Go on a Tangram Holiday

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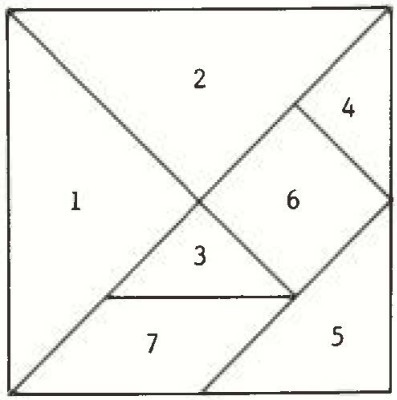
A tangram is a puzzle composed of seven parts called tans. Tans are formed by cutting a square and its interior into five triangles, a square, and a parallelogram as shown. The angles of the tans are 45, 90, or 135 degrees. The area of the large triangle is four times that of the small triangle.

Materials

Construction paper with the tan square on it and a pair of scissors to cut out the different tans.



Aims

1. To have some fun.
2. To review some basic geometric shapes.
3. To discover the relationship among the shapes.
4. To review Pythagorus' Theorem.
5. To review perimeter and area.
6. To introduce radicals.
7. To recall the concept of congruence.

Activity 1

Fold a square piece of paper to form the seven tan shapes. This is a good activity for shape recognition. This activity has been successfully done by a mixed class of Grade 7 and 8 students as well as by Grade 9 general and advanced classes.

Activity 2

The objective is to discover the relationship among the shapes and introduce the concept of congruence, equal in all respects.

Note $\triangle 1 \cong \triangle 2$, $\triangle 3 \cong \triangle 4$, $\triangle 5 = 2\triangle 4$, $\triangle 5 = 2\triangle 3$,
$\triangle 5 = 1/2\triangle 2$, $\triangle 5 = 1/2\triangle 1$, $\triangle 3 = 1/4\triangle 2$,
or $\triangle 2 = 4\triangle 3$

Prove that figures 5, 6, and 7 have the same area. Hint: use $\triangle 3$ and $\triangle 4$; with these tans you can build figures 5, 6, and 7.

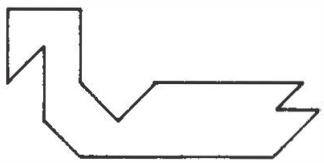
Activity 3

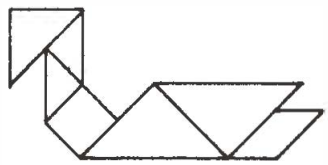
The object of this activity is to have some fun and to give an opportunity to some of the lesser lights to shine in class.

Materials needed: the seven tans and a tangram picture.

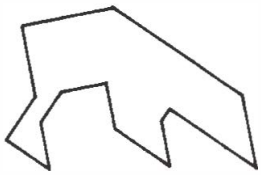
The object of the puzzle is to put the seven tans together to form outlines of all sorts. You must use all seven pieces and you may not overlap the tans. Look at the tangram pictures below. How can you arrange the seven tans to form each of them?

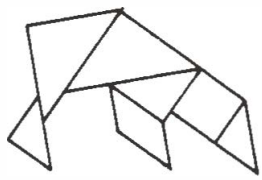
I encourage the students to do this activity at the kitchen table with the whole family participating. In this way, the student is back for more puzzles to complete. Some educators would argue that this is an excellent right-brain hemisphere activity.





duck





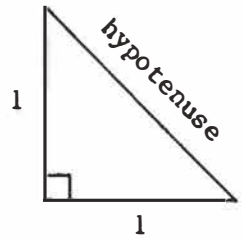
bear

The completed tangram puzzles make for a colorful bulletin board. Once the student has outlined the seven tans, have him or her color them.

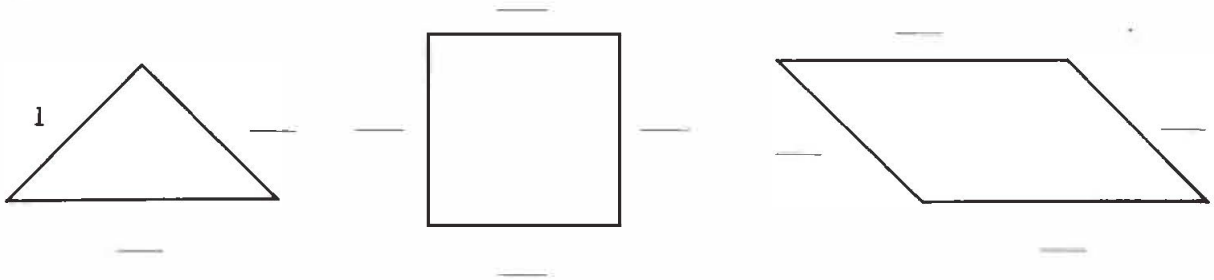
Activity 4

The objectives here are, first, to review Pythagorus' Theorem: for any right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides; and, second, to establish the relationship of the different sides of the tans.

$h^2 = 1^2 + 1^2$
$h^2 = 1 + 1$
$h^2 = 2$
$|h| = \sqrt{2}$
$|h| = 1.414$



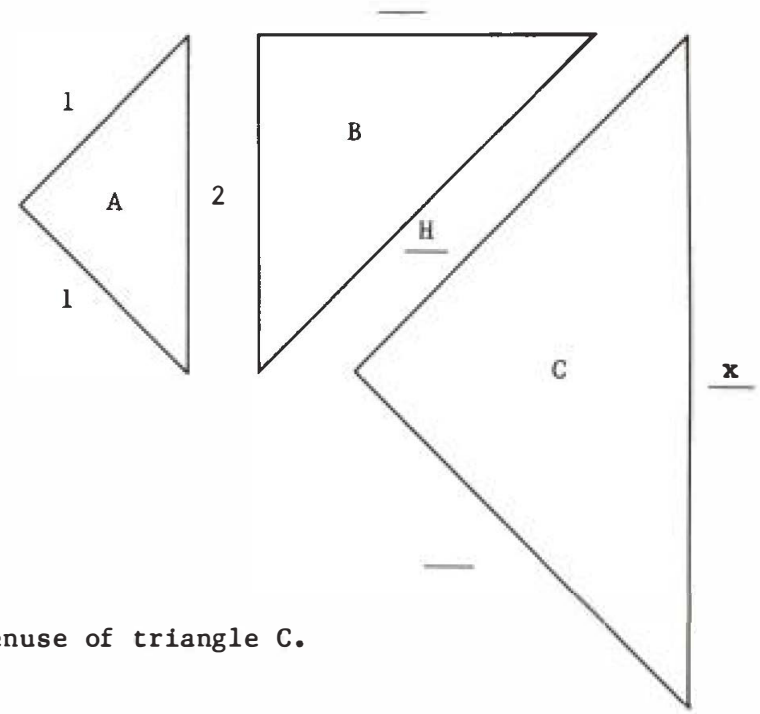
Complete the labeling.



Use Pythagorus to find the hypotenuse of triangle B.

$H2 = \sqrt{2}^2 + \sqrt{2}^2$
$H^2 = 2 + 2$
$H^2 = 4$
$|H| = \sqrt{4}$
$|H| = 2$

Note $\sqrt{2}^2 = \sqrt{2} \times \sqrt{2}$
$= \sqrt{2 \times 2}$
$= \sqrt{4}$
$= 2$



Use Pythagorus to find the hypotenuse of triangle C.

$x2 = 2^2 + 2^2$
$x^2 = 4 + 4$
$x^2 = 8$
$|x| = \sqrt{8}$ ⟶ Read: the length of x is the square root of eight.

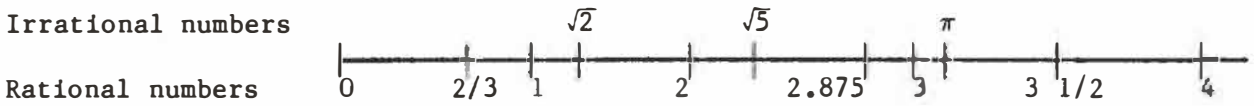
$|x| = \sqrt{4} \times \sqrt{2}$ ⟶ Read: the length of x is the root of four, times the root of two.

$|x| = 2 \times \sqrt{2}$ ⟶ Read: the length of x is two roots of two.
$x = 2 \times 1.414$
$x = 2.828$

Activity 5

The objective here is to get a feel for irrational numbers. Hold up the small triangle from the ends of the hypotenuse; between your fingers is the length of an irrational number. Is it real? It's between my fingers and I can feel it; you're darn right it's real. It is the square root of two, $\sqrt{2}$.

What are radicals? Radicals are irrational numbers. Place the following on the real number line: 2/3, $\sqrt{2}$, $\sqrt{5}$, 2.875, and 3 1/2.



The irrationals in union with the rationals comprise the reals.

Activity 6

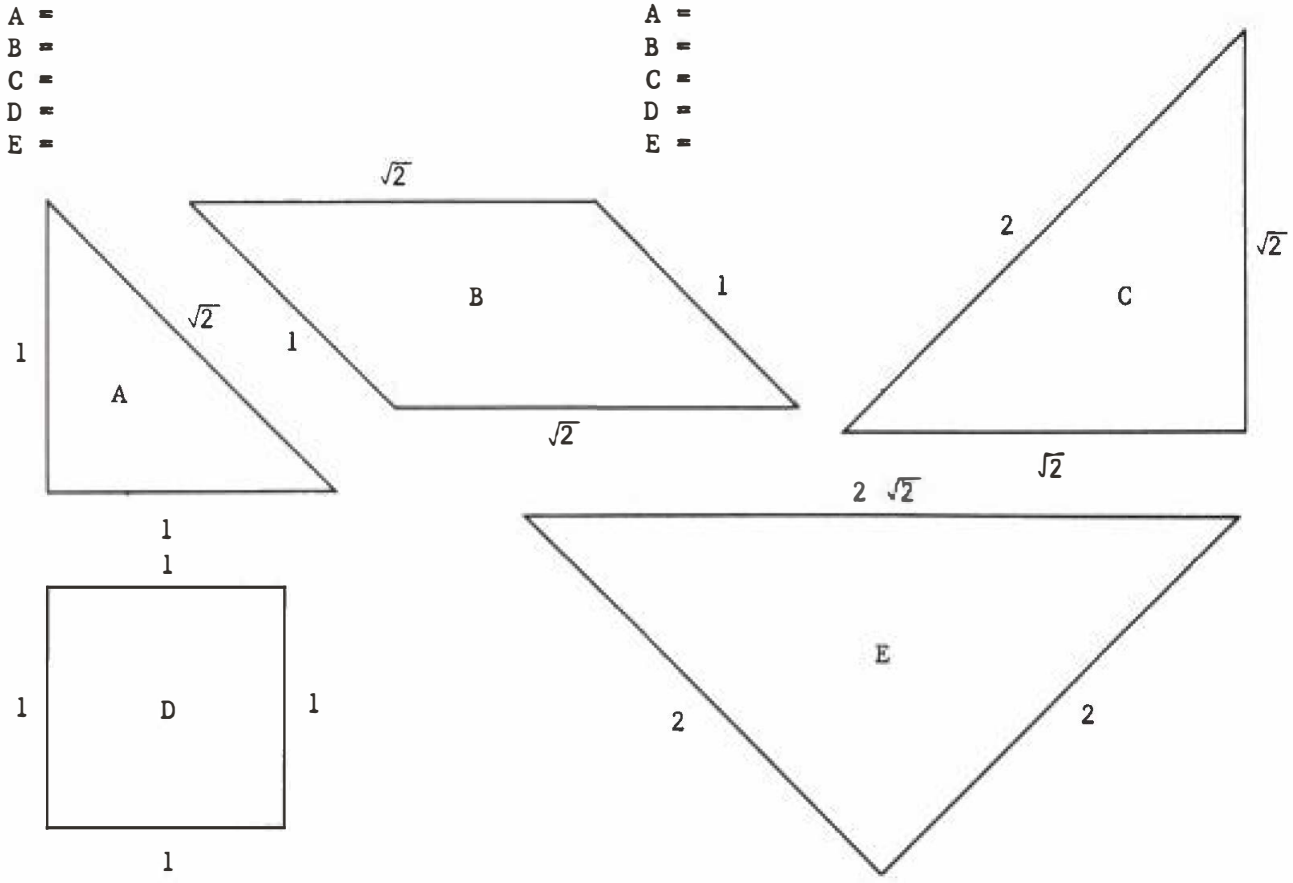
The objective here is to get comfortable with radicals while finding the perimeter and area of the tan shapes. See the figures below to complete the chart.

Perimeter of

A =
B =
C =
D =
E =

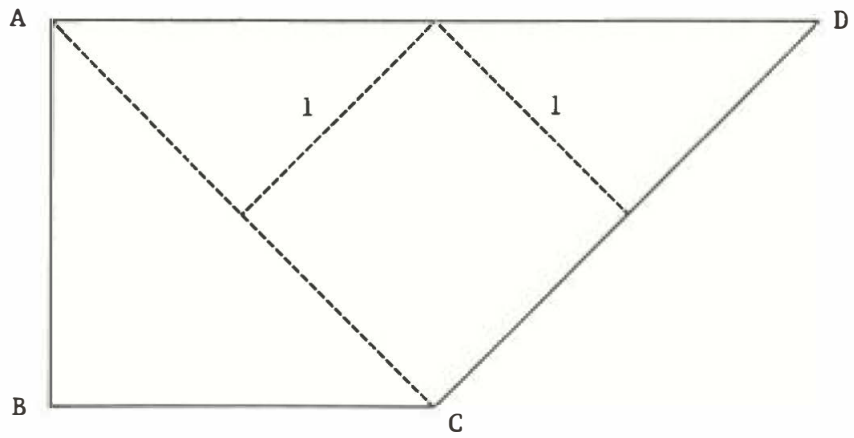
Area of

A =
B =
C =
D =
E =



Activity 7

First, do some mental calculations or manipulations of the shapes below in order to find the area of the figure ABCD. Second, recall the area formula for a trapezoid and use it to verify your answer.



The answer is three. The area of the square is one, the area of the two small triangles is one, and the area of the middle-sized triangle is one; therefore, the area of the trapezoid is three.

Use the formula to verify this answer. The area of a trapezoid is the average of the two parallel sides times the perpendicular distance between them.

$$\frac{AD + BC}{2} \cdot AB = \frac{2\sqrt{2} + \sqrt{2}}{2} \cdot \sqrt{2} = \frac{3\sqrt{2}}{2} \cdot \sqrt{2} = \frac{3 \times 2}{2} = 3$$

References

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