Broadening Horizons – Individual Student Projects in Mathematics

Joan Haig
Lethbridge Collegiate Institute

During the past few years, I have toyed with ways to tap student interest in mathematics and to encourage students to go beyond what is required by the curriculum. Time pressures are such that there is not a great deal of class time available for this. A club works well for the students who can fit it into their busy schedules. What to do? Back to the old project idea.

My first serious assignment of projects was made at the Math 20 level in an honors class. We talked about possible topics, possible sources of information, people who would be helpful, and time limits. The students were then given a two-week period to investigate and select a topic they were interested in, and a further six weeks to develop their idea.

Since this class was quite highly motivated and of a very verbal nature, it was decided that there would be three aspects to each project. The project chosen would have to:

1. demonstrate or explain some mathematical principle or be related to a specific branch of mathematics;

2. be presented to the class and have a visual, as well as a verbal, aspect;

3. be handed in as a text.

The marks were divided among the three aspects, and the results were very exciting.

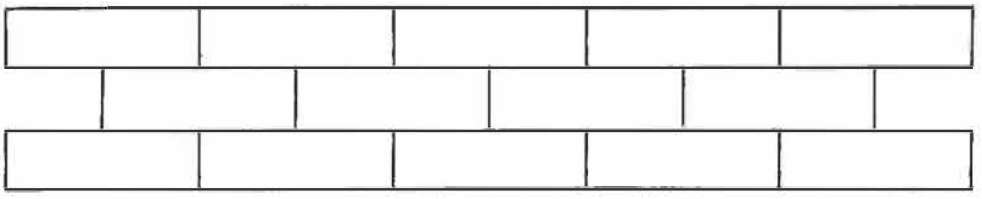
Please bear in mind that these students were in their first semester of Grade 11. The projects varied widely in scope and in method of presentation. The range included paper folding geometry, a project that mapped tone patterns of different musical scales, a computer version of slot machines, models of a stellated dodecahedron, and a flexihexagon. I was very impressed with these young people, in both their choice of topics and their presentations.

I have selected for your enjoyment one project that reflected the student's personal interest, was very well presented, and did not involve a great deal of complicated paraphernalia. Kathy Pratt's investigation of tessellations is an example of a personal interest topic that would not otherwise have been explored during high school. The following represents the handed-in portion of Kathy's assignment.

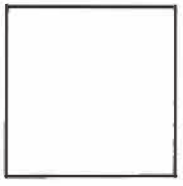
Tessellations

The Romans decorated their buildings and towns with mosaic floors and pavements made of very small tiles called tessellae. From this word comes the word tessellation, which is used to describe ways of filling space. The study of these is one of the bridges between mathematics and art. Such decorations have been used for centuries, but today, artists, arthitects, and designers are making more and more use of simple geometric shapes and the ways in which they fit together.

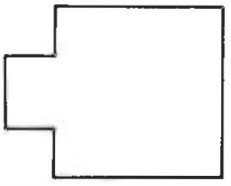
A definition of a tessellation is a repeated pattern of shapes, which are often polygons, that will completely cover a flat surface, leaving no gaps or overlaps. The following is an example of a tessellation of rectangles:

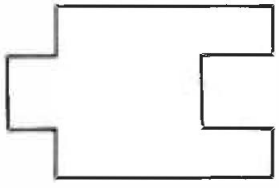


Shapes in a tessellation must be capable of fitting together. An example would be a square that is changed slightly. To make the new shape tessellate, the added feature must be taken off the opposite side. The new shape will tessellate.

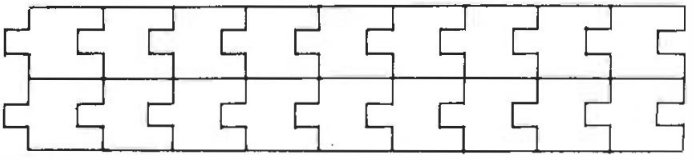


The square is changed to



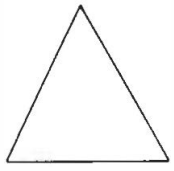


Take the added feature off the opposite side.

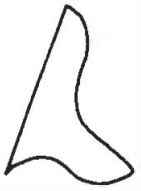


The new shape tessellates.

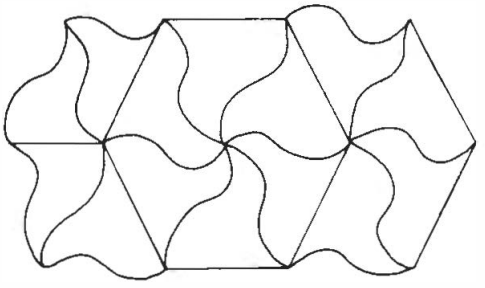
More complex shapes can be made using curved lines as well as straight lines:



Alter two sides of a triangle.

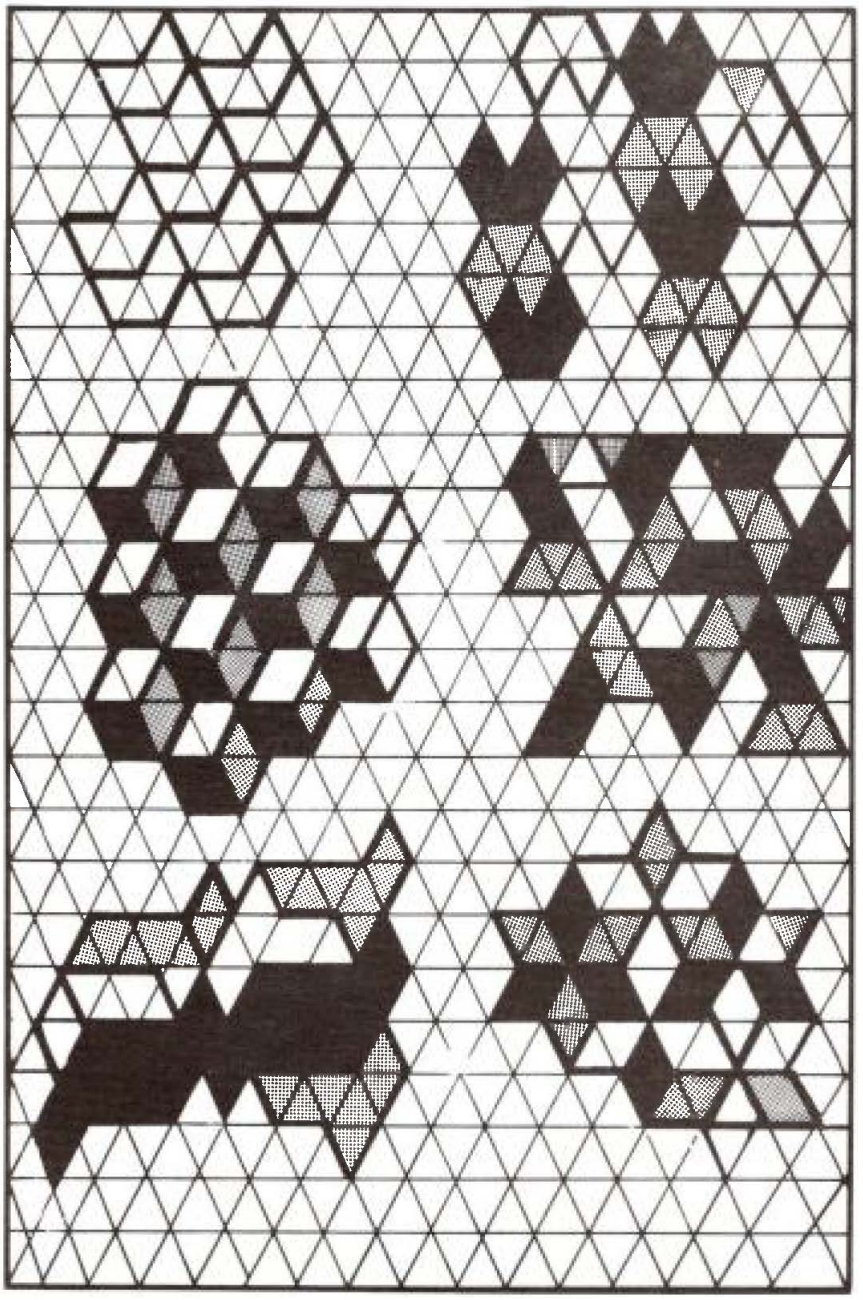


This figure will have the same area as the original triangle.

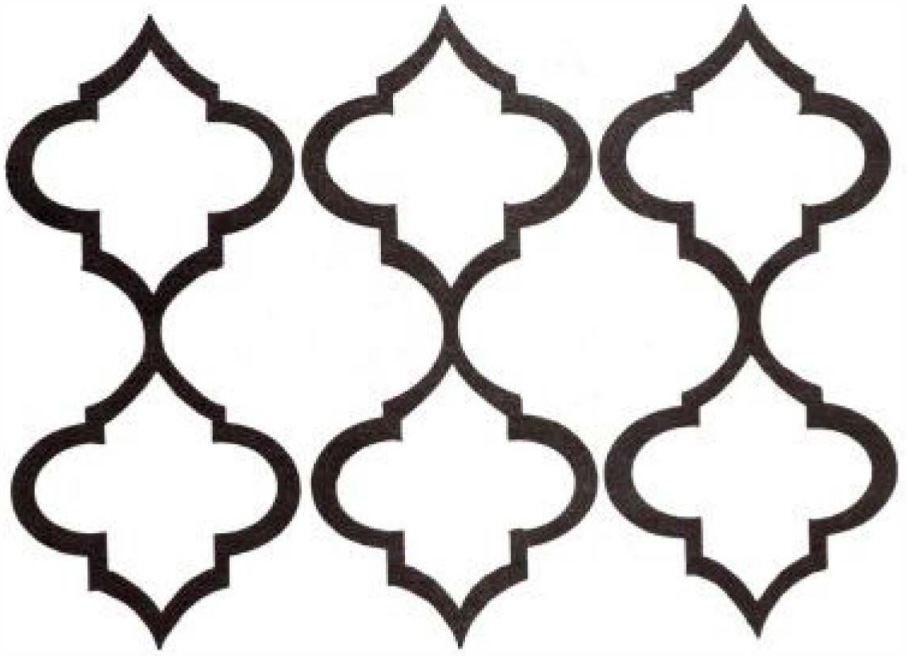


Tessellations can be used to create curved lengthening, shortening, and three-dimensional effects. An important aspect of tessellations is color, because a tessellation can appear one way and entirely different in another way just by the way it is colored.

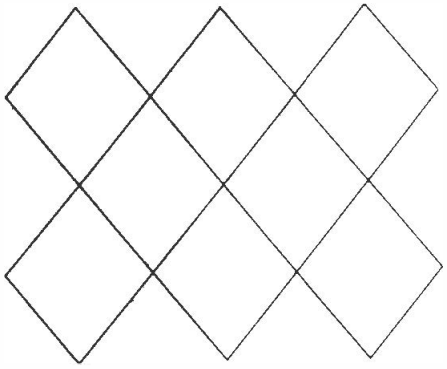
These are triangular-based tessellations. Some are the same tessellations colored differently.

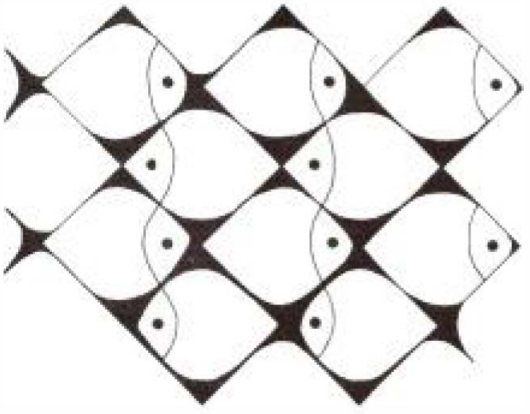


Tessellations can be found in buildings, on material, wallpaper, floor coverings, wrapping paper, and quilts. A tessellation of hexagons appears in honeycomb and chicken wire. The following pattern is common on floor coverings:

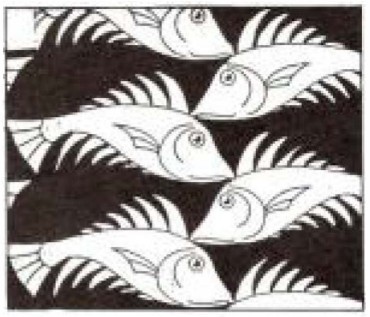


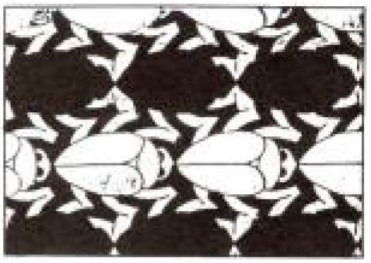
Shapes within tessellations can easily be adapted. The basic quadrilateral tessellation could be turned into two shoals of fish swimming in opposite directions:

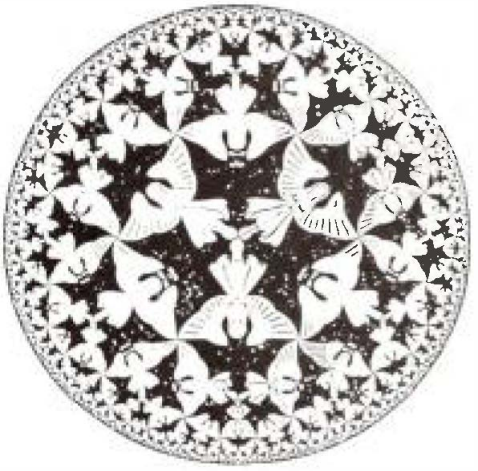




The Dutch artist Mauritz Escher visited the Alhambra Palace in Spain and was fascinated by the tiling patterns on the walls and floors. He began to experiment to find how a surface might be regularly divided and filled with congruent figures without leaving any gaps. This led him to devote his life to developing more and more complex tessellations.







Even though we don't usually notice tessellations around us, they are a big part of our lives. They affect our architecture, fabric, wallpaper, floors, quilts, and art. They are also a part of our history. Tessellations have been around for a long time, but there are always new and fascinating discoveries to be made. No matter how often they are used, they never fail to be interesting and remarkable.

Joan Haig is head of the Mathematics Department at Lethbridge Collegiate Institute. Joan has been active in the South West Regional of MCATA.

REFERENCES

Mold, Josephine. Tessellations. Cambridge University Press, 1969.

Pittman, Clark. Mathways. Toronto, Ontario, 1979, pp. 110-17.

The Graphic Art of M.C. Escher.