Hindu Inversion in Present-Day Algebra Classrooms

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Hindus used inversion to solve number problems that were posed for intellectual pleasure. The method, known as "vilomagati" (working backwards), was a favorite method of Hindus and was commonly used by them since early years of the Christian era. Beginning from the end, the suggested operations were inversed: addition was changed to subtraction, multiplication changed to division, square to square root, and vice versa. Bhascara's Lilavati gave the rule of the method as:

> To investigate a quantity, one being given, make the divisor a multiplicator, and a multiplier a divisor; the square a root, and the root a square; turn the negative into positive, and a positive into negative. If a quantity was to be increased or diminished by its own proportionate part, let the lower (denominator) be increased or diminished by its numerator, and the numerator remains unchanged; and then proceed with the other operations of inversion, as before directed.
> (Brahmagupta, 1817)

I have made use of inversion in my elementary algebra classes with excellent results. The procedure reinforces basic algebraic operations, in addition to providing an insightful review of the solution of linear and quadratic equations.

Consider the following example:

> **What is that quantity which, when divided by seven, then multiplied by three, then squared, then increased by five, then divided by three-fifths, then halved, and then reduced to its square root, happens to be the number five?**
> (Datta and Singh, 1935)

Beginning from the end, the solution by inversion proceeds as follows:

Number	5
Squared	25
Doubled	50
Multiplied by 3/5	30
Decreased by 5	25
Root	5
Divided by 3	$\pm \frac{5}{3}$
Multiplied by 7	$\pm \frac{35}{3}$

Hence, Quantity $= \pm \frac{35}{3}$

The problem is solved using algebraic procedures as follows:

Let the number	=	x
Divide by 7	=	$\frac{x}{7}$
Multiply by 3	=	$\frac{3x}{7}$
Square	=	$\frac{9x^2}{49}$
Add 5	=	$\frac{9x^2}{49} + 5$

Divide by $\frac{3}{5}$ = $\frac{5}{3}\left(\frac{9x^2}{49} + 5\right)$

Halved = $\frac{5}{6}\left(\frac{9x^2}{49} + 5\right)$

Square root = $\sqrt{\frac{5}{6}\left(\frac{9x^2}{49} + 5\right)}$

By question:

$$\sqrt{\frac{5}{6}\left(\frac{9x^2}{49} + 5\right)} = 5$$

To solve this, we square both sides to get:

$$\frac{5}{6}\left(\frac{9x^2}{49} + 5\right) = 25$$

Multiply both sides by 6/5, and we have:

$$\frac{9x^2}{49} + 5 = 30$$

Subtract 5:

$$\frac{9x^2}{49} = 25$$

Multiply by 49/9:

$$x^2 = \frac{49(25)}{9}$$

Take square root of both sides:

$$x = \pm\frac{7(5)}{3} = \pm\frac{35}{3}$$

which is the required quantity.

Steps of inversion and the algebraic solution are compared, and the technique of inversion of operations is emphasized.

Students are encouraged to do similar problems by both procedures. They are asked to construct their own problems and solve them by both methods. The results of the strategy are very interesting. Students develop a better understanding and appreciation of the algebraic operations.

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EDITORS' COMMENT: Dr. Bhalla's article caused the editors to look at the process of teaching solving equations. Two parallel but reversed processes are given. Should students be provided the opportunity to develop equations, as well as solve equations?

Developing an Equation

$x = 2$
$x + 3 = 5$ (Add 3)
$3(x + 3) = 15$ (Multiply by 3)
$\frac{3(x + 3)}{5} = 3$ (Divide by 5)

Solving an Equation

$\frac{3(x + 3)}{5} = 3$
$3(x + 3) = 15$ (Multiply by 5)
$x + 3 = 5$ (Divide by 3)
$x = 2$ (Subtract 3)

REFERENCES

Brahmagupta. Algebra, with Arithmetic and Mensuration, from the Sanscrit of Brahmagupta and Bhascara. Translated by Henry Thomas Colebrooke. London: J. Murray, 1817, pp. 21-22.

Datta, B., and A.N. Singh. History of Hindu Mathematics. Bombay: Asia Publishing House, 1935, p. 232.