Evaluating Problem Solving

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As problem solving becomes a greater part of mathematics education in Alberta schools, its overall complexity will increase. Rather than giving students instruction about single problems that have single answers, they will be provided with complex problems that require a more complete effort to solve. Here is an example of a complex problem.

> Consider 12. It has four numbers less than it that only have a factor of one in common with 12 (11, seven, five, and one). These are called *monodivisors* of 12. Similarly, six has two monodivisors (five and 1). Investigate the number of monodivisors for different numbers.

The above problem is taken from some British "O" level investigations. It is obvious that a student's completion of this problem will require greater effort than more traditional problems. After students are finished answering complex problems, the teacher is left with the difficult task of evaluating the student's work. The amount and type of work accomplished throughout the problem-solving process needs to be acknowledged and recorded. The student's work will vary in the type of solution and level of completion. This type of problem requires a "holistic" approach to evaluation. In this article, an Impressionistic Scoring Criteria scheme will be presented, which will give educators a valid basis for problem-solving evaluation.

Problem Complexity

A better understanding of problem-solving evaluation can be developed if there is an appreciation of how problems vary in complexity. Relative measurement of complexity can be done using six factors. These factors are:

1. stage implementation
2. stage depth
3. strategy implementation
4. strategy depth
5. open-endedness
6. duration

Stage implementation refers to the number of stages the student is required to execute for the successful completion of a problem. Alberta Education suggests a framework of four stages. These are:

- understanding the problem
- developing a plan
- carrying out the plan
- looking back

Complex problems usually use four stages. Less complex problems use one, two, or three of the stages.

Stage depth refers to the degree with which each stage of the four-stage framework is implemented for the successful completion of a problem. Complex problems require that stages be fully implemented. Less complex problems require only partial implementation of a stage.

Strategy implementation refers to the number of strategies that can be or must be used to successfully complete a problem. Some examples of strategies as outlined in Alberta Education documents include:

- looking for patterns
- identifying key words
- using a simpler problem
- applying patterns
- looking for alternative ways to solve the problem
- making diagrams and models

Complex problems tend to have more possible strategies or require the implementation of more than one strategy for their successful completion. Less complex problems use fewer strategies (usually one).

Strategy depth refers to the degree of implementation of the strategy such that the problem is completed. More complex problems have higher degrees of strategy implementation than less complex problems.

Open-endedness has primary application to the "looking back" stage of the problem-solving framework. If the completion of a problem leads to the exploration of many facets of that problem or problems like it, then the problem is said to be open-ended. Complex problems tend to be more open-ended than less complex problems.

Duration refers to the amount of time and effort required to complete a problem. Complex problems have higher duration levels than less complex problems.

Complex problems have high levels of stage implementation, stage depth, strategy implementation, strategy depth, open-endedness, and duration. Complex problems have an inherent "creative" aspect to them, which allows students to explore problems rather than just find right answers. This creative element of mathematics motivates and inspires teachers and students. The inclusion of problem solving (especially open-ended complex problems) in the mathematics curriculum can enhance the environment for creative learning in the mathematics classroom.

Evaluation of Problem Solving

The evaluation of complex problems requires a different approach than traditional right-answer or part-mark approaches. Ideally, recording a student's performance and behavior during a complex problem-solving session would give a teacher the best data for making accurate and objective evaluations. However, the logistics and time required for such an evaluation technique is limited by the time needed to observe and interact with each student. There are two possible solutions: (1) use an instrument that records objective and observable student behavior data quickly and efficiently, or (2) evaluate the written work and make inferences about the student's ability at completing each complex problem. An Impressionistic Scoring Criteria scheme assists the teacher in evaluation of student behavior and written material.

The Impressionistic Scoring Criteria scheme, developed by the writer and outlined on page 26, was borrowed heavily from a scheme previously developed by Dr. Terry Rusnack and the writer for evaluating process skills in science. It rates students on a scale that is linked to observable student behaviors. The teacher uses it in checklist fashion. It looks at he complete problem-solving process rather than at just evaluating students on the correctness of their answers.

The criteria were developed in four categories:

1. problem-solving stages
2. strategies
3. solution, and
4. participation.

These categories can be changed to fit various teaching situations. All of them can be used at once, or they can be used individually. In addition, they are not exhaustive, and educators could easily develop other categories.

In using the scheme for observable student behavior, a teacher could circulate around the room and record data as students do their work. Also, it is sufficiently flexible that teachers could use it to evaluate students' written solutions.

Teachers who have used it have reported that it is quick and accurate. In addition, these teachers have reported an increased understanding of problem solving because they had been forced to look at the complexity of student behavior in solving these problems. They have found there is more to problem solving than just finding solutions.

In conclusion, complex problems require more involvement from teachers so that students will receive fair and just evaluation. The Impressionistic Scoring Criteria scheme can assist teachers with evaluating student solutions to complex problems.

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EDITORS' NOTE: Readers may be interested to compare the Impressionistic Scoring Criteria scheme presented in this article with the Mathematics Problem-Solving Behavior Scale distributed by Alberta Education. This rating scale is provided on page 27.

Impressionistic Scoring Criteria

	Problem-Solving Steps	Strategies (?)	Solution	Participation
5	made an effort using four of the steps	has considerable depth and expertise in using the strategy	- complete - correct solution - used more than one strategy - manipulated the problem and solution	- involves oneself quickly
4	made an effort using three of the steps	has expertise in using the strategy	- complete - correct solution - used more than one strategy or manipulated the problem	- needs a start
3	made an effort using two of the steps	has some expertise in using the strategy	- somewhat complete - correct solution	- needs periodic assistance
2	made an effort using one of the steps	needs to develop expertise in the strategy	- incomplete - has errors	- needs constant attention
1	Excused Absence	Excused Absence	Excused Absence	Excused Absence
	COMMENTS:	COMMENTS:	COMMENTS:	COMMENTS:

Mathematics Problem-Solving Behavior Rating Scale

1. *CIRCLE the number indicating your rating for each of the four indicators, based on the defined observable behaviors.*

2. *Add the circled numbers to determine the score.*

Understanding the Problem

- Asks the questions to clarify the problem
- States relevant facts in the problem
- Perceives implied relationships

Observed to a High Degree				Not Exhibited
5	4	3	2	1

Devising **a Plan**

- Summarizes data by making a table, graph, or diagram
- Develops approaches (looks for patterns, works backwards, makes predictions and verifies, decomposes problem into parts)
- Recalls related problems previously solved
- Estimates solution

Observed to a High Degree				Not Exhibited
5	4	3	2	1

Carrying Out the **Plan**

- Uses a table or diagram to arrive at solution
- Applies a formula
- Performs computation required for solution
- Decides where to begin
- Switches strategy when it is no longer applicable

Observed to a High Degree				Not Exhibited
5	4	3	2	1

Looking Back

- Describes strategy used in solving the problem
- Verifies that solution satisfies conditions of the problem
- Looks for alternative ways to solve the problem
- Creates applications or related story problems

Observed to a High Degree				Not Exhibited
5	4	3	2	1

SCORE: ________