Investigating Number Relationships

Oscar Schaaf
University of Oregon

$$1 + 2 = 3$$
$$4 + 5 + 6 = 7 + 8$$
$$9 + 10 + 11 + 12 = 13 + 14 + 15$$
$$16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$$

1. Study the number patterns in the above sequence of equations.
   (a) Write the next two equations.
   (b) What would be the first number in the tenth row? The last number?
   (c) What would be the first number in the nth row? The last number?

2. In the second row, the sum of the consecutive numbers on each side of the equal sign is 15.
   (a) Write another set of consecutive numbers whose sum is 15.
   (b) Are there other such sets whose sum is 15? Explain.

3. The third row shows two sets of consecutive numbers with a sum of 42; the fourth row shows two sets with a sum of 90.
   (a) Is it possible for every integer less than 35 to be a sum of two or more consecutive positive integers? Make a systematic list.
   (b) Which numbers, if any, cannot be sums of consecutive numbers?
   (c) Which numbers can be sums of two consecutive positive integers? Give an important characteristic or property of these numbers. Write an algebraic expression which describes these numbers.
   (d) Which numbers can be the sums of three consecutive positive integers? Give an important property of these numbers. Write an algebraic expression that describes these numbers.

4. Write two questions involving consecutive numbers for other class members to answer. Be certain you can answer your own questions!

Many mathematics educators consider the study of relations and functions as the heart of algebra. Students should be expected to search for relationships in their physical environment. In mathematics itself, they should express these relationships using word descriptions, tables, graphs, equations, and other algebraic expressions, and use these expressions in solving problems and making predictions. This lesson on investigating number relationships can be done in an algebra class early in the year while students are becoming familiar with algebraic expressions and variables.

Comments

Have students study the lesson sheet on their own for several minutes before allowing them to work together in groups of three or four. Encourage students in the group to answer their own questions before seeking your help. Questions 1(c), 2(b), 3(c), and 3(d) should be discussed during the lesson's culminating session.

ANSWERS:

1. (a) 25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35.
   36 + 37 + 38 + 39 + 40 + 41 + 42 = 43 + 44 + 45 + 46 + 47 + 48.
   (b) 100, 120.
   (c) n, (n + 1) −1 or n + 2n.
   Ask pupils to discuss the method they used to get the last number in the nth row. Several methods will likely be mentioned.

2. (a) 1 + 2 + 3 + 4 + 5.
   (b) Not with positive integers. There are several more if students choose to use negative integers and zero; for example, −3 + −2 + −1 + 0 + 1 + 2 + 3 + 4 + 5 + 6. There are only three solutions − one with two consecutive numbers, one with three, and one with five. The last sum with six consecutive positive integers is 21.

3. (a) No.
   (b) 1, 2, 4, 8, 16, and 32.
   (c) 3, 5, 7, 9, 11, 13, 15 . . . 35. Odd numbers from 3 through 35.
   2n + 1 where n is a positive integer.
   (d) 6, 9, 12 . . . 33. Numbers divisible by 3 from 6 through 33.
   3n + 3 where n is a positive integer.

4. Answers will vary. These questions should be used as a follow-up lesson.

Oscar Schaaf is Professor Emeritus, College of Education, University of Oregon. He has been a speaker at many NCTM meetings, including those held in Alberta. Dr. Schaaf was director of the Lane County Mathematics Project, which focused on problem solving. He suggests this problem-solving lesson is suitable for use at the junior and senior high school level.

EDITORS' NOTE: Students are encouraged to submit questions to the editors at the following address:

The Editors
Delta-K
c/o 2510 - 22 Avenue S
Lethbridge, Alberta
T1K 1J5