HT REPEAT x [what]	Explore
PU PD PE <CTRL> S <CTRL> L <CTRL> T	Using transparencies of lakes, take the turtle for a walk.
Procedures TO Name (What the turtle is to do) END	Write procedures to make squares and/or triangles and polygons.
Editor ED or EDIT "Name - to enter CTRL C - to close or exit CTRL N, P, B, F, A, E, D, K, ●	

Variables	Use appropriate names
:SIZE	Provides flexibility
:NUM	Provides counters
:COUNTER	

File Handling
SAVE "Name
LOAD "Name
POTS
POPS
ERALL

Launching the First Problem-Solving Task

A total of three to three and a half hours was spent on the first problem. The problem was:

Making a Mountain

> Randy's whole class is making a human pyramid. Seven students are on the bottom layer, six on the next layer, and one student less on each layer above that. How many students are there in Randy's class?

Computer Solution to the Pyramid Problem

1. Understanding the Problem
   a) Understand the mathematics problem
   b) Develop a plan--diagram
                  --simplify
   c) Carry out the plan

2. Developing a Plan
   a) Design a square
   b) Design a row of squares
   c) Move and repeat rows (one square less each time)
   d) Position first square
   e) Count squares and print out total

3. Carrying Out the Plan

4. Looking Back

To become acquainted with the problem, students discussed it and drew a diagram of it. At this point, the mathematical problem could easily have been solved. Pupils appeared to be at Step 3 in Polya's model, but were actually still at Step 1 in terms of finding a computer solution. We must agree, then, that there is value in finding a computer solution beyond just finding the mathematical solution. There is value, for example, in having students find a computer solution and then apply the method to other problems. There is value also in having students think logically and solve the subproblems as the computer solution is being designed and written.

Once the plan started to develop, pupils were anxious to carry it out. Thus, the plan developed as far as Step 2b, and then that part was implemented. This went smoothly, and at the end of the 45-minute period, everyone had succeeded in designing a row of squares on the screen.

The next part of the plan, Step 2c, was obvious. The "what" was obvious, but not the "how"! At this point, there was much traversing between Step 2 and Step 3. Before the hour was done, further LOGO instruction was needed in how to assign a variable to count the blocks in each row. Lots of group discussion took place, and everyone except "G" had a superprocedure that would draw a pyramid with any number of blocks in the base. Step 2c of the plan was now complete.

A half-hour session the next morning was needed to carry out Part 2d of the plan. The SETPOS primitive was introduced and a chart of the screen grid explored. This allowed for proper placement of the first block of the pyramid.

Carrying out Step 2e of the plan required another variable to count the actual number of blocks used. Some time and experimentation were required to locate this counter correctly. The primitive PRINT was introduced and used. The need to set variables in the superprocedure was discussed. There was joy and relief when all worked. (At this point, "G" was the only one who had not completed the procedures.) Students spent some time "looking back," but not a lot of interest or enthusiasm was shown for this step.

An extra half hour was given to "G", during which time he completed his procedures--a worthy piece of work. His was the only one that allowed the user to determine the size of the blocks in the pyramid, as well as their number! His procedures and a graphics dump of his solution are included in Appendix B.

Additional Concepts Students Learned About LOGO and Mathematical Manipulations

Additional LOGO Concepts	**Mathematical Ideas**
MAKE "NUM :NUM - 1	Counting, number patterns
IF :NUM = 0 [STOP]	Four basic operations
PRINT	Coordinate grid system and x, y values
TYPE	Degrees and angles
SETPOS [x y]	Formation of geometric shapes
SETX	
SETY	
WAIT	

Time Allotment

Most students spent 14 to 16 hours on the project, although some students spent more. Seven hours were spent initially on introductory knowledge of LOGO. Three to three and a half hours were spent on the first mathematics problem to be solved. An additional four to six hours were spent on subsequent problems. After solving the pyramid problem students could work on the same problem as another student or work on a problem of their own.

Difficulties Encountered

Need for Many Variables
Right from the beginning, pupils wanted to make their solutions as universal as possible. Considering the time required to write computer solutions, this was a

wise decision. Pupils very quickly were involved in handling variables and enjoying the flexibility that resulted. However, they found the task of identifying, defining, and applying necessary variables a bit onerous at times.

Applying an Old Procedure to a New Problem
Trying to adapt an old procedure to fit a new, similar problem appears to be difficult and undesirable. A fresh start, rather than editing the first attempt, was more appealing. Of course, there was much content from the first attempt repeated in the second.

Need for Counters
Using one counter was not that difficult, but having a number of counters did cause some head scratching. Often, more than one attempt at placement was required in order to have the counter function as desired.

Using Recursion
Using tail-end recursion was understood and enjoyed. Using a recursive procedure within a recursive procedure (Could this be called an embedded recursion?) was a challenge. The use of a REPEAT within a REPEAT was previously used.

Conclusions

If at any point along the way the project had been stopped, the students would still have benefited from the project. Much more time could profitably have been spent on the project in furthering the understanding my students and I have of LOGO and problem solving.

Writing computer solutions to the problems chosen definitely provided a challenge to the students with whom I worked. I would not want to tackle this project within similar time constraints with the slower students in the class. However, there are many applications of LOGO that would be very profitable for the average student.

Working through this project has revealed many areas of the mathematics program that are involved in problem solving with LOGO. Further time for exploration in other areas, such as geometry, coordinate geometry, and number patterns, would be beneficial. Throughout the sessions, pupils were enthusiastic and ambitious. After the third hour of classes, an 8 a.m. session was established at the request of the students. During the fifth hour of classes, there was much excitement generated when Mike, a student, discovered that spinning a square filled a circle.

In the eighth hour of classes, when our first problem-solving problem (We had truly solved many problems by this time!) was introduced, we had what was probably our most verbal session. There were times of excitement and of discouragement. The level of the group's interest, enthusiasm, and effort stayed high throughout the project.

Some opinions about the project were given by students. Although neither LOGO nor BASIC was clearly the preferred language in which to program, and although neither was better liked, LOGO was the one seen to relate closer to the rest of the school work. All students enjoyed their work in LOGO. The only dissatisfaction expressed was by one student in regard to a difficult problem. Also, the majority of students stated that they would prefer to work with a partner.

Appendix A

Problems the Group Solved

Pyramid Building. Randy's whole class is making a human pyramid. They put seven students at the bottom, six students on the next layer, and one student less on each layer above that. How many students are there in Randy's class?

A man has 55 concrete blocks. He wishes to build a set of stairs by piling them up so there is no empty space under each step. How many steps will there be in the stairway?

Ball Falls and Bounces. A rubber ball is dropped from a height of 16 m. Each time it lands, it bounces to a height half the distance from which it last fell. The ball is finally caught when it bounces to a height of 1 m. Find the total distance the ball travels.

Mix and Match (Branching). A couple has three children. Each child has two children. How many grandchildren are there?

Willy has four pairs of sweatpants and three different T-shirts in his dresser. How many different outfits can he wear?

Squares on a Checkerboard. How many squares are there in an 8 X 8 checkerboard?

Handshake. The Bear family has a family reunion. Each member of the family arrives separately. As they arrive, they shake the paw of each bear already there. If there are 11 bears in the family, how many paw shakes occur?

Pigs and Chickens. Farmer Brown has some pigs and chickens, 18 in all. If the animals have 48 legs altogether, how many pigs and how many chickens does Farmer Brown have?

Appendix B

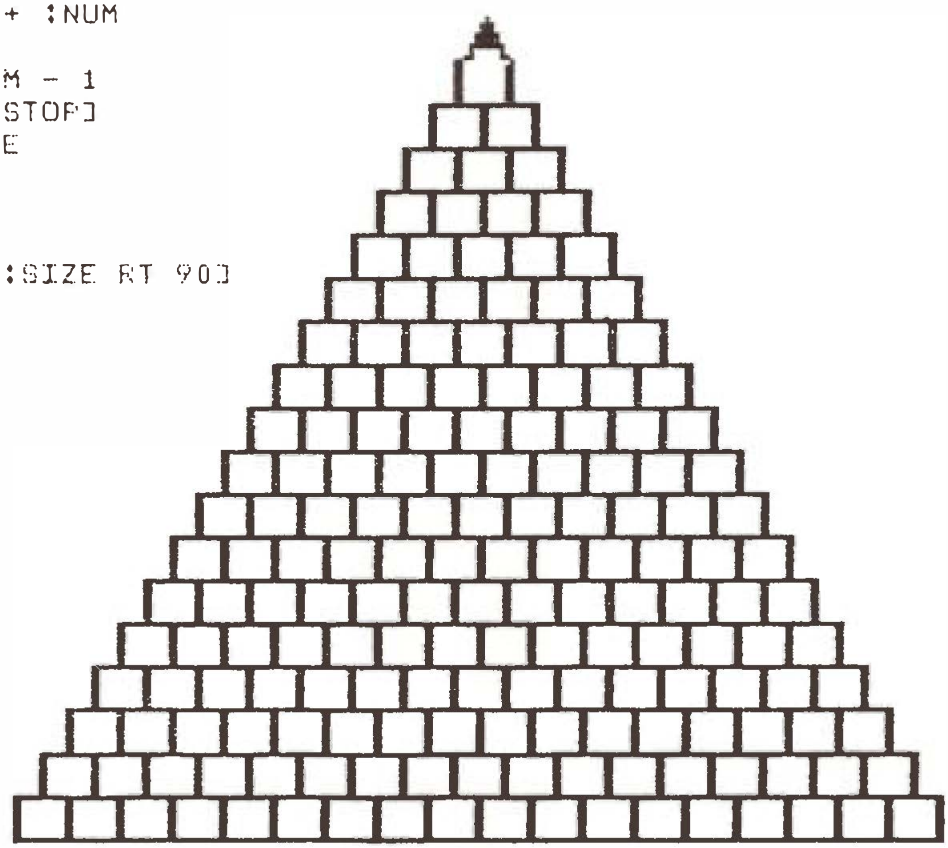
Solution to the Pyramid Problem

NOTE: The solutions to the other problems have not been included so that teachers and their students will have the opportunity to experience the thrill of learning how to develop LOGO solutions.

```
?PO [PYR ROW SQ]
TO PYR :NUM :SIZE
MAKE "BL 0
PU HT
SETPOS [-115 -75]
PD
ROW :NUM :SIZE
ST
END

TO ROW :NUM :SIZE
REPEAT :NUM [SQ :SIZE RT 90 FD :SIZE LT 90]
FD :SIZE LT 90 FD :SIZE * :NUM - .5 * :SIZE
RT 90
MAKE "BL :BL + :NUM
PRINT :BL
MAKE "NUM :NUM - 1
IF :NUM = 0 [STOP]
ROW :NUM :SIZE
END

TO SQ :SIZE
REPEAT 4 [FD :SIZE RT 90]
END
```



```
PYR 18 10
PRINT :BL 171
```

```
?.PRINTER 0?PYR 7 25
7
13
18
22
25
27
28
```