Families of Problems: Changing Conditions, Variables, and Information Sought

John B. Percevault
University of Lethbridge

Since 1980, mathematics curricula have shown an increased emphasis on problem solving. Problems differing from the problems in the text and non-routine problems have been presented as alternatives. How successful has this emphasis on problem solving been in developing better thinkers and better problem solvers?

The problems noted below are being presented to upper-elementary students who have not learned to translate, that is, to develop an equation. Often the problem is presented in isolation, solved, and subsequently forgotten.

Alternatively, groups of related problems can be developed and solved. Teachers can share their thought processes with students by asking questions as problems are developed and solved. Relationships and patterns can be explored. The process of thinking, the problem-solving strategy, can be emphasized as the process is used in solving the related problems. Data from the problems can be organized to facilitate the ability to translate. Students become problem solvers, not just solvers of a problem.

The process used throughout this article involves the development of an organized list. Admittedly, other strategies could be used. Sample questions, indicators of the teacher's thought, are posed. Relationships are probed. As the problem varies, the teacher and the student note what has changed, namely, the *condition*, the *variable*, or the *information sought*. Finally, the organized list is used as the basis for translation.

Sample Problem

A farmer has a unique manner of determining how many animals - chickens or cows - he has. He counts the total number of legs and the number of heads. From this, he can determine the number of chickens or cows. One day, he counted 26 legs and 10 heads. How many of each animal were there? The solution, obtained through use of an organized list, follows:

Figure 1.

No. of Chickens	No. of Cows	Chickens' Legs	Cows' Legs	Total No. of Legs
0	10	0	40	40
1	9	2	36	38
2*	8	4	32	36
3	7	6	28	—
	6	8	24	—
5	5	10	20	—
6		12	16	—
7	3	14	12	26
8	2	16	8	—
9	1	18	4	—
10	0	20	0	—

*At this point, students could be challenged to look for patterns.

Sample questions include:

- How is the number of chickens changing?
- How is the number of cows' legs changing?

- How is the total number of legs changing?

Other questions that model the teacher's thought processes could include:

- Could we have a total of 33 legs?
- What is the largest total of legs?
- What is the smallest possible total of legs?
- How many addition facts (two addends) are there?
- Can you develop another problem from this data?
- How many problems could be developed?

Note that the basis for translation is being developed. Figure 1 is completed below, including two sample translations.

	No. of Chickens	No. of Cows	Chickens' Legs	Cows' Legs	Total No. of Legs
	x	10-x	2x	+ 4(10-x)	= 26
OR					
	x	y (x+y=10)	2x	+ 4y	= 26

Variation #1

The next time the students meet this type of problem, the farmer is sitting on his veranda. He counts wheels of vehicles and notes that the vehicles are either cars or bicycles. One evening, he counts 10 vehicles and 26 wheels, or any of 11 possible totals of wheels. The question is obvious.

A *condition* has changed. The *process* used to solve the problem remains the same and transfers to another problem. Furthermore, when the student is taught to *transform*, the thought process remains constant.

The process applies to other problems as well. Consider the following problem:

> Ten kilograms of a mixture of candy sells at $2 and $4 per kilogram. The value of the 10 kilograms is $26 (or $38, or ____). How many kilograms of each type of candy are there?

Or, substitute:

> Twelve coins - nickels or dimes. The value of the coins is $1.

Variation #2

Change the location to a furniture shop that specializes in making chairs (four legs) and stools (three legs). During one hour, 10 articles were produced that had a total of 34 legs. How many of each, chairs and/or stools, were produced?

Questions that could be asked include:

- As the number of chairs increases and the number of stools decreases, how does the sum of the legs differ?
- How many sets, each containing two addends, are there that have a sum of 10?
- How many possible totals of legs are there?
- How many different questions could have been asked?

Another change in variables could roduce the following problem:

> Another day, the carpenter shop produced sofas (six legs) and stools. Twelve articles were produced using a total of 51 legs (or any other suitable replacement). The question is obvious.

Questions, as well as those previously posed, could include:

- How will the total number of legs vary over the 13 possibilities?

The teacher may also wish to use the table that could be produced to develop the equation or system of equations. The process of developing an organized list transfers to a seemingly different problem.

Figure 2.

No. of Stools	No. of Sofas	Total Legs (Stools)	Total Legs (Sofas)	Total Legs
0	12	0	72	72
1	11	3	66	69
—	—	—	—	—
4	?	—	—	—
		—	—	—
—	—	—	—	—
?	5	—	—	—
		—	—	—
—	—	—	—	—
—	—	—	—	—
11	—	—	—	39
12	0	36	0	36
x	12-x	3(x) +	6(12-x)	= 51
OR				
x	y	3x +	6y	= 51
(x + y = 12)				

A variable, a number, has changed; the process remains constant.

Variation #3

The basic problem is varied again. The information sought is changed.

> One day, the carpenter shop sold furniture orders to 11 customers. Each customer bought 10 articles, either sofas and/or stools, chairs and/or stools, sofas and/or chairs. None of the 11 orders was the same. How many legs were required to complete all the orders?

All of the questions posed previously are appropriate. Further questions may need to be asked, namely:

- Have you solved a problem similar to this one?
- How did you solve that problem?
- Can you develop a chart? An organized list?
- What must be done to answer the question posed in the problem?

Variation #4

Some problems, such as the following, have multiple solutions:

> One day, the farmer counted 32 legs of chickens and/or cows, but forgot to count the number of heads. What combination of chickens and/or cows could have a total of 32 legs?

The conditions of the problem are satisfied with 16 chickens or eight cows, the largest and smallest total of animals. Many of the questions previously presented may be used. Once again, the strategy of a logically organized list will allow students to determine the other possible solutions.

Conclusion

As the student experiences problem-solving situations, opportunity should be given to examine groups of related problems. Problems may be altered by changing conditions, by changing the variables, and by changing the information sought. The process of problem solving, the thought process, remains constant. Such a planned approach should allow students to solve problems, not just to arrive at the solution. Finally, problems may also be selected to assist the students in translation, developing equations.

The final problem posed in this article could be used to relate to information from other subjects or to have students seek information.

> Among the inhabitants of the planet of Skol are giant spiders, each of which is fed by little men called Roods. Each day, the spiders require exactly 48 fly legs. The flies of Skol are abnormal compared to the flies on planet Earth. Each female fly has two more legs, and each male fly has two less legs than the flies on Earth. How many combinations of female and/or male flies does the Rood need to catch to ensure his and the spider's survival? If fed too many fly legs, the spider, Phoenix-like, explodes, and the little spiders devour the Rood. If fed too few legs, the Rood is eaten to complete the spider's meal.

Embellish the conditions more if you want to. By now, you probably have had enough of legs.

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The writer acknowledges the influence of John LeBlanc, in his address at the 1981 NCTM annual meeting. The schema presented by LeBlanc follows:

Effective **Teachers Structure** Questions –

For Understanding
(a) Questions related to given information.
(b) Questions related to stated conditions.
(c) Questions related to questions asked (main idea).
(d) Questions related to a proposed solution.

For Extension (Modifying Problems)
(a) Modify variables.
(b) Modify conditions.
(c) Modify information sought.

John Percevault is Associate Professor, Faculty of Education, University of Lethbridge. He served as a teacher, principal, superintendent of schools, and mathematics consultant for the province before joining the Faculty of Education. Mr. Percevault represents the Alberta Faculties of Education on the Mathematics Council, ATA. He has been a member of Alberta Education's Mathematics Curriculum Coordinating Committee. His current interests include teaching thinking, a broad issue that includes problem solving.

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