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EDITORIAL

Direction

Delta-K has two new editors, Art Jorgensen and John Percevault. At least, the initial issue in this volume has been published under their editorship.

As editors of *Delta-K*, we will endeavor to feature articles and ideas that will be useful to teachers in the classroom, to provide an opportunity for the participation of Alberta students through the "Student Problem Corner," and most importantly, to provide an outlet for Alberta mathematics educators to exchange ideas. These three objectives are facets of improving mathematics education.

We anticipate publishing three issues of *Delta-K* during 1985-86. The next issue should reach you by the end of January, and the third issue by the end of May. However, whether or not this is realistic could depend on the contribution of articles. In addition, you can look forward to receiving the 1985 issue of *The Canadian Mathematics Teacher*.

Comment

In this issue, **Al Anderson** challenges teachers to examine whether or not problem solving is being taught in their classrooms. The implication that teachers can arrange time, organization of students, and teaching strategies is clear. **Walter Szetela** enlarges on the theme that problem solving can be a practical exercise, utilizing the newspaper and students' identification with a popular figure. **G.S. Bhalla** utilizes working backwards. The "Hindu Inversion Method" is intriguing. Does the strategy of working backwards apply to geometric proofs as well? **Joan Haig** shares with us an example of how she involves honors mathematics students in exploring content beyond the prescribed curriculum. **John Percevault** suggests that teachers develop a set of problems that are related to a particular problem-solving strategy. A thought process is applied to "families of problems" and is extended to translation. The central question posed by **Hank Boer** challenges teachers to examine the difference between evaluating the solution to a problem and evaluating the problem-solving process. A schema is developed. **Oscar Schaaf and Ian Beattie** contribute ideas that you can try in the classroom Monday morning. Student responses to Schaaf's and Beattie's contributions will be published.

Share *Delta-K* with your colleagues. Reactions will be appreciated.

In Appreciation

The executive of MCATA and the editors wish to acknowledge the contribution of the previous editor, Gordon Nicol.

- Coeditors Arthur O. Jorgensen
and John B. Percevault

How Are We Doing – Now That We Know What It's All About?

A. Anderson
County of Vermilion River

No other topic in mathematics education, involving curriculum development, implementation, or instructional inservice, has received more attention than that of problem solving. It has been defined, redefined, exemplified, and "workshopped" into submission, it would seem. Publishers of resources, from monographs to computer software, have left us with no excuses. Every mathematics teacher and administrator must know what it is and why it is, and so the big question remains: *What have we been doing with problem solving, now that we know what it is all about?*

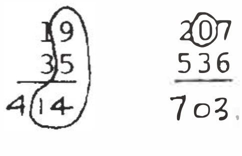
The assumption in this article is that we have passed the "understanding the problem" and the "developing and carrying through the plan" stages, and we are now looking back. This point is can be argued. However, the question remains: "Are children actually experiencing anything different in mathematics than they did before the topic emerged into prominence?" My "guess and check" is no. For the believer, this is tough medicine, but if we were to look at an overall time-on-task analysis of Alberta elementary children's mathematics engagements, it would be as follows: operations and properties - 60 percent; numeration - 25 percent; measurement - 10 percent; geometry - four percent; graphing - one percent; and problem solving - zero percent.

This is another story of curriculum and implementation similar to that of the "new mathematics." Problem solving will also fail for the same reasons. The topic and content has not had an impact on the belief system of teachers and administrators. The word fell on unprepared soil, and the intensity of the normal classroom snuffed it out. We gave teachers seed packages, but neglected the planting and the nurturing. Our workshops and monographs gave episodes of problem solving, but did not confront the issues of where, when, and how in the context of the total mathematics program. Anyone can set up an appropriate problem and get people all excited and involved in finding solutions. To help in this regard, we even provided a good list of skills. (See *Let Problem Solving Be the Focus for the 1980s*, an Alberta Education monograph published in 1983.)

We also failed in not recognizing that change comes only when that which is new is perceived as being more appropriate than that which is present, and when the new can be accommodated within the teacher's conceptual and operational plan.

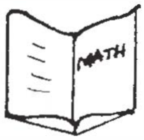
We are almost to the point of waiting for something new to come along. While we seem to need that, let's not give up on problem solving yet. One of the big reasons for not including problem solving in the mathematics program has been time. "How can I fit more into my current program?" has been the question.

Here are a few ways of recapturing this precious commodity. Note the savings:



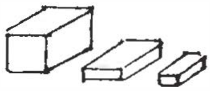
If students are having difficulty, diagnose and treat the specific errors. If they are succeeding, restrict practice and review to reasonable limits.

Save 15%



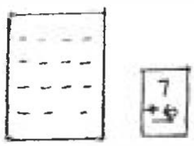
If you teach the content of an authorized textbook, you are keeping children overly busy, including nonprogram objectives and missing others. Design and utilize a meaningful plan.

Save 15%



If you can teach the concept of skill manipulatively, don't do it abstractly. We know this, but fail so often to apply this most important principle.

Save 10%



No more worksheets for basic fact recall - use student-made flash cards, and personal and home contracts for mastery. Time saved includes helping students overcome dependency habits.

Save 5%

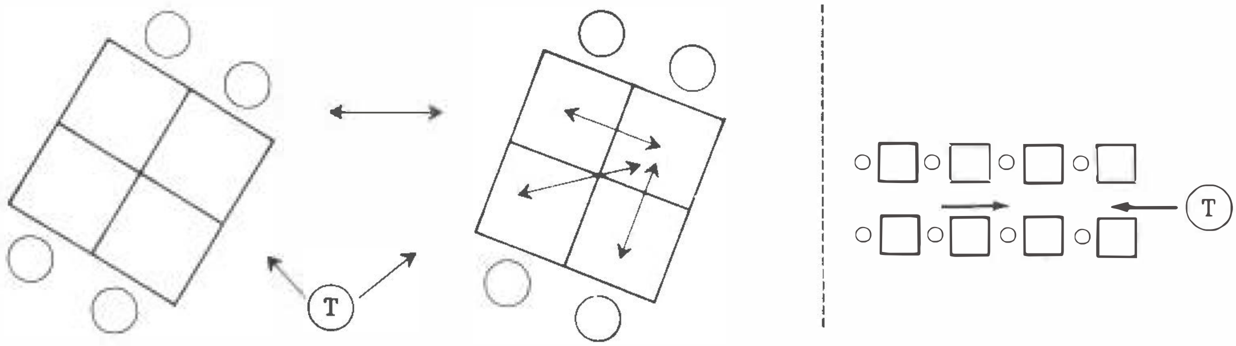
We can be more comprehensive in dealing with the components of effective lessons and increase the positive attitude toward mathematics by 50 percent. To illustrate, teachers could take any mathematics objective at any grade level - as an example, "compare two or more objects as shorter, longer, thinner, thicker, heavier, lighter than" (Grade 1) - and balance one week of instruction as follows:

Instructional Program Emphasis in Minutes

	Day 1	Day 2	Day 3	Day 4	Day 5	Total
Goal-Objective Orientation	10				5	15
Introduction Activity	10	5	5	5	5	30
Lesson Development	15	15	10			40
Practice Application		10	10		10	30
Problem Solving			10	35		45
Assessment Evaluation		10			10	20
Closure Activity	5		5	5	10	25

In this model, problem solving requires but one period per 205-minute week, still allowing time for mastery of the core program objectives. Implementing the plan will involve describing the specific activities under each of the instructional components for the week. Each week is best tied to an overall yearly outline.

Reorganize your physical classroom space while you are at it. The interactive and integrative nature of good mathematics problems requires that communications flow multidirectionally. Clustered desk arrangements allow for the needed get-togethers to include you, the teacher. Let's face it, a good percentage of a student's learning is from other students. One step back, and you have your individual child work space.



It is critical that problem solving become a part of instruction in the mathematics program in the elementary school. Alberta Education will soon release a monograph on the topic as it applies to the junior high level.

So the question remains: "What are we going to do with problem solving now that we know what it is all about?"

Dr. Anderson is Assistant Superintendent of Schools for the County of Vermilion River. He previously served as Coordinator of Mathematics for the Medicine Hat Public School system and as Math Consultant for Alberta Education, Grande Prairie Regional Office.

The Newspaper Wedge: Motivate Students to Solve and Create Problems

Walter Szetela
University of British Columbia

Motivation is a daily problem for teachers who want their students to be active learners, especially with the competition of television, rock video, space exploration, and so forth. The problem is perhaps more severe with the learning of mathematics and, in particular, with problem solving. Students will have more desire and expend greater effort in the solution of problems that they can relate to their real-life experiences than most of those found in textbooks.

Teachers need to be alert to events that appeal to students and which can be used as a realistic context for learning. Newspaper reports, advertisements, sports news, travel information, financial pages, and even the comic strips can help teachers to motivate students and can serve as a vehicle for interesting problem-solving activities. Such problems can provide opportunities for a variety of solutions and stimulate additional related problems. Even more important, discussions about methods of solutions and the solutions themselves are more likely to generate new ideas and enable students to approach other problems with more confidence.

Current Events Context

Consider, for example, an event such as Steve Fonyo's "Journey for Lives" across Canada. One newspaper report gave the following information about Fonyo:

His daily 32-kilometre distance is marked by a series of orange cones placed at one-kilometre intervals.* Between one set of two cones, he took 1,179 steps. At that rate, how many steps will he have taken on his 7,320-kilometre cross-Canada journey?

This bit of information can be used to devise interesting mathematics activities and problem-solving situations for students, who are likely to show more interest and effort because of their knowledge, awareness, and admiration of a young Canadian hero. Children will be eager to solve the problem suggested by the news item. It is preferable to have students compute the answer with a calculator after they have determined that the product of 1,179 and 7,320 will provide the answer. This problem becomes a lead-in to several other interesting problems, such as the following.

1. How many **steps** do ***you*** take to walk **a kilometre?**

This is a good problem situation in which there are no numbers. The first thing that must be determined is what information is needed. Next, how will the information be obtained? These conditions make the problem more realistic than the standard textbook variety.

*The actual report had the cones at one-mile intervals. The number of steps has been adjusted for one-kilometre intervals.

It is an excellent situation in which to involve the class in a discussion of how such a problem might be solved for each individual student. In such a discussion, teachers can expect some impractical suggestions amid some unusual and productive ones, but no suggestion should be dismissed without allowing the student to see why the idea may not be helpful. Sometimes, when that opportunity is given, a seemingly unproductive idea actually turns out to be a brilliant one. Class discussions that encourage students to speak without threat of embarrassment may do much to encourage confidence and persistence in problem solving, instead of frustration and futility.

It is my belief that if you try out such a discussion with the above or similar problems, you will reap a harvest of more interesting ideas than you expect. Eventually, the discussion will conclude with an agreement that, in order to solve the problem, each student must find the length of his step, or pace. Further discussion will result in decisions on methods and materials to be used for determining pace length. When each student's pace length has been determined, there are several ways in which the problem might be solved.

Determination of Pace Length

Method 1. Have students work in pairs. One student should take a few steps and then freeze in position. The other student should use a metric tape to measure the distance from toe to toe or heel to heel.

Method 2. Have the class work outdoors. Working in pairs, one student starts from a marker, walks 10 steps, stops, and marks the terminal spot. The two students use a long metric tape to measure the distance. The pace length is determined by dividing the distance by 10.

Solving the Problem

Using a pace length of 40 centimetres:

Method 1. There are 1,000 metres in a kilometre. We also know there are 100 centimetres in a metre. Convert 1,000 metres to 1,000 X 100, or 100,000 centimetres. Divide 100,000 by 40. The number of paces = 2,500.

Method 2. Convert 40 centimetres to 0.4 metres. Divide 1,000 by 0.4 to get 2,500.

Method 3. Make a table:

Steps	Distance
10	4 metres
100	40 metres
1,000	400 metres
2,000	800 metres

Note that with 2,000 steps, you would travel 800 metres, leaving only 200 metres more to go. Note also that with 1,000 steps, you travel 400 metres. Then, to cover 200 metres more, you need 500 more steps, or a total of 2,500.

Method 4. Guess and test. The goal is to walk 1,000 metres, or 100,000 centimetres.

FIRST GUESS: 1,000 steps.
This takes you 1,000 X 40 centimetres, or 40,000 centimetres. This is much too small.

SECOND GUESS: 2,000 steps.
This takes you 2,000 X 40 centimetres, or 80,000 centimetres. This is much closer to our goal.

THIRD GUESS: 2,500 steps.
This takes you 2,500 X 40 centimetres, or 100,000 centimetres. That's just right.

These various solutions are given to show that there are many ways of looking at a problem, any one of which may fit a student's learning style better than a preconceived conventional solution. Teachers who invite such a variety of solutions will find that students will be more successful and more positive in their problem-solving behavior.

The above was a detailed discussion of just one problem. Solving such a problem may actually whet the appetites of students for additional related problems, such as the following:

2. If you take 1,850 steps to walk to school, about how far is your home from school?
3. How many steps would you take to *run* one kilometre? (Is your pace length for walking the same as for running?)
4. A marathon distance is about 42 kilometres. About how many steps would you take to walk that distance?
5. The distance around the earth at the equator is about 40,000 kilometres. About how many steps would you have to take to go that distance?

After problems like this, you may find that your students wish to create problems of their own. Such problems might be created individually or in groups for a "best problem of the day" contest.

Advertisement Context

Advertisements offer many opportunities for motivating students. How many different problems could your students construct and solve using the information shown in the ad at the bottom of this page? Here are a few examples:

1. How far would the Mercedes Benz 190 E 2.3 go in 24 hours?
2. How long would it take the car to go across Canada?
3. How much fuel would the car use in one hour?
4. How long would it take the car to travel 50,000 kilometres? Could this be done in eight days and nights?
5. How much fuel was used to travel 50,000 kilometres?
6. The world's fastest human can run 100 metres in 10 seconds. How far would the Mercedes Benz travel in 10 seconds?

Some of the above problems have insufficient information. Such problems provide an excellent basis for developing ideas about the problem conditions and estimation skills. For example, the question about fuel consumption at 248 kilometres per hour compared to consumption at 100 kilometres per hour should be discussed.

For students who are interested in things other than automobiles, numerous other ads will provide them with interesting and challenging problem situations.

In the past nine years, Mercedes-Benz has raced the clock four times— and set forty-one international records.

ON A TRACK IN NARDO, ITALY, on a sweltering August day, a 16-valve four-cylinder Mercedes-Benz 190 E 2.3 crosses its mark at 248 kilometres an hour. A cheer goes up from the gathered technicians.

Not because the car is traveling at 248 kilometres an hour.

Because the car has been *averaging* 248 kilometres an hour for eight days and nights.

Comic Strip Context

The comic strip illustrated at the bottom of this page provides another example of a situation which can be a highly motivating instrument for the construction of problems by students. The task is to formulate problems for which the answer is 11 potatoes.

Here are a few examples, although your students will likely invent more interesting ones:

1. For lunch, I ate four potatoes, and for dinner, I ate seven potatoes. How many potatoes did I eat altogether?
2. There were 24 potatoes in a bag. My mother cooked 13 of them. How many were left?
3. Polly filled five bags with 55 potatoes. Each bad contained the same number of potatoes. How many were in each bag?
4. On Sunday, Mike ate one potato. He ate three potatoes on Monday, five potatoes on Tuesday, and seven potatoes on Wednesday. If he continues this pattern, how many potatoes will he eat on Friday?
5. Two carrots have the same mass as one potato. How many potatoes will it take to balance 22 carrots?
6. Instead of candles, my mother puts potatoes on my birthday cake. Last year, I was 10. What will my mother put on my next birthday cake?

Sports Context

It is not difficult to see how sports stories and statistics provide a wide variety of problem possibilities that have wide appeal. Consider, for example, the daily standings of professional hockey teams. What problems might be suggested by the following year-end standings?

	Won	Lost	Ties	Points	Goals	Goals Against
Edmonton	57	18	5	--	446	314
Calgary	34	32	14	82	311	314
Vancouver	32	39	9	73	306	328
Winnipeg	31	38	11	73	340	374
Los Angeles	23	44	13	59	309	376

These figures provide an illustration of well-organized data and provide practice for identification and selection of relevant data in various problems. Here are some possible problems students or teachers might construct at different grade levels.

1. How many games did Winnipeg play? (Primary)
2. How many more games did Edmonton win than it lost? (Primary)
3. The number of points for Edmonton has been left out. What should Edmonton's point total be? (Intermediate)
4. If the number of wins and losses for Vancouver were reversed, how

PEANUTS

many points would they have earned? (Intermediate)

5. What was the average number of goals scored per game by Calgary? (Upper Intermediate)

6. Vancouver and Winnipeg had the same number of points. If tie games were not counted, which team had the better percentage of games won? (Upper Intermediate)

Feature Article Context

Newspapers often contain articles which highlight a seasonal, recreational, or historical topic of special interest. An article written to commemmorate the fiftieth anniversary of the legendary indestructible airplane, the Douglas DC-3, gave the following information about one particular airplane built in 1942, which is still in service with the French navy. The plane's log book shows that it has used up 700 tires, 35,000 sparkplugs, and 160 engines. Following are some examples of problems that could be constructed and solved.

1. For every tire that had to be replaced, how many sparkplugs needed replacement?

2. For each engine replacement, how many sparkplugs were replaced?

3. What is the average number of tires, sparkplugs, and engines replaced each year since the plane was built?

4. If the plane lasts five more years, about how many sparkplugs will be needed to keep it in service?

A useful and interesting assembly of newspaper problems, appropriately titled "Newspaper Math," is available from Collier Macmillan Canada Ltd. (Fraser, 1980).

Summary

The main purpose of this article is to illustrate the wealth of problem-solving material in daily newspapers that has appeal for students. Such problems bring the real world into the classroom and motivate students better than textbook problems. Sometimes the problem situations do not contain enough information; other times, they contain more than enough. In either case, students should be challenged to construct problems.

It is also suggested that students should be given every opportunity to solve problems by different methods. Even if a method given by a student seems cumbersome, it may have the virtue of being understood by that student, and comprehension in problem solving is more important than churning out meaningless answers. Teachers who risk some loss of control in a truly open problem-solving atmosphere may be surprised by the growth in problem-solving skills and the improvement in attitude toward problem solving.

Walter Szetela is Associate Professor, Department of Mathematics and Science Education, University of British Columbia. He has chaired sessions and given presentations at the annual meeting of the NCTM. Dr. Szetela is the author of "Problem Solving in Mathematics: Are Reading Skills Important?" published in MCATA Math Monograph No. 6, Reading in Mathematics.

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Hindu Inversion in Present-Day Algebra Classrooms

G.S. Bhalla
Bronx Community College

Hindus used inversion to solve number problems that were posed for intellectual pleasure. The method, known as "vilomagati" (working backwards), was a favorite method of Hindus and was commonly used by them since early years of the Christian era. Beginning from the end, the suggested operations were inversed: addition was changed to subtraction, multiplication changed to division, square to square root, and vice versa. Bhascara's Lilavati gave the rule of the method as:

> To investigate a quantity, one being given, make the divisor a multiplicator, and a multiplier a divisor; the square a root, and the root a square; turn the negative into positive, and a positive into negative. If a quantity was to be increased or diminished by its own proportionate part, let the lower (denominator) be increased or diminished by its numerator, and the numerator remains unchanged; and then proceed with the other operations of inversion, as before directed.
> (Brahmagupta, 1817)

I have made use of inversion in my elementary algebra classes with excellent results. The procedure reinforces basic algebraic operations, in addition to providing an insightful review of the solution of linear and quadratic equations.

Consider the following example:

> **What is that quantity which, when divided by seven, then multiplied by three, then squared, then increased by five, then divided by three-fifths, then halved, and then reduced to its square root, happens to be the number five?**
> (Datta and Singh, 1935)

Beginning from the end, the solution by inversion proceeds as follows:

Number	5
Squared	25
Doubled	50
Multiplied by 3/5	30
Decreased by 5	25
Root	5
Divided by 3	$\pm\frac{5}{3}$
Multiplied by 7	$\pm\frac{35}{3}$

Hence, Quantity $= \pm\frac{35}{3}$

The problem is solved using algebraic procedures as follows:

Let the number	=	x
Divide by 7	=	$\frac{x}{7}$
Multiply by 3	=	$\frac{3x}{7}$
Square	=	$\frac{9x^2}{49}$
Add 5	=	$\frac{9x^2}{49} + 5$

Divide by $\frac{3}{5}$	=	$\frac{5}{3}(\frac{9x^2}{49} + 5)$
Halved	=	$\frac{5}{6}(\frac{9x^2}{49} + 5)$
Square root	=	$\sqrt{\frac{5}{6}(\frac{9x^2}{49} + 5)}$

By question:

$$\sqrt{\frac{5}{6}(\frac{9x^2}{49} + 5)} = 5$$

To solve this, we square both sides to get:

$$\frac{5}{6}(\frac{9x^2}{49} + 5) = 25$$

Multiply both sides by 6/5, and we have:

$$\frac{9x^2}{49} + 5 = 30$$

Subtract 5:

$$\frac{9x^2}{49} = 25$$

Multiply by 49/9:

$$x^2 = \frac{49(25)}{9}$$

Take square root of both sides:

$$x = \pm\frac{7(5)}{3} = \pm\frac{35}{3}$$

which is the required quantity.

Steps of inversion and the algebraic solution are compared, and the technique of inversion of operations is emphasized.

Students are encouraged to do similar problems by both procedures. They are asked to construct their own problems and solve them by both methods. The results of the strategy are very interesting. Students develop a better understanding and appreciation of the algebraic operations.

Dr. Bhalla teaches in the Department of Mathematics at Bronx Community College, Bronx, New York.

EDITORS' COMMENT: Dr. Bhalla's article caused the editors to look at the process of teaching solving equations. Two parallel but reversed processes are given. Should students be provided the opportunity to develop equations, as well as solve equations?

Developing an Equation

$x = 2$

$x + 3 = 5$ (Add 3)

$3(x + 3) = 15$ (Multiply by 3)

$\frac{3(x + 3)}{5} = 3$ (Divide by 5)

Solving an Equation

$\frac{3(x + 3)}{5} = 3$

$3(x + 3) = 15$ (Multiply by 5)

$x + 3 = 5$ (Divide by 3)

$x = 2$ (Subtract 3)

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Broadening Horizons – Individual Student Projects in Mathematics

Joan Haig
Lethbridge Collegiate Institute

During the past few years, I have toyed with ways to tap student interest in mathematics and to encourage students to go beyond what is required by the curriculum. Time pressures are such that there is not a great deal of class time available for this. A club works well for the students who can fit it into their busy schedules. What to do? Back to the old project idea.

My first serious assignment of projects was made at the Math 20 level in an honors class. We talked about possible topics, possible sources of information, people who would be helpful, and time limits. The students were then given a two-week period to investigate and select a topic they were interested in, and a further six weeks to develop their idea.

Since this class was quite highly motivated and of a very verbal nature, it was decided that there would be three aspects to each project. The project chosen would have to:

1. demonstrate or explain some mathematical principle or be related to a specific branch of mathematics;

2. be presented to the class and have a visual, as well as a verbal, aspect;

3. be handed in as a text.

The marks were divided among the three aspects, and the results were very exciting.

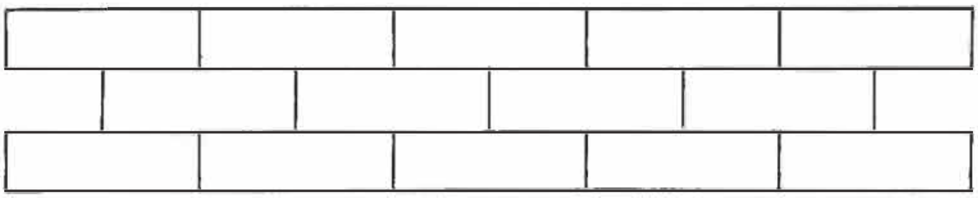
Please bear in mind that these students were in their first semester of Grade 11. The projects varied widely in scope and in method of presentation. The range included paper folding geometry, a project that mapped tone patterns of different musical scales, a computer version of slot machines, models of a stellated dodecahedron, and a flexihexagon. I was very impressed with these young people, in both their choice of topics and their presentations.

I have selected for your enjoyment one project that reflected the student's personal interest, was very well presented, and did not involve a great deal of complicated paraphernalia. Kathy Pratt's investigation of tessellations is an example of a personal interest topic that would not otherwise have been explored during high school. The following represents the handed-in portion of Kathy's assignment.

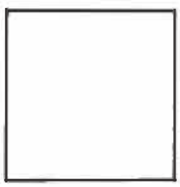
Tessellations

The Romans decorated their buildings and towns with mosaic floors and pavements made of very small tiles called tessellae. From this word comes the word tessellation, which is used to describe ways of filling space. The study of these is one of the bridges between mathematics and art. Such decorations have been used for centuries, but today, artists, arthitects, and designers are making more and more use of simple geometric shapes and the ways in which they fit together.

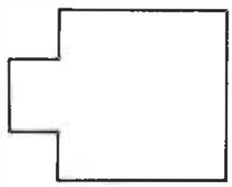
A definition of a tessellation is a repeated pattern of shapes, which are often polygons, that will completely cover a flat surface, leaving no gaps or overlaps. The following is an example of a tessellation of rectangles:

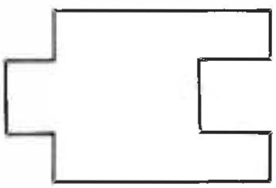


Shapes in a tessellation must be capable of fitting together. An example would be a square that is changed slightly. To make the new shape tessellate, the added feature must be taken off the opposite side. The new shape will tessellate.

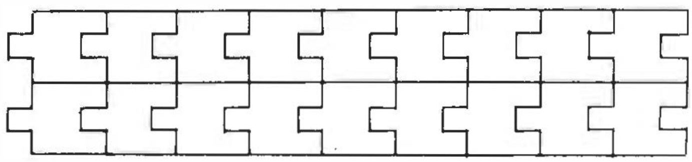


The square is changed to



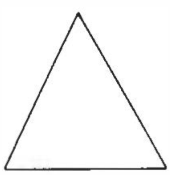


Take the added feature off the opposite side.

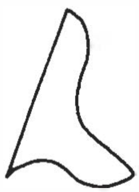


The new shape tessellates.

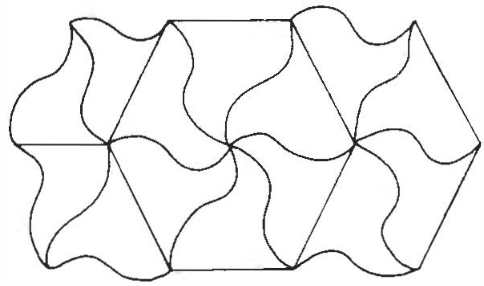
More complex shapes can be made using curved lines as well as straight lines:



Alter two sides of a triangle.

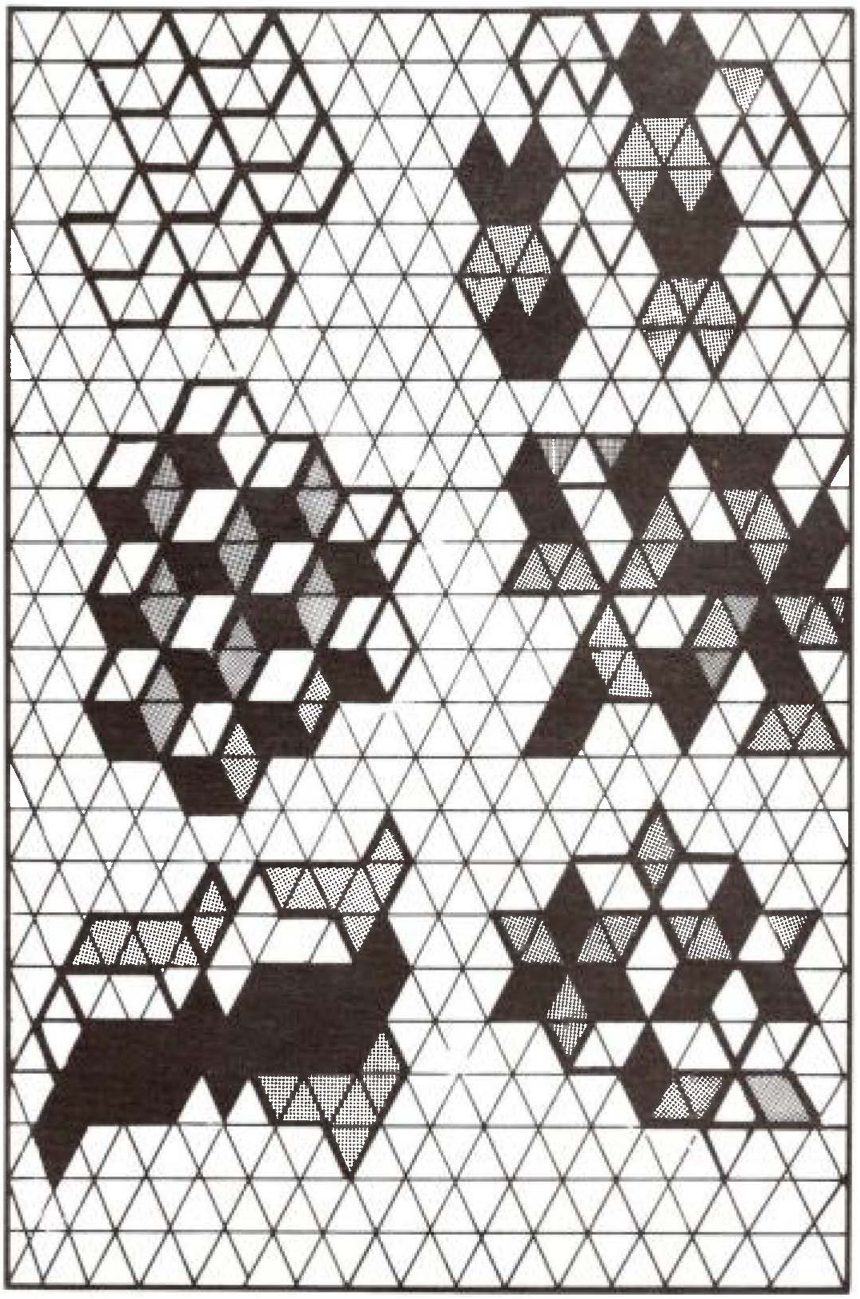


This figure will have the same area as the original triangle.

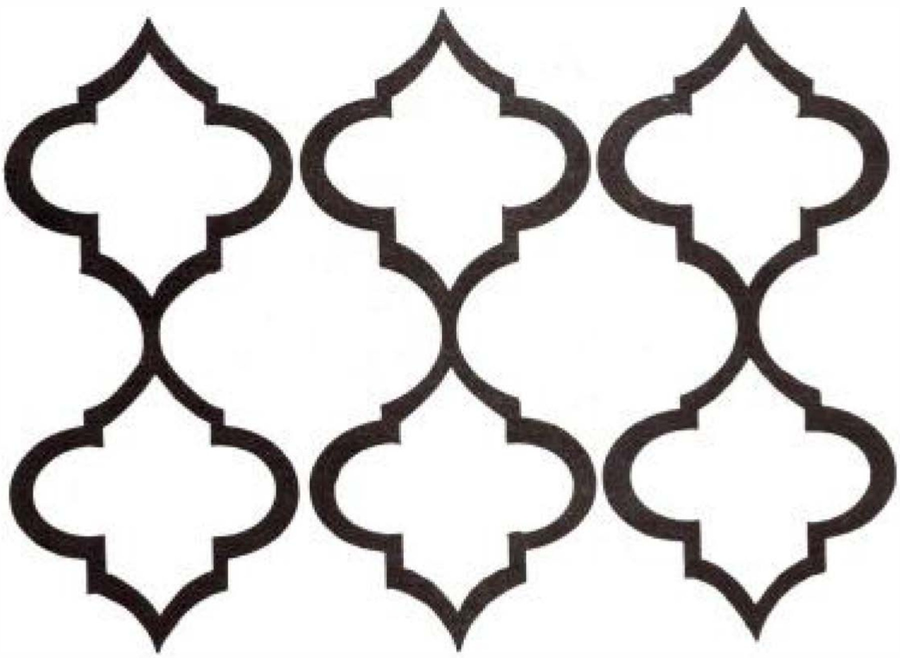


Tessellations can be used to create curved lengthening, shortening, and three-dimensional effects. An important aspect of tessellations is color, because a tessellation can appear one way and entirely different in another way just by the way it is colored.

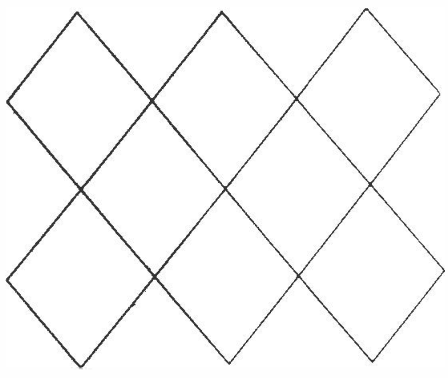
These are triangular-based tessellations. Some are the same tessellations colored differently.

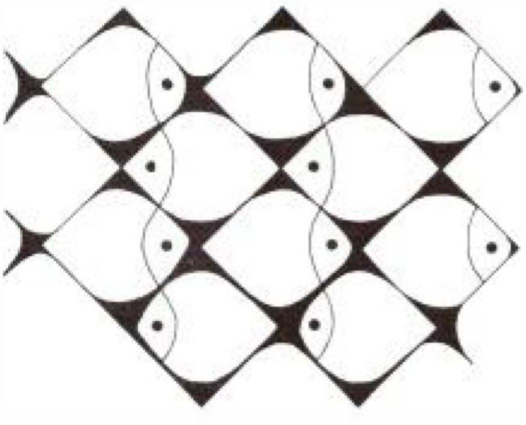


Tessellations can be found in buildings, on material, wallpaper, floor coverings, wrapping paper, and quilts. A tessellation of hexagons appears in honeycomb and chicken wire. The following pattern is common on floor coverings:

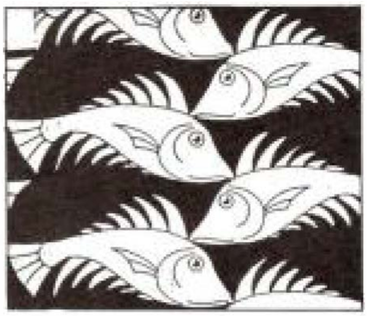


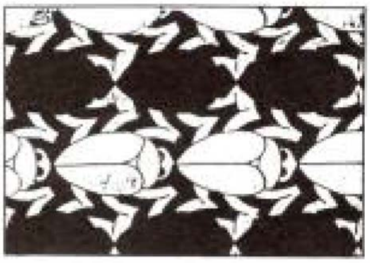
Shapes within tessellations can easily be adapted. The basic quadrilateral tessellation could be turned into two shoals of fish swimming in opposite directions:

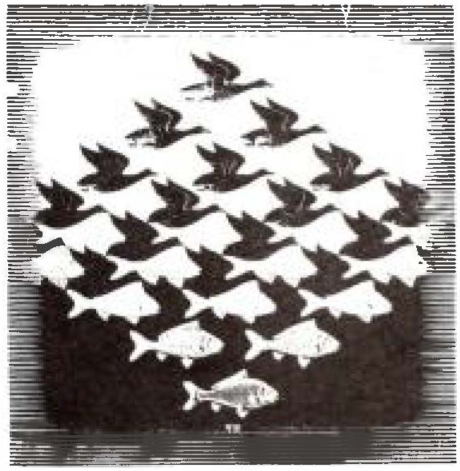


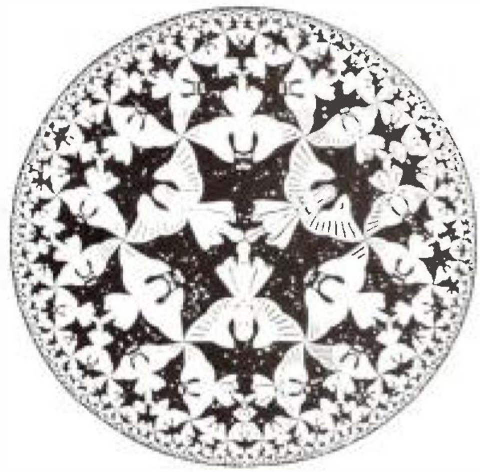


The Dutch artist Mauritz Escher visited the Alhambra Palace in Spain and was fascinated by the tiling patterns on the walls and floors. He began to experiment to find how a surface might be regularly divided and filled with congruent figures without leaving any gaps. This led him to devote his life to developing more and more complex tessellations.









Even though we don't usually notice tessellations around us, they are a big part of our lives. They affect our architecture, fabric, wallpaper, floors, quilts, and art. They are also a part of our history. Tessellations have been around for a long time, but there are always new and fascinating discoveries to be made. No matter how often they are used, they never fail to be interesting and remarkable.

Joan Haig is head of the Mathematics Department at Lethbridge Collegiate Institute. Joan has been active in the South West Regional of MCATA.

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Families of Problems: Changing Conditions, Variables, and Information Sought

John B. Percevault
University of Lethbridge

Since 1980, mathematics curricula have shown an increased emphasis on problem solving. Problems differing from the problems in the text and non-routine problems have been presented as alternatives. How successful has this emphasis on problem solving been in developing better thinkers and better problem solvers?

The problems noted below are being presented to upper-elementary students who have not learned to translate, that is, to develop an equation. Often the problem is presented in isolation, solved, and subsequently forgotten.

Alternatively, groups of related problems can be developed and solved. Teachers can share their thought processes with students by asking questions as problems are developed and solved. Relationships and patterns can be explored. The process of thinking, the problem-solving strategy, can be emphasized as the process is used in solving the related problems. Data from the problems can be organized to facilitate the ability to translate. Students become problem solvers, not just solvers of a problem.

The process used throughout this article involves the development of an organized list. Admittedly, other strategies could be used. Sample questions, indicators of the teacher's thought, are posed. Relationships are probed. As the problem varies, the teacher and the student note what has changed, namely, the *condition*, the *variable*, or the *information sought*. Finally, the organized list is used as the basis for translation.

Sample Problem

A farmer has a unique manner of determining how many animals - chickens or cows - he has. He counts the total number of legs and the number of heads. From this, he can determine the number of chickens or cows. One day, he counted 26 legs and 10 heads. How many of each animal were there? The solution, obtained through use of an organized list, follows:

Figure 1.

No. of Chickens	No. of Cows	Chickens' Legs	Cows' Legs	Total No. of Legs
0	10	0	40	40
1	9	2	36	38
2*	8	4	32	36
3	7	6	28	___
___	6	8	24	___
5	5	10	20	___
6	___	12	16	___
7	3	14	12	26
8	2	16	8	___
9	1	18	4	___
10	0	20	0	___

*At this point, students could be challenged to look for patterns.

Sample questions include:

- How is the number of chickens changing?
- How is the number of cows' legs changing?

- How is the total number of legs changing?

Other questions that model the teacher's thought processes could include:

- Could we have a total of 33 legs?
- What is the largest total of legs?
- What is the smallest possible total of legs?
- How many addition facts (two addends) are there?
- Can you develop another problem from this data?
- How many problems could be developed?

Note that the basis for translation is being developed. Figure 1 is completed below, including two sample translations.

	No. of Chickens	No. of Cows	Chickens' Legs	Cows' Legs	Total No. of Legs
	x	10-x	2x	+ 4(10-x)	= 26
OR					
	x	y (x+y=10)	2x	+ 4y	= 26

Variation #1

The next time the students meet this type of problem, the farmer is sitting on his veranda. He counts wheels of vehicles and notes that the vehicles are either cars or bicycles. One evening, he counts 10 vehicles and 26 wheels, or any of 11 possible totals of wheels. The question is obvious.

A *condition* has changed. The *process* used to solve the problem remains the same and transfers to another problem. Furthermore, when the student is taught to *transform*, the thought process remains constant.

The process applies to other problems as well. Consider the following problem:

> Ten kilograms of a mixture of candy sells at $2 and $4 per kilogram. The value of the 10 kilograms is $26 (or $38, or ____). How many kilograms of each type of candy are there?

Or, substitute:

> Twelve coins - nickels or dimes. The value of the coins is $1.

Variation #2

Change the location to a furniture shop that specializes in making chairs (four legs) and stools (three legs). During one hour, 10 articles were produced that had a total of 34 legs. How many of each, chairs and/or stools, were produced?

Questions that could be asked include:

- As the number of chairs increases and the number of stools decreases, how does the sum of the legs differ?
- How many sets, each containing two addends, are there that have a sum of 10?
- How many possible totals of legs are there?
- How many different questions could have been asked?

Another change in variables could roduce the following problem:

> Another day, the carpenter shop produced sofas (six legs) and stools. Twelve articles were produced using a total of 51 legs (or any other suitable replacement). The question is obvious.

Questions, as well as those previously posed, could include:

- How will the total number of legs vary over the 13 possibilities?

The teacher may also wish to use the table that could be produced to develop the equation or system of equations. The process of developing an organized list transfers to a seemingly different problem.

Figure 2.

No. of Stools	No. of Sofas	Total Legs (Stools)	Total Legs (Sofas)	Total Legs
0	12	0	72	72
1	11	3	66	69
—	—	—	—	—
4	?	—	—	—
—	—	—	—	—
?	5	—	—	—
—	—	—	—	—
—	—	—	—	—
11	—	—	—	39
12	0	36	0	36
x	12-x	3(x) +	6(12-x)	= 51
OR				
x	y	3x +	6y	= 51
(x + y = 12)				

A variable, a number, has changed; the process remains constant.

Variation #3

The basic problem is varied again. The information sought is changed.

> One day, the carpenter shop sold furniture orders to 11 customers. Each customer bought 10 articles, either sofas and/or stools, chairs and/or stools, sofas and/or chairs. None of the 11 orders was the same. How many legs were required to complete all the orders?

All of the questions posed previously are appropriate. Further questions may need to be asked, namely:

- Have you solved a problem similar to this one?
- How did you solve that problem?
- Can you develop a chart? An organized list?
- What must be done to answer the question posed in the problem?

Variation #4

Some problems, such as the following, have multiple solutions:

> One day, the farmer counted 32 legs of chickens and/or cows, but forgot to count the number of heads. What combination of chickens and/or cows could have a total of 32 legs?

The conditions of the problem are satisfied with 16 chickens or eight cows, the largest and smallest total of animals. Many of the questions previously presented may be used. Once again, the strategy of a logically organized list will allow students to determine the other possible solutions.

Conclusion

As the student experiences problem-solving situations, opportunity should be given to examine groups of related problems. Problems may be altered by changing conditions, by changing the variables, and by changing the information sought. The process of problem solving, the thought process, remains constant. Such a planned approach should allow students to solve problems, not just to arrive at the solution. Finally, problems may also be selected to assist the students in translation, developing equations.

The final problem posed in this article could be used to relate to information from other subjects or to have students seek information.

> Among the inhabitants of the planet of Skol are giant spiders, each of which is fed by little men called Roods. Each day, the spiders require exactly 48 fly legs. The flies of Skol are abnormal compared to the flies on planet Earth. Each female fly has two more legs, and each male fly has two less legs than the flies on Earth. How many combinations of female and/or male flies does the Rood need to catch to ensure his and the spider's survival? If fed too many fly legs, the spider, Phoenix-like, explodes, and the little spiders devour the Rood. If fed too few legs, the Rood is eaten to complete the spider's meal.

Embellish the conditions more if you want to. By now, you probably have had enough of legs.

Acknowledgment

The writer acknowledges the influence of John LeBlanc, in his address at the 1981 NCTM annual meeting. The schema presented by LeBlanc follows:

Effective Teachers Structure Questions -

For Understanding
(a) Questions related to given information.
(b) Questions related to stated conditions.
(c) Questions related to questions asked (main idea).
(d) Questions related to a proposed solution.

For Extension (Modifying Problems)
(a) Modify variables.
(b) Modify conditions.
(c) Modify information sought.

John Percevault is Associate Professor, Faculty of Education, University of Lethbridge. He served as a teacher, principal, superintendent of schools, and mathematics consultant for the province before joining the Faculty of Education. Mr. Percevault represents the Alberta Faculties of Education on the Mathematics Council, ATA. He has been a member of Alberta Education's Mathematics Curriculum Co-ordinating Committee. His current interests include teaching thinking, a broad issue that includes problem solving.

***This article was also published in the* Canadian Mathematics Teacher.**

Evaluating Problem Solving

Hank Boer
Lethbridge School District No. 51

As problem solving becomes a greater part of mathematics education in Alberta schools, its overall complexity will increase. Rather than giving students instruction about single problems that have single answers, they will be provided with complex problems that require a more complete effort to solve. Here is an example of a complex problem.

> Consider 12. It has four numbers less than it that only have a factor of one in common with 12 (11, seven, five, and one). These are called *monodivisors* of 12. Similarly, six has two monodivisors (five and 1). Investigate the number of monodivisors for different numbers.

The above problem is taken from some British "O" level investigations. It is obvious that a student's completion of this problem will require greater effort than more traditional problems. After students are finished answering complex problems, the teacher is left with the difficult task of evaluating the student's work. The amount and type of work accomplished throughout the problem-solving process needs to be acknowledged and recorded. The student's work will vary in the type of solution and level of completion. This type of problem requires a "holistic" approach to evaluation. In this article, an Impressionistic Scoring Criteria scheme will be presented, which will give educators a valid basis for problem-solving evaluation.

Problem Complexity

A better understanding of problem-solving evaluation can be developed if there is an appreciation of how problems vary in complexity. Relative measurement of complexity can be done using six factors. These factors are:

1. stage implementation
2. stage depth
3. strategy implementation
4. strategy depth
5. open-endedness
6. duration

Stage implementation refers to the number of stages the student is required to execute for the successful completion of a problem. Alberta Education suggests a framework of four stages. These are:

- understanding the problem
- developing a plan
- carrying out the plan
- looking back

Complex problems usually use four stages. Less complex problems use one, two, or three of the stages.

Stage depth refers to the degree with which each stage of the four-stage framework is implemented for the successful completion of a problem. Complex problems require that stages be fully implemented. Less complex problems require only partial implementation of a stage.

Strategy implementation refers to the number of strategies that can be or must be used to successfully complete a problem. Some examples of strategies as outlined in Alberta Education documents include:

- looking for patterns
- identifying key words
- using a simpler problem
- applying patterns
- looking for alternative ways to solve the problem
- making diagrams and models

Complex problems tend to have more possible strategies or require the implementation of more than one strategy for their successful completion. Less complex problems use fewer strategies (usually one).

Strategy depth refers to the degree of implementation of the strategy such that the problem is completed. More complex problems have higher degrees of strategy implementation than less complex problems.

Open-endedness has primary application to the "looking back" stage of the problem-solving framework. If the completion of a problem leads to the exploration of many facets of that problem or problems like it, then the problem is said to be open-ended. Complex problems tend to be more open-ended than less complex problems.

Duration refers to the amount of time and effort required to complete a problem. Complex problems have higher duration levels than less complex problems.

Complex problems have high levels of stage implementation, stage depth, strategy implementation, strategy depth, open-endedness, and duration. Complex problems have an inherent "creative" aspect to them, which allows students to explore problems rather than just find right answers. This creative element of mathematics motivates and inspires teachers and students. The inclusion of problem solving (especially open-ended complex problems) in the mathematics curriculum can enhance the environment for creative learning in the mathematics classroom.

Evaluation of Problem Solving

The evaluation of complex problems requires a different approach than traditional right-answer or part-mark approaches. Ideally, recording a student's performance and behavior during a complex problem-solving session would give a teacher the best data for making accurate and objective evaluations. However, the logistics and time required for such an evaluation technique is limited by the time needed to observe and interact with each student. There are two possible solutions: (1) use an instrument that records objective and observable student behavior data quickly and efficiently, or (2) evaluate the written work and make inferences about the student's ability at completing each complex problem. An Impressionistic Scoring Criteria scheme assists the teacher in evaluation of student behavior and written material.

The Impressionistic Scoring Criteria scheme, developed by the writer and outlined on page 26, was borrowed heavily from a scheme previously developed by Dr. Terry Rusnack and the writer for evaluating process skills in science. It rates students on a scale that is linked to observable student behaviors. The teacher uses it in checklist fashion. It looks at he complete problem-solving process rather than at just evaluating students on the correctness of their answers.

The criteria were developed in four categories:

1. problem-solving stages
2. strategies
3. solution, and
4. participation.

These categories can be changed to fit various teaching situations. All of them can be used at once, or they can be used individually. In addition, they are not exhaustive, and educators could easily develop other categories.

In using the scheme for observable student behavior, a teacher could circulate around the room and record data as students do their work. Also, it is sufficiently flexible that teachers could use it to evaluate students' written solutions.

Teachers who have used it have reported that it is quick and accurate. In addition, these teachers have reported an increased understanding of problem solving because they had been forced to look at the complexity of student behavior in solving these problems. They have found there is more to problem solving than just finding solutions.

In conclusion, complex problems require more involvement from teachers so that students will receive fair and just evaluation. The Impressionistic Scoring Criteria scheme can assist teachers with evaluating student solutions to complex problems.

Hank Boer is Coordinator of Mathematics and Science, Lethbridge School District No. 51. Currently, Mr. Boer is president of the South West Regional, MCATA.

EDITORS' NOTE: Readers may be interested to compare the Impressionistic Scoring Criteria scheme presented in this article with the Mathematics Problem-Solving Behavior Scale distributed by Alberta Education. This rating scale is provided on page 27.

Impressionistic Scoring Criteria

	Problem-Solving Steps	Strategies (?)	Solution	Participation
5	made an effort using four of the steps	has considerable depth and expertise in using the strategy	- complete - correct solution - used more than one strategy - manipulated the problem and solution	- involves oneself quickly
4	made an effort using three of the steps	has expertise in using the strategy	- complete - correct solution - used more than one strategy or manipulated the problem	- needs a start
3	made an effort using two of the steps	has some expertise in using the strategy	- somewhat complete - correct solution	- needs periodic assistance
2	made an effort using one of the steps	needs to develop expertise in the strategy	- incomplete - has errors	- needs constant attention
1	Excused Absence	Excused Absence	Excused Absence	Excused Absence
	COMMENTS:	COMMENTS:	COMMENTS:	COMMENTS:

Mathematics Problem-Solving Behavior Rating Scale

1. *CIRCLE the number indicating your rating for each of the four indicators, based on the defined observable behaviors.*

2. *Add the circled numbers to determine the score.*

Understanding the Problem

- Asks the questions to clarify the problem
- States relevant facts in the problem
- Perceives implied relationships

Observed to a High Degree				Not Exhibited
5	4	3	2	1

Devising **a Plan**

- Summarizes data by making a table, graph, or diagram
- Develops approaches (looks for patterns, works backwards, makes predictions and verifies, decomposes problem into parts)
- Recalls related problems previously solved
- Estimates solution

Observed to a High Degree				Not Exhibited
5	4	3	2	1

Carrying Out the **Plan**

- Uses a table or diagram to arrive at solution
- Applies a formula
- Performs computation required for solution
- Decides where to begin
- Switches strategy when it is no longer applicable

Observed to a High Degree				Not Exhibited
5	4	3	2	1

Looking Back

- Describes strategy used in solving the problem
- Verifies that solution satisfies conditions of the problem
- Looks for alternative ways to solve the problem
- Creates applications or related story problems

Observed to a High Degree				Not Exhibited
5	4	3	2	1

SCORE: ________

Investigating Number Relationships

Oscar Schaaf
University of Oregon

$$1 + 2 = 3$$
$$4 + 5 + 6 = 7 + 8$$
$$9 + 10 + 11 + 12 = 13 + 14 + 15$$
$$16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$$

1. Study the number patterns in the above sequence of equations.

   (a) Write the next two equations.

   (b) What would be the first number in the tenth row? The last number?

   (c) What would be the first number in the nth row? The last number?

2. In the second row, the sum of the consecutive numbers on each side of the equal sign is 15.

   (a) Write another set of consecutive numbers whose sum is 15.

   (b) Are there other such sets whose sum is 15? Explain.

3. The third row shows two sets of consecutive numbers with a sum of 42; the fourth row shows two sets with a sum of 90.

   (a) Is it possible for every integer less than 35 to be a sum of two or more consecutive positive integers? Make a systematic list.

   (b) Which numbers, if any, cannot be sums of consecutive numbers?

   (c) Which numbers can be sums of two consecutive positive integers? Give an important characteristic or property of these numbers. Write an algebraic expression which describes these numbers.

   (d) Which numbers can be the sums of three consecutive positive integers? Give an important property of these numbers. Write an algebraic expression that describes these numbers.

4. Write two questions involving consecutive numbers for other class members to answer. Be certain you can answer your own questions!

Many mathematics educators consider the study of relations and functions as the heart of algebra. Students should be expected to search for relationships in their physical environment. In mathematics itself, they should express these relationships using word descriptions, tables, graphs, equations, and other algebraic expressions, and use these expressions in solving problems and making predictions. This lesson on investigating number relationships can be done in an algebra class early in the year while students are becoming familiar with algebraic expressions and variables.

Comments

Have students study the lesson sheet on their own for several minutes before allowing them to work together in groups of three or four. Encourage students in the group to answer their own questions before seeking your help. Questions 1(c), 2(b), 3(c), and 3(d) should be discussed during the lesson's culminating session.

ANSWERS:

1. (a) 25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35.
   36 + 37 + 38 + 39 + 40 + 41 + 42 = 43 + 44 + 45 + 46 + 47 + 48.
   (b) 100, 120.
   (c) n, (n + 1) -1 or n + 2n.
   Ask pupils to discuss the method they used to get the last number in the nth row. Several methods will likely be mentioned.

2. (a) 1 + 2 + 3 + 4 + 5.
   (b) Not with positive integers. There are several more if students choose to use negative integers and zero; for example, -3 + -2 + -1 + 0 + 1 + 2 + 3 + 4 + 5 + 6. There are only three solutions - one with two consecutive numbers, one with three, and one with five. The last sum with six consecutive positive integers is 21.

3. (a) No.
   (b) 1, 2, 4, 8, 16, and 32.
   (c) 3, 5, 7, 9, 11, 13, 15 . . . 35. Odd numbers from 3 through 35.
   2n + 1 where n is a positive integer.
   (d) 6, 9, 12 . . . 33. Numbers divisible by 3 from 6 through 33.
   3n + 3 where n is a positive integer.

4. Answers will vary. These questions should be used as a follow-up lesson.

Oscar Schaaf is Professor Emeritus, College of Education, University of Oregon. He has been a speaker at many NCTM meetings, including those held in Alberta. Dr. Schaaf was director of the Lane County Mathematics Project, which focused on problem solving. He suggests this problem-solving lesson is suitable for use at the junior and senior high school level.

EDITORS' NOTE: Students are encouraged to submit questions to the editors at the following address:

The Editors
Delta-K
c/o 2510 - 22 Avenue S
Lethbridge, Alberta
T1K 1J5

STUDENT PROBLEM CORNER

Students are encouraged to examine the problem presented below. Why does it work? Send your explanation to:

The Editors
Delta-K
c/o 2510 - 22 Avenue S
Lethbridge, Alberta
T1K 1J5

The names of students who successfully explain the trick will be published.

Lightning Addition

Ian D. Beattie
University of British Columbia

For many years, magicians have used mathematics to entertain and puzzle their audiences with amazing feats of mind reading, prediction, and rapid calculation. Most of these feats can be explained simply and learned quickly. Those who know the "trick" can derive great satisfaction from mystifying their friends, or, for teachers, their pupils!

The Trick

1. Ask a friend to write down any two one- or two-digit numbers, one below the other.

2. Tell him to add these two numbers and write the result below the other two.

3. Now, ask him to add this new number to the one above it to get a fourth number.

4. Ask him to keep doing this until he has 10 numbers written in a column.

5. Make sure that, while he is doing this, you cannot see the numbers. When he has obtained the 10 numbers, ask to see them.

6. You then quickly write in the total of the 10 numbers.

7. How? The total of the numbers is found by multiplying the fourth last number by 11.

This works no matter what numbers your friend chooses. The request that he use one- or two-digit numbers is just to keep his addition simple. It is helpful to do this on a blackboard so that classmates can check your friend's addition as he goes along.

An Example

Let's say the numbers 21 and 32 are chosen. Then the 10 numbers are: 21, 32, 53, 85, 138, 223, 361, 584, 945, 1,529. Their sum is 3,971. The fourth last number is 361. Eleven times 361 is 3971.

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What Is the School?

Who Is the Pupil?
A child of God, not a tool of the state.

What Is the Teacher?
A guide, not a guard.

What Is the Faculty?
A community of scholars, not a union of mechanics.

What Is a Principal?
A master of teaching, not a master of teachers.

What Is Learning?
A journey, not a destination.

What Is a Discovery?
Questioning the answers, not answering the question.

What Is the Process?
Discovering the ideas, not covering the content.

What Is the Goal?
Opened minds, not closed issues.

What Is the Test?
Being and becoming, not remembering and reviewing.

What Is the School?
Whatever we choose to make it.

- Source Unknown

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