Fraction and Decimal Numeration Suggestions for Curriculum Revision

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During the last five years, significant studies have produced convincing evidence that we are doing a rather poor job of connecting conceptual meanings or understandings with the symbolization of fractions and decimals in the fourth through seventh grades. Peck and Jencks (1981) found that out of 20 sixth graders, nine could draw pictures of simple fractions like 1/4, 1/3, 1/5, and 3/4. Only five of the nine could use these ideas to compare fractions correctly, and only two of the five could use a conceptual approach to add fractions such as 2/3 and 1/4. Peck and Jencks maintain that this small sample is representative of hundreds of children they interviewed.

In another study, less than 30 percent of entering freshman at City University of New York could correctly select the smallest of a list of five decimal numbers, while much higher percentages were able to compute accurately with decimals (Grossman, 1983). The Second National Assessment of Educational Progress reported that less than 30 percent of 13-year-old children selected the correct decimal equivalent for simple fractions such as 5/8, and 38 percent selected .5 as an equivalent to 1/5 (Carpenter, et al., 1981). These and many other researchers and reviewers have concluded that significantly more time must be spent on conceptual development of both fractions and decimals.

Hiebert points out in an excellent review of current research (1984) that a major flaw in children's mathematical learning is a failure to make connections between understanding and symbols. According to Hiebert, connections are possible at three sites: meaning of symbols, understanding of procedural rules, and consideration of the reasonableness of solutions (a sort of real world understanding linked to symbolic manipulation). The first and third sites have received the least amount of attention in curriculum development. He contends that "if students can be assisted in developing rich meanings for the symbols and in recognizing that solutions to written problems should make sense, their struggle to link form and understanding, to learn mathematics in a meaningful way, would be greatly enhanced."

Perhaps the most detailed and extensive investigation into what children know about fractions and how they learn fraction concepts is being done in a series of studies by Post, Wachsmuth, Lesh, and Behr (1985). Their teaching experiments and interviews with children have revealed that the concept of fraction is a complex and slowly evolved construct. Their work, however, is beginning to lead the way for curriculum reform in the area of fraction concept development. Many of the ideas presented as suggestions in this paper are derived from their studies.

Less research seems to have been done on the use of models such as place value materials to develop decimal concepts or on ways to help children connect fraction and decimal concepts. The suggestions in the activities section of this paper are based on extension of the fraction work by Post, et al., and on my own experience with fifth grade children earlier this year.

The current curriculum simply does not permit teachers the time to do required concept development.

The overriding suggestion is that curriculum revisions must be made to create a significant period of time for students in the fourth through sixth grades to work with a variety of both fraction models and place value models. This would allow them to develop not only the concepts of rational numbers, but also the connections that these concepts have with the symbolism of fractions and decimals. This time, which seems essential from both a developmental standpoint and from results of empirical research, can be created by postponing all rules for fraction and decimal symbol manipulation until at least the latter half of the sixth grade. The current curriculum simply does not permit teachers the time to do required concept development.

In a fifth grade class in which I was working this year, students at one point had not begun the study of decimals and were well into multiplication and division algorithms for fractions. Yet only one of the seven children I interviewed had a firm concept of fraction. None of those I talked with had even a vague idea of the meaning of decimal numerals, and none could give any conceptual explanation of the fraction algorithms with which they were becoming quite adept. About three weeks later, the study of decimals had progressed to the multiplication algorithm. However, in a class discussion which I conducted, there was virtually no indication that students understood decimals as representations of simple fractions.

To change this relatively absurd situation, the curriculum must shift from an emphasis on symbolic rules, which appear meaningless to children. Also, activities must be developed so that teachers will know what to do with the various models for fractions and decimals.

The activities suggested in the following pages are offered as ideas. The objective of these activities is to develop an understanding of rational number concepts and to connect these ideas to the symbolism of fractions and decimals. Special attention is given to the effort of relating the symbolism of fractions to that of decimals. While many of the activities are based on recent research, many remain to be tested and further developed. The intent is to get more physical models and activities with models into the classroom. Whenever possible, connection of an oral form of the concept is connected with the models prior to symbolization. Symbolic rules for equivalent fractions are not discussed, although some activities guide students to develop these ideas in an intuitive manner.

Another focus of these activities is to utilize a variety of models whenever appropriate. Evidence suggests that a well-formed mathematical concept becomes model free. Dependence on a single model has a tendency to leave a concept tied inflexibly to the model. One observes this quite readily in children who seem to have their concept of fraction firmly tied to circular regions.

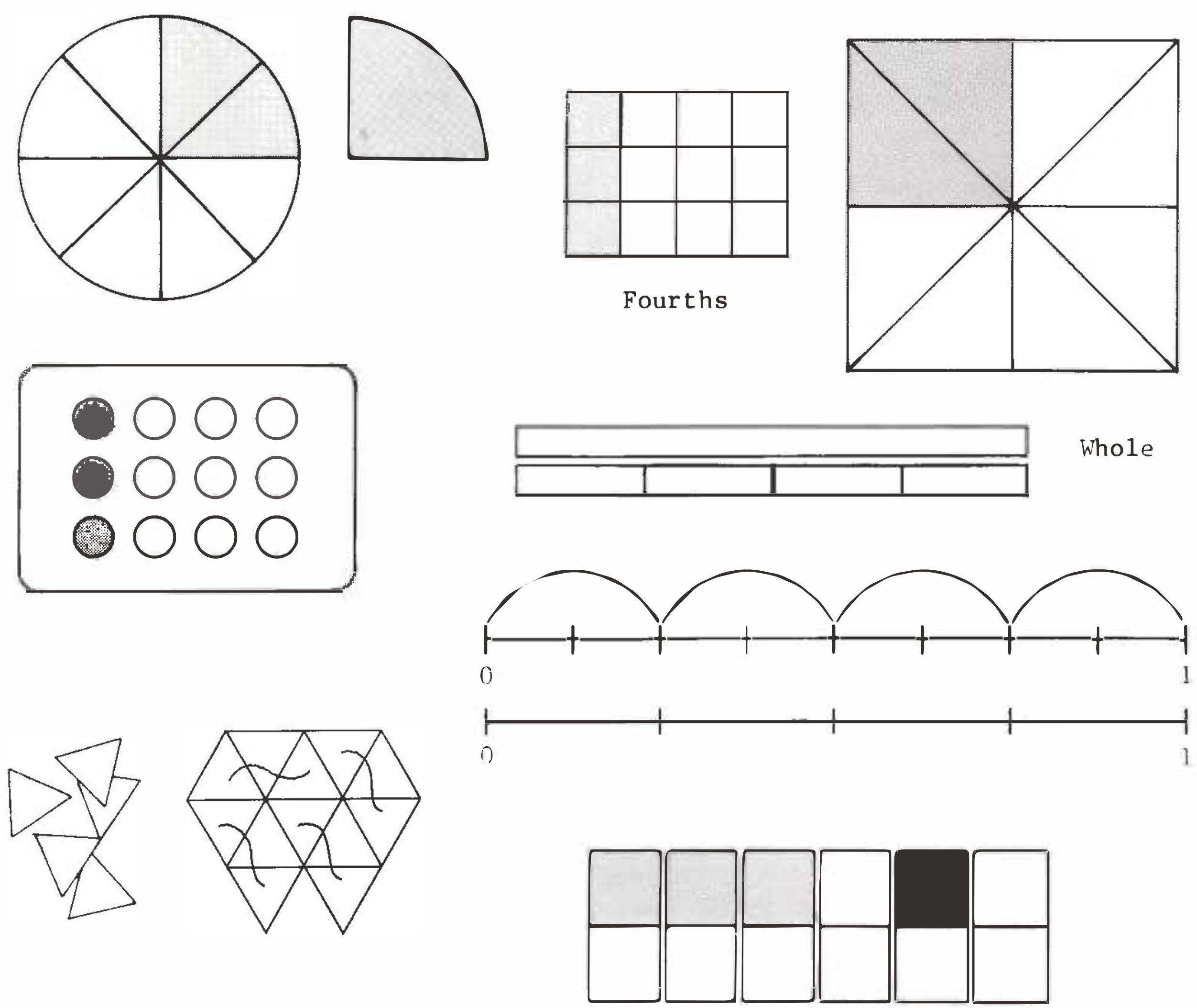
Suggested Activities

OBJECTIVE:
To make meaningful transformations between models and symbols.

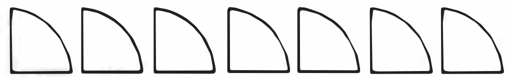
1. Develop fraction "words" (halves, thirds, fourths, fifths, and so on) in connection with assorted models. For each fraction word, there are two factors that are essential: the correct number of parts must be presented, and all parts must be the same size.

   Introduce this notion with as many models as possible:

   - circular pies
   - squares and rectangles subdivided into smaller parts for shading
   - sets of two-color or two-sided counters
   - colored fraction strips or Cuisenaire rods
   - number lines, subdivided into various subunits
   - regions made from posterboard squares or equilateral triangles

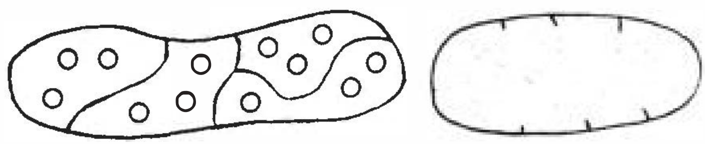


2. Orally, count unit fractions with each model.



"One-fourth, two-fourths . . . seven-fourths."

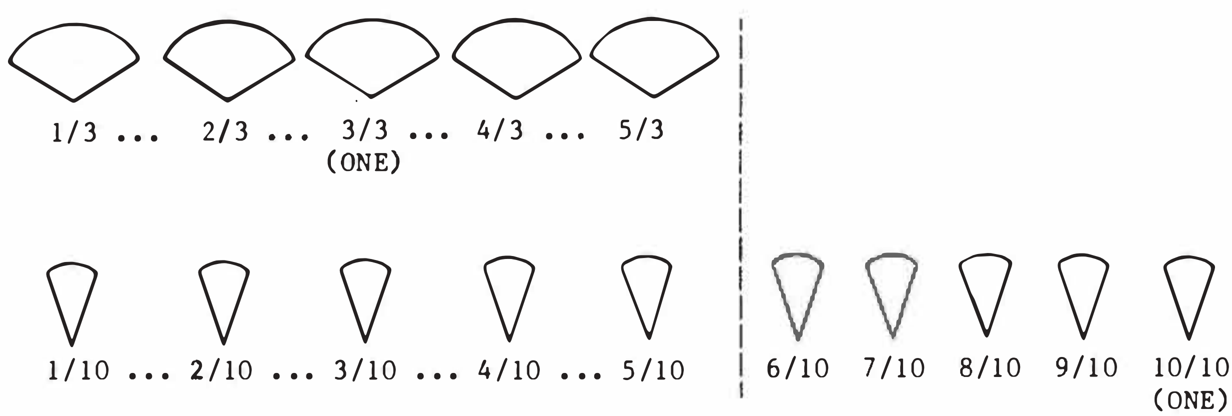
"One-third, two-thirds . . . five-thirds."



"One-fourth, two-fourths . . . six-fourths."

3. When children can identify fractional words and can count them accurately, introduce the written form for fractions as the counting is done. Use as many models as possible.

It is significant to count several different fractional parts in a parallel fashion. This helps to develop the difficult inverse relationship between number of parts to make a whole, and size of the parts. In comparison of 3/4 with 5/12, for example, children need to see that while five is more parts than three, the fourths are much larger. This type of reasoning is slow to develop and requires a significant amount of varied experience.



4. Use models to go from either a given unit or whole to a given fraction and vice versa. A progressively difficult series of five question types is suggested.

(a) **Given a unit fraction, find the whole.**

EXAMPLE: If the red (2) strip is 1/5, what strip is the whole?
Answer: Orange (10).

EXAMPLE: If three counters makes 1/5, how many beans in a whole set?
Answer: 15.

(b) **Given a unit fraction, find a nonunit fraction.**

EXAMPLE: If the light green (3) strip is 1/4, what strip is 3/4?
Answer: Blue (9).

EXAMPLE: If four counters are 1/2 of a set, how many counters in 3/2 of a set?
Answer: 12.

(c) **Given a nonunit fraction, find the unit fraction.**

EXAMPLE: If the dark green (6) strip is 3/4, what strip is 1/4?
Answer: Red (2).

EXAMPLE: If 12 counters are 3/4 of a set, how many counters in 1/4 of a set?
Answer: 4.

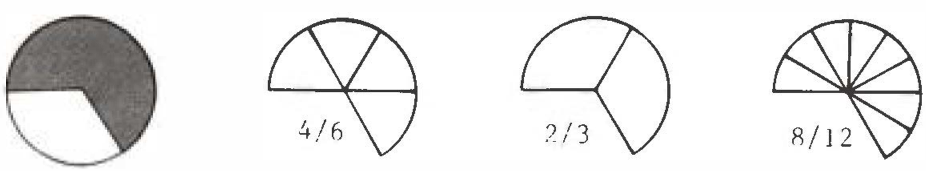
(d) **Given a nonunit fraction, find the whole.**

(e) **Given the whole, find a nonunit fraction.**

NOTE: The above questions cannot be done with pie pieces, nor any model in which the whole is a fixed size.

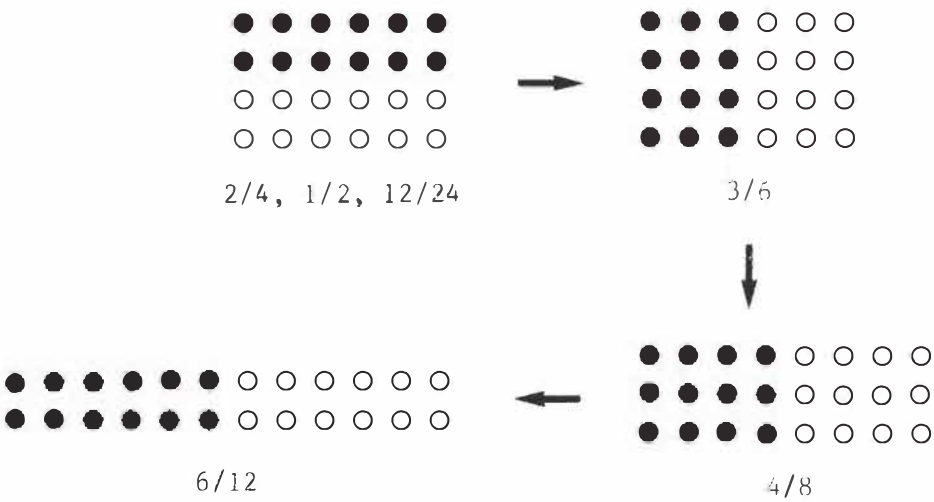
OBJECTIVE:
Develop the basic concept of equivalence. That is, multiple fraction names can be used to represent the same amount.

1. Provide drawings or cutouts of a fractional part of a region, and have children find fraction names for the region by covering it in as many ways as possible using pieces of the same size. Children should write all results.



Later, extend the exercise to include pieces not in the set of manipulatives. For example, "What if all of our 1/8 pieces were cut in half. What could we call this part?"

2. Provide sets of counters or drawings of counters in two different colors. What fraction is each color? By rearranging the pieces, children can find different fraction names.



OBJECTIVE:
To make comparisons between two or more fractions.

1. Find a way to show two fractions of the same unit or whole at the same time. (This activity is only useful with strips or counters, or with other models in which the size of the whole can vary.

2. Fraction pairs should be given to children in written form. Children use models to determine which is greater. Many children will "know" (either correctly or incorrectly) which is greater without recourse to models. These children should use models to confirm or check their thoughts. Fraction pairs can be given in three different categories:

   (a) like denominators and unlike numerators
   (b) like numerators and unlike denominators
   (c) both numerator and denominator different

   Select fraction pairs so that it is possible within the available model to form equivalent fractions with like denominators.

The last two activities provide experience with all of the concepts that are necessary for creating a general equivalent fraction concept. However, no algorithm nor symbolic rule is developed (such as multiplying or dividing top and bottom number by a constant), nor should such a rule be introduced. Experience indicates that as soon as such a symbolic rule exists, children will mindlessly use the rule and will ignore the conceptual referent no matter how poorly their ideas are formed. The next activity will challenge children even further toward development of their own symbolic rule for equivalent fractions. No rule should be provided.

OBJECTIVE:
To investigate in a symbolic mode the concept of equivalent fractions.

Provide models for children to use in completing equations of the type shown. Notice that there are four different types of exercises. Each type should be worked on within the context of several different models.

$\frac{6}{8} = \frac{\square}{4}$ $\frac{2}{3} = \frac{\square}{9}$ $\frac{3}{5} = \frac{9}{\square}$ $\frac{8}{12} = \frac{2}{\square}$

OBJECTIVE:
To help with the connection between fraction and decimal notation.

The models described here permit fractions with denominators of 10 and 100 to be shown easily. Some models even show thousandths.

- Circular pie pieces which include tenths and fifths. (If not used earlier, they should be included prior to work with decimals, since children's connections with fractional parts of pies is very strong.)
- Larger interlocking pie pieces marked around the edge in tenths and hundredths.
- Squares drawn on paper cut into tenths, hundredths, and thousandths. (Models are given on page 43.)
- Metre sticks with decimetre strips and centimetre squares of tagboard.
- Place value pieces, such as centimetre strips and squares.

These materials can all be used as fraction models, and many of the activities described earlier should be done with these. It is important to use some of the same models for fractions and decimals so that connections between the two can be developed.

OBJECTIVE:
To extend previously learned fraction concepts to fractions with denominators of 10, 100, and 1000.

1. Using all of the models above, go from fraction to model and from model to fraction. That is, given a model, name the fraction and vice versa.

2. Emphasize fraction equivalences between tenths, hundredths, and thousandths. Show by means of the models that 23/100 is the same as 2/10 and 3/100. (Do not use any reference to common denominators or symbolic addition rules. Use only models.)

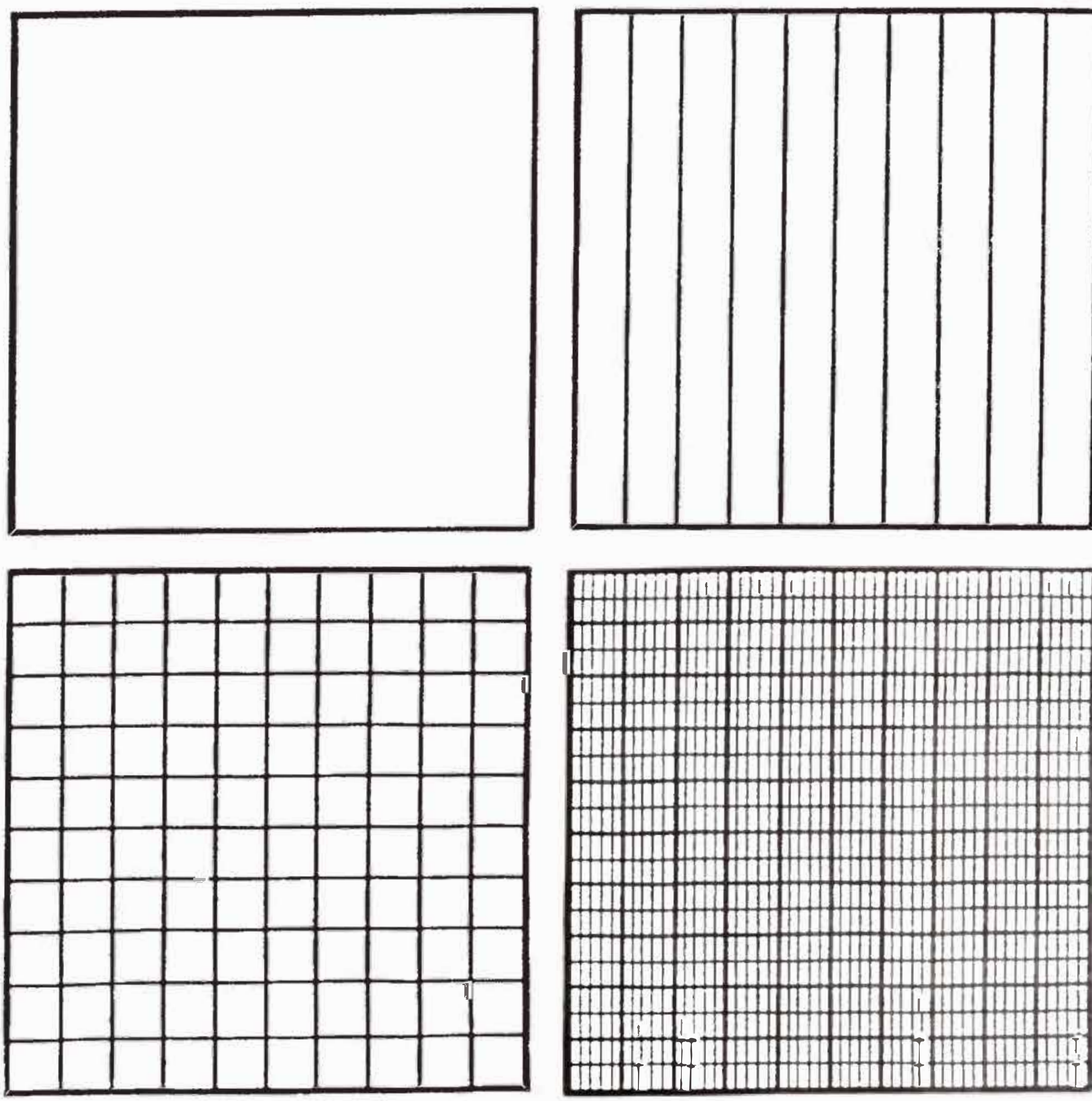
   The following examples are to be done with children first in an oral mode using models.

27/100 = □/10 and □/100 and 4/10 and 3/100 = □/100

6/10 = □/100 and 40/100 = □/10

Use similar examples with thousandths when appropriate models are available. Children apparently do not automatically extend these ideas to thousandths without working with models.

Decimal Squares



OBJECTIVE:
To develop equivalences between simple and familiar fractions and fractions with 10, 100, and 1000 as denominators.

1. Using the decimal fraction models, have children find equivalent names for these simple fractions:
   - halves, fourths, fifths (requires only tenths and hundredths)
   - eighths (requires thousandths)
   - thirds (requires a discussion of continuous subdivision by tens)

2. Provide common and familiar models (counters, pies, and strips) for fractions in the above families, and ask students to show the same amount using one of the decimal models.

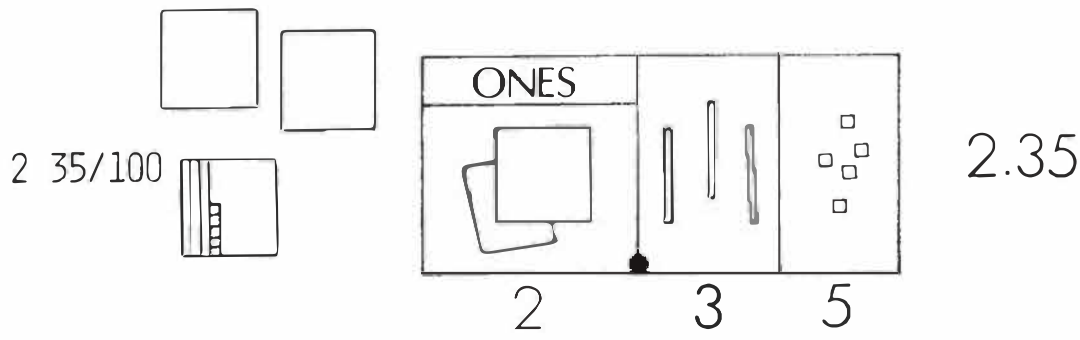
OBJECTIVE:
To introduce decimal symbolism.

The following activities are based on children's familiarity with the whole number place value system. In particular, children should know that 10 of anything in one position is the same as one in the position to the left. Ten ones makes one 10, 10 tens makes a hundred, and so on. Likewise, one in any position can be exchanged for 10 in the position to the right.

With this understanding, decimals may be introduced as another convention for writing fractions. That is, we can show children a way to write simple fractions using "regular" numbers. Decimal numeration in this scheme comes after an understanding of fractions and even fractions with "decimal" denominators (10, 100, 1000 . . .). The reverse approach is essentially to assume that children can somehow extend the whole number decimal concept to place values less than one. Later, we try to explain that these are really just fractions. The latter way seems developmentally awkward, if not backward.

1. Using centimetre strips and squares, agree that the 10 by 10 square will represent one. Discuss briefly how a ten would be made of 10 of these squares (a 10 by 100 strip), and a hundred made of 100 of these squares (a 100 by 100 square). Discuss where each would go on a place value chart. If you wanted to place the strips (that the children already know to be 1/10), where would they go? Clearly, to the right of the ones (by pattern or progression). Now ask students to show, using their place value pieces, a number such as 3 7/10. Place the pieces on the place value chart, and write down what the chart shows. Enter the decimal point, because without the decimal, 3 7/10 looks like 37. The decimal is needed to indicate which digit is the ones digit.

   With this introduction, have children first model with the strips and squares various fractions and mixed numbers that are tenths and hundredths. The pieces are then moved to a place value chart and the decimal number written.



Model as a fraction. Put on place value chart. Write decimal.

2. Use other fraction models subdivided in tenths and hundredths (from the list provided earlier), and have children translate fractions illustrated with these models to decimal numeration. To do this, they first show the same fraction using strips and squares, and then move these pieces to a place value chart.

3. Give children fractions with the familiar fraction models (pies, fraction strips, sets of counters), and have them show the same fraction with a decimal fraction model.

4. Starting with nondecimal fractions from the list of familiar fractions in the previous set of activities (halves, fourths, fifths, eighths, and thirds), have students translate these to decimal numerals. This is done by modeling the fraction with strips and squares, translating it to a place value chart, and then writing the decimal numeral.

5. Give children decimal numerals, and have them show these with a decimal fraction model (the reverse of activity #2 above).

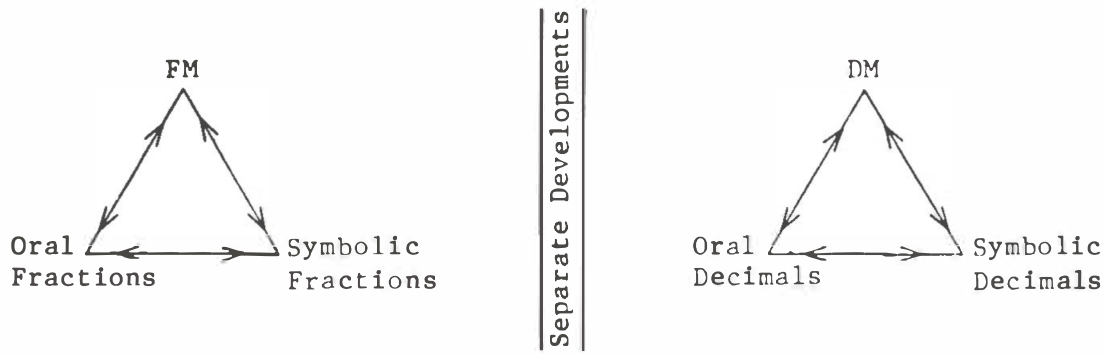
6. Give children decimal numerals, and have them show these using regular fraction models (the reverse of activity #3 above).

7. Have children move back and forth between decimal and fraction equivalents using assorted models to verify their reasoning.

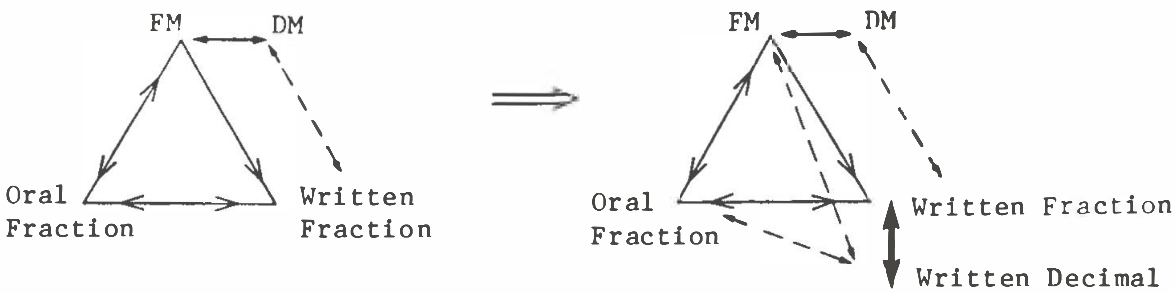
8. Use decimal models to illustrate decimals that are not "nice" fractions, but which are close to more familiar fractions. For example, which of these fractions is .23 close to: 1/2, 1/3, 1/4, or 1/5? Similar approximation exercises should be done with a wide assortment of one, two, and three-place decimal numbers.

Discussion

The traditional curriculum has kept the subjects of fractions and decimals fairly separated. Even in the best developments, oral and written names for fractions are connected to a very special set of fraction models (FM). Similarly, decimal numeration in both oral and written form is related to a very different set of models for decimals (DM). Later, children are taught decimal-fraction equivalents. This is frequently done through an unfamiliar meaning of fractions, division. Bottom numbers are divided into top numbers, and decimal equivalents somewhat magically appear. The meanings that children give to decimals in this approach is based largely on their ability to extend the whole number system and understand the words tenths, hundredths, and thousandths. Evidence suggests that this understanding of decimals is weak, and that there is very little understanding of the relationships between fractions and decimals.



The objective proposed throughout the sequence of introductory activities for decimals is essentially to firmly develop fraction concepts with standard fraction models. These concepts must be developed to the point that the notion of fraction becomes model free. That is, children will demonstrate an ability to translate fraction concepts from one model to another, and to use any model to illustrate meaning behind symbolic activities. When this is done, models that readily illustrate tenths, hundredths . . . (DM) are introduced for familiarity within the already established concept scheme. Next, the symbolic scheme for decimal numeration is introduced and connected with the concepts via the already familiar model. Decimals are, in this development, simply a new way of writing about ideas children already understand.



Notice that oral fractions and oral decimals are essentially indistinguishable. A new oral language does not need to be developed.

Clearly, this is not a complete development of decimals. Comparison of decimals is a conceptual skill that is important, but was not addressed here specifically. Nor was a general conversion scheme between fractions and decimals (if that is, in fact, desired). The ideas here are presented for trial introduction and discussion as this complex area of numeration is researched further in the classroom.

Dr. John Van de Walle is a professor of education at Virginia Commonwealth University, Richmond, Virginia. Dr. Van de Walle's article was presented to the NCTM annual meeting in San Antonio, Texas, in April 1985.

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