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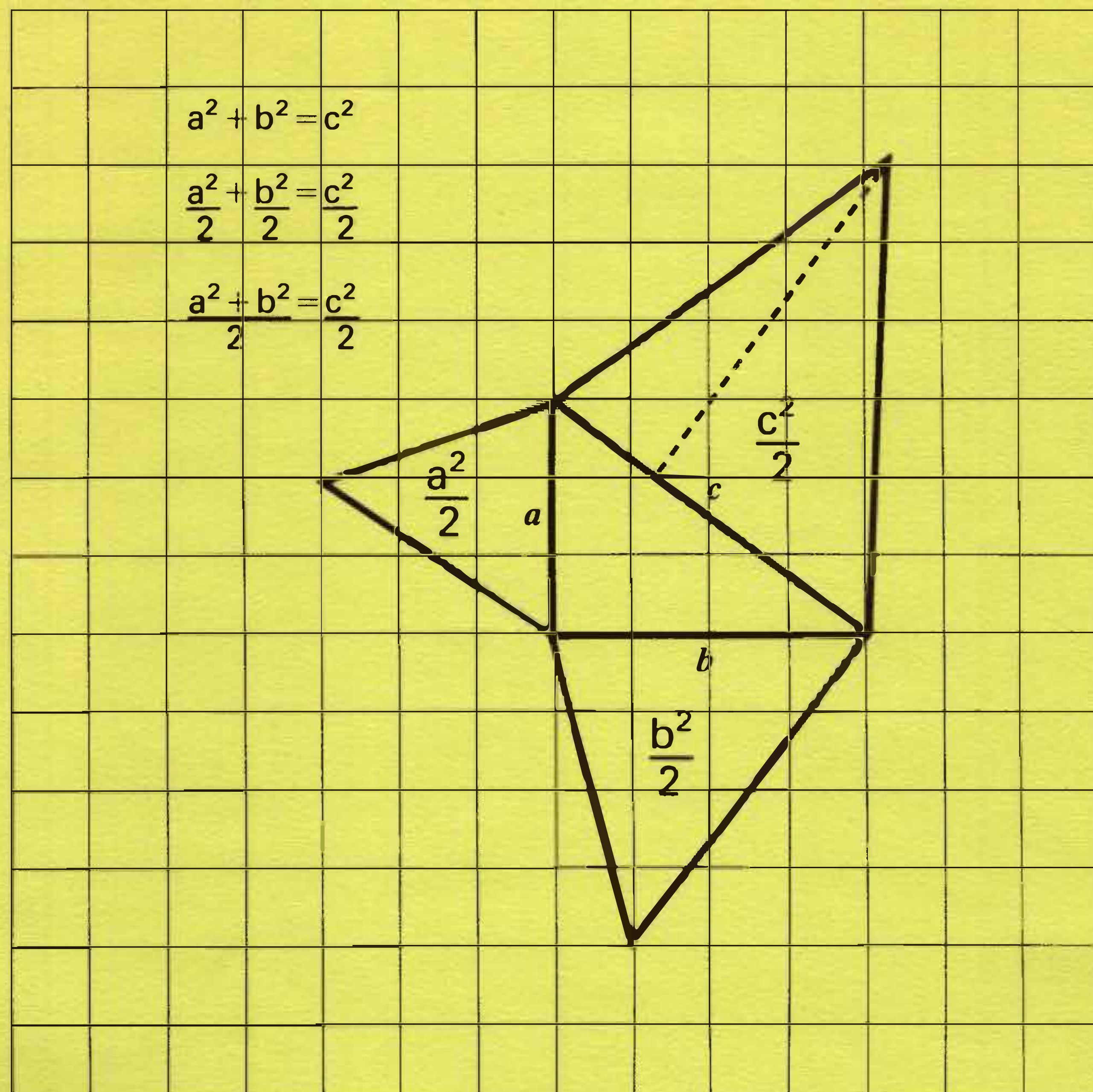

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EDPRESS

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EDITORIAL

Direction

The editor remains committed to the three objectives for *Delta-K* stated in the last issue. Were the articles in the October 1985 issue useful to classroom teachers? Student solutions to the problems posed have not been received. Apparently, this objective may have to be reexamined. The increasing number of Alberta mathematics educators whose articles appear in this issue is evidence that the third objective is realized.

Comments

Dick Holmes, a friend and professional colleague of Marshall Bye, has prepared a tribute to the first recipient of the Mathematics Educator of the Year Award. Marshall was presented with the award at the MCATA Annual Conference held in Red Deer in October 1985.

William Cooke discusses three types of reluctant learners found in mathematics classrooms, and suggests approaches for therapeutic intervention. The results of a survey on the status of implementing problem solving into the curriculum in southern Alberta schools is presented by **Ron Cammaert**. **A. H. Skolrood** and **M. Jo Maas** draw parallels between problem solving in mathematics and the inquiry (problem solving) approach in social studies. **D. Sawada** probes the thought processes of a young problem solver, Marvin. All mathematics teachers who have a "Marvin" in their classroom would consider themselves fortunate. (Or would they?) **Don Kapoor** examines reading skills necessary for the successful teaching and learning of mathematics. Theory and practice are interrelated.

The next group of articles present ideas teachers may wish to use. **Joe Krywolt** describes how he has organized his classroom for instruction in problem solving. **M. Jo Maas** offers suggestions for incorporating drill and practice with problem solving. The extensions she provides for a basic problem are interesting and useful. **John Van de Walle** reviews pertinent literature and offers suggestions that could improve instruction in fractions and decimal numerals. Three solutions to a calculus problem are presented by **D. Duncan** and **B. Litwiller**.

The contributors to the Student Problem Corner are **L. Esposito** and **John Percevault**. Try the problems with your students, and encourage students to forward their solutions. The next issue of *Delta-K* will focus on "Technology in Mathematics Teaching."

Change in Editorship

Dr. Arthur Jorgensen has accepted a position in Jamaica for 18 months, commencing January 1986, to work on developing an elementary mathematics curriculum. Art will be missed as co-editor of your journal. Good luck, Art!

- John Percevault

A Tribute to Marshall Bye

Dick Holmes
University of Calgary

Marshall Bye

Marshall Bye, the first recipient of the Mathematics Educator of the Year Award, was born and raised in the Peace River region of Alberta. He worked on the farm and in the mill camps of British Columbia to help finance his education.

Marshall has an outstanding background of academic achievement. He earned his B.Ed. degree from the University of Alberta, and in 1962, received a Shell Merit Fellowship in mathematics to Stanford University. In 1968, he was awarded a National Science Foundation Scholarship to study mathematics at Wayne State University in Detroit, were he earned an M.Ed. degree. In 1972, while on sabbatical leave from the Calgary Board of Education, he studied in London, England, at the University of London Institute of Education and was named as Associate of the University of London.

The beginnings of an illustrious teaching career began in Peace River, where he taught elementary and junior high school.

In 1952, Marshall left the Peace River country and moved to southern Alberta. He taught in the County of Wheatland and there, in 1959, assumed his first principalship. He remained with the County of Wheatland until 1964, when he moved to Bowness High School in Calgary. He served as mathematics department head at Bowness High until 1967, when he became a member of the central office curriculum staff as secondary mathematics consultant. One year later, he was named mathematics supervisor, a position he held until his retirement in July 1983.

Marshall's interest in, and dedication to, mathematics is reflected in the many responsibilities he has assumed in this area. He was president of MCATA in 1966 and most recently served on the executive of the National Council of Supervisors of Mathematics. He has presented sessions at conferences and conventions all over North America, is a successful author of mathematics textbooks, has taught mathematics methods courses at the university level in Calgary and Montana, and has been a member of many provincial mathematics committees and research teams.

Now that he has retired (at least, from regular duties with the Calgary Board of Education), he hopes to find time to pursue some of his interests. He likes photography, loves camping and hiking, and takes many small vacations in his motor home with his lovely wife, Evelyn. Marshall dotes on his grandchildren, the offspring of his daughters Carole and Joanne.

We wish him well in his retirement and congratulate him on his most recent and well-deserved honor.

Dick Holmes is a faculty associate of the University of Calgary.

Seven-Link Chain Problem

Basic Problem

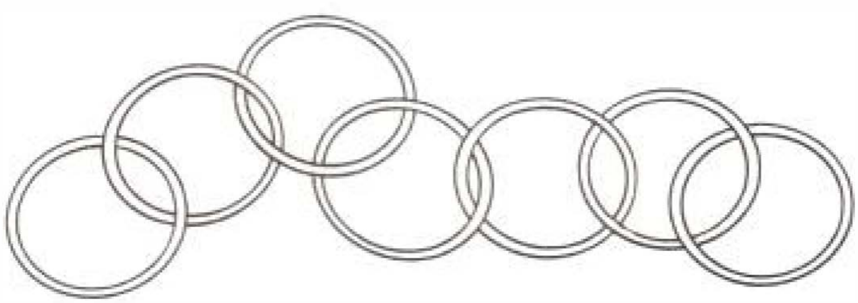
During a revolution, a prince escapes with an expensive seven-link gold chain. He finds a haven at the home of a trusted peasant, and the prince and peasant agree to the following conditions:

1. One link of the chain will provide one day of protection.

2. A link must be given at the start of each day. Stated another way, there will be no prepayments or arrears.

3. The prince and peasant agree that the chain should be cut in such a manner that a minimum number of links are cut.

How many links need to be cut so that the prince may pay the peasant one link per day with no prepayment or arrears occurring?



Variations of this problem are given on pages 14 and 35. The problems require the same basic analytical skills, but different questions are presented in each.

Mathematics and the Reluctant Learner

William A. Cooke
University of Lethbridge

The following data will examine the problem of motivation as it relates to reluctant learners, particularly those elementary school children who have developed serious emotional blocks to the learning of mathematics. The degree of success of teachers depends directly upon the degree to which they understand *why* such motivational blocks occur. The children discussed in this paper are for all intents and purposes mentally capable of learning, are of perfectly adequate intelligence, and there is no indication of learning disabilities of a "specific" nature, no sensory deprivation, neurological impairment, nor any identifiable cerebral dysfunction.

Motivational Blocks: Three Types

Mathematics blocks are more frequently described in terms of adjustment problems than in terms of learning problems (Beilin). The difficulty generally stems from the affective domain, rather than resulting from some primary cognitive deficit. Numerous descriptors have been ascribed to children experiencing this type of blocking. They have been described as anxious and defensive (Hewitt), as aggressive and contentious (Lilly), and as socialized deviants (Crow). These sets of descriptors differentiate three of the more significant causes of prevalent mathematics blocks.

The Anxious Defensive Child

This child might be clssified as emotionally disturbed. The block appears to be psychopathological in nature because he has experienced repeated failure in mathematics. He has not satisfied his need to successfully perform even simple arithmetic tasks. Assignments cause him to feel fear, nervousness, and tenseness. Anxiety and defensiveness increase each time a mathematics stimulus is introduced. Mathematics, although not necessarily the exclusive cause for his neurotic abreactions, seems to have become the recipient of the child's fixated fears. His mathematics block takes on the form of a phobic reaction (Keogh, Erickson, Long).

The Aggressive Contentious Child

The second child might be identified as having a character problem. This child is one who, from early childhood, has been spoiled, pampered, and overindulged. He has not been provided with the necessary guidelines for developing responsible behavior. Somehow he lacks that "built-in" regulator of behavior most children have developed (Rosenberg, McCord). He has not successfully internalized usual moral and ethical precepts upheld in the *normal* family unit. Instead, he has come to think of himself as the centre of the universe and is impulsive, demanding, selfish, and ill-tempered. He has never been encouraged to practice need-gratification-delay. He is a completely self-centred, hedonistic little outlaw, who spends his time and energy seeking after that which is pleasurable and avoiding that which is in the least way distasteful, mathematics not being the least of these things. His mathematics block takes on the form of conscious, deliberate avoidance (Szurek, Werry).

The Socialized Deviant Child

The third child is categorized as being socially disadvantaged. His emotional block is due primarily to social alienation. He is the child who comes from a background which deviates considerably from the norm - socially, culturally, morally, and ethically (Passow). His deviation from the norm is so marked that he is impervious to what is being taught most of the time. With repetitive regularity, he misinterprets and misunderstands. There are just too many linguistic gaps. His mathematics block results primarily from communication problems (White, Cheyney, Nazzaro, Fuchigami).

Intervention Procedures

With this only too brief overview of the three identified syndromes, what of a practical nature can the mathematics teacher do to alleviate these learning blocks and facilitate effective locomotion?

The first and perhaps most important suggestion is that any therapeutic approach must be implemented with the child's *particular* learning block in mind. Thus, motivational remediation is *differential* in nature. What works beautifully for Child A will not meet the needs of Child B, and furthermore, what works so well with Child B will not succeed with Child C, and so on.

The Anxious Defensive Child

With reference to the anxious defensive child, what is called for primarily is a warm, accepting, and trusting relationship. This child needs to be supported and encouraged. Because of the intense fear he associates with mathematics, he finds it impossible to relax to the point where he can understand the concepts being taught. The teacher's motivational responsibility is to help reduce his anxiety level by making him feel safe and secure. The notion that he is unconditionally accepted must be reinforced. He must *never* feel rejected because of his performance level.

In order to break his failure expectancy, the teacher will guarantee ongoing success by designing learning encounters that will encourage him to read and interpret problems, work through concrete experiences, and express mathematical concepts. He should be encouraged to talk about his successes in mathematics, as well as many other things which might be troubling him. When the child recognizes that the teacher cares, he will feel more free to ventilate his feelings. As negative energy is drained off, his negative feelings about mathematics can be neutralized. With each success, the child's concept of self is enhanced. With increased self-esteem and confidence, defensive reactions disappear. For this type of mathematics block, the child must enjoy an empathic relationship with the teacher. The more often the teacher can listen as the child expresses his suppressed frustrations, the less negative feeling will be left to fixate on mathematics.

For therapeutic intervention to be successful, the teacher should be sensitive to the following:

1. Allow the child to move easily into a relationship because he feels safe and wants to be there, rather than because the teacher insists upon it.
2. Offer support and encouragement without praising too much. Excessive praise can cause undue stress.

3. Encourage a bond of trust by listening effectively. Avoid registering shock or surprise.

4. Move slowly and carefully. Exercise patience to avoid pressuring.

5. Feed back positive information making it clear that progress has been made because of the child's initiative and success, rather than because of what the teacher has done.

6. Identify the nature of the child's success, and point out how he can repeat it.

7. Help the child to understand why he experienced those old blocks and how to deal with negative stimuli in the future.

The whole therapeutic process of reciprocating positive locomotion for negative will take time. As emotional overlay is alleviated through threat reduction, trust and security will emerge from the affective domain, and his mathematics blocks will have a better opportunity to be removed.

The Aggressive Contentious Child

With specific reference to our second child, an altogether different approach is needed. The character problem child refuses to participate in mathematics simply because it does not please him. He becomes resistant and reluctant when he thinks there is nothing particularly worthwhile in it for him. He is accustomed to satisfying his ego needs for recognition and power without putting forth much effort. If things don't go right for him, he simply walks away or attacks and blames. Failure is never his fault. He is quite capable of concocting elaborate rationalizations to protect his own narcissistic need to be in a preferred position (Boyer, Rosenberg, Werry). He is an impulsive, demanding, and impatient learner. Because he finds the going rough in grasping mathematical concepts, he feels uncomfortable. He has been conditioned all through his life to expect immediate need gratification. He has never learned to struggle, to persist, nor to wait for positive returns.

Character problem children cannot tolerate any degree of discomfort. They cannot cope with difficulty. Probably one of the most accurate descriptors for this type of child is that he is excessively *hedonistic*. He continually seeks after that which is self-aggrandizing and pleasurable, while avoiding anything which will insult his sense of self and cause him pain. As a result, he has learned how to beat the system and con his way out of tight situations. He can come up with hundreds of excuses explaining why he can't cope with mathematics, all of which place the blame directly upon someone else, usually the teacher.

Therapeutic method for this child requires the following dictates on the part of the teacher:

1. Approach the child with a straightforward, businesslike attitude, firmly establishing yourself as the authority figure. Refuse to listen to his cock-and-bull stories.

2. Explain clearly and succinctly to the child exactly why he is experiencing difficulty in mathematics by reviewing the nature of his problems.

3. Appeal to his hedonistic nature by complimenting excessively for work well done. Even for the slightest effort in mathematics, provide special recognition, privileges, and rewards.

4. When he decreases his diligence and effort, remove special privileges and attention.

5. Maintain strict, hardline expectancy. The child is clever, and he needs to be challenged.

6. Assign the child to help others with math problems. The attention and power will reinforce the need to learn himself so that he can tutor those who will give him needed recognition.

Again, working with the character problem child will take time and patience. However, it can be successful providing we remember that these children are quite capable of learning. They are not suffering emotional trauma, nor are they mentally deficient. They are usually just plain lazy and will put the necessary effort into improving their mathematical skills only when they come to realize that the personal rewards they gain are truly worth the effort.

The Socialized Deviant Child

This child's problem stems from a deficiency in the communications and expectancy areas. Although studies have shown that, within the socially disadvantaged population, mathematics reasoning tops the list insofar as intellectual functioning is concerned, the child appears most of the time to be uninvolved and detached. His attitude is one of being uninterested. He seems to be peripheral to the other students. The simple fact is that he has practically nothing in common with his peers, nor with the teacher. His social distance stems from a background so entirely different from the norm that it is improbable that he will accommodate many of the concepts being presented (Jones, Deutsch). Also, he lacks the perseverance to focus on any formal learning task for a prolonged period of time (Beilin). Other problems including self-discipline, social graces, deviant behaviors, and social attitudes, although quite acceptable at home, cause him to be seriously out of step in the regular classroom.

By the time they reach pubescence, children from a disadvantaged home milieu are usually so limited in intellectual skills that a reversal of their condition is very difficult. The problem with these children is not that they can't learn nor do not wish to learn, but rather is one of motivating them to feel less and less removed from the group socially, emotionally, and physically. Their specific need is to be included and to become active participants in the learning process.

Therapeutic support for the disadvantaged child might include the following:

1. Involving the child in novel, creative, and exciting learning encounters so he can avoid the negative aversions formerly experienced within formal learning environments.

2. Speaking the child's language so communication can be effective through common vocabulary meanings, thus reducing confusion.

3. Developing learning packages that do not depend upon homework. Materials should be as independently self-instructional as possible so the child can mark his own work and enjoy immediate feedback.

4. Teaching through games and problem solving activities that are fun in order to maximize the child's inherent problem solving abilities.

5. Employing concrete learning through sensory motor activities, introducing mathematical concepts deliberately through manipulation of materials, rather than expecting high level abstract, symbolic, and representational conceptualizations.

6. Providing for the most minimal cognitive stimulation to make up for lack of perceptual discrimination skills learned in the home.

The secret in working with the disadvantaged child is in knowing the degree to which he lags behind the average child verbally and socially. Generally, if the teacher can communicate with the child and involve him to actively participate in the group, the teaching act will prove successful. As the child becomes more and more sure of his environment and more and more understanding of what is expected of him, he will adjust and learn, and his success rate will increase significantly.

Finally, relative to a practical approach for removing blocks with reluctant learners, what is called for is a differential approach. Ninety-nine percent of the problem is resolved when we are successful in diagnosing the nature of the mathematics block. When we clearly understand exactly what underlies the child's reluctance to participate, an effective therapeutic design can be implemented. With the emotionally disturbed child, love and support reduce defensiveness and permit the teacher to reach him. In the case of the character problem child, a highly structured, behavioristic reward approach works best. Concerning the disadvantaged child, what is called for is developing an effective communication system and making the child feel an integral part of the group.

Professor Cooke teaches at the University of Lethbridge. His major interests lie in the fields of child guidance and special education. He is a registered psychologist in the province of Alberta and is actively involved in consulting, advising, and programming for children experiencing a wide variety of adjustment problems.

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Implementation and Evaluation of Problem Solving in Elementary Mathematics

Ron Cammaert
Alberta Education

This paper is an initial evaluation of the implementation of a curriculum model in elementary mathematics in the province of Alberta. The paper also examines the possible reasons for success or failure of that implementation in various jurisdictions in southern Alberta.

Implementation Model

A revised elementary mathematics program was implemented in Alberta in September 1982. Revisions to the 1977 program were seen to be minor in nature (Alberta Education, 1982). The 1982 program placed a greater emphasis on problem solving to reflect recommendations made by the National Council of Teachers of Mathematics and information gathered from school systems and provincial testing.

To assist school jurisdictions and teachers with implementation of the revised program, a curriculum guide was developed and distributed to each school in the province. A series of one-day workshops dealing with the problem solving component of the program were sponsored by Alberta Education. Two teachers from each jurisdiction were sent to this workshop with the expectation that they would, in turn, conduct workshops for the teachers in the various schools within their systems.

To further assist implementation of the problem solving component, a monograph entitled "Let Problem Solving Be the Focus for the 1980s" was published by Alberta Education in September 1983. One copy of this monograph was sent to each jurisdiction central office and school in the province. Additional copies could be ordered as needed.

Evaluation Procedures

Interviews were conducted with the superintendent or delegate responsible for curriculum in each of the jurisdictions in southern Alberta. To obtain responses from teachers, a questionnaire was prepared and distributed to approximately 30 percent of the teachers. One hundred and seventy-two questionnaires were returned. Comments on observations made during 11 school evaluation visits over the 1983-84 and 1984-85 school years were also included.

Survey Results

Results obtained from interviews with superintendents, questionnaires completed by teachers, and classroom observation seem to be fairly consistent. All three sources tend to give the picture of an initial effort and of some awareness being developed by teachers, but not a great deal of "institutionalization" of the change on the part of teachers. Almost half of the teachers responding to the poll indicated that they had not received inservice, and of those who had attended inservice workshops, twice as many indicated dissatisfaction with the inservice than indicated satisfaction. It would seem that the inservice delivery system did not meet the needs of the teachers.

Most teachers reported having a copy of "Let Problem Solving Be the Focus for the 1980s." Unfortunately, they were not asked to rate the effectiveness of the document.

The majority of teachers indicated that they teach problem solving, feel comfortable with the model, and allocate time to the instruction of problem solving. One of the problems connected to a discussion of this concept is that there are several interpretations a teacher may give to the words "problem solving." The concept is very different from traditional word problems found in most textbooks, but "problem solving" can refer to both. When a teacher indicates comfort with the concept, there is no way of knowing if the teacher is referring to the old or new version. Based on interviews with superintendents, classroom observations, and given the fact that almost half of the teachers reported that they had not been given inservice orientation, one would be skeptical that the majority of teachers actually do teach problem solving in the manner being discussed.

Teacher comments were reflected in statements made by superintendents. Both groups desire more inservice assistance and feel that more resources should be made available.

Discussion of the Implementation Model

In Alberta, the development of provincial programs is centralized, but includes broad consultation. Centralized development is favored for economic efficiency and to ensure structured uniformity. In this particular case, the need for change was recognized at the provincial level as a result of a thrust in mathematics education in North America. The National Council of Teachers of Mathematics, among other groups in mathematics education, has identified the teaching of problem solving as highly important in the curriculum of our classrooms. This perception comes from analysis of the needs of society. It is desirable to have people who are able to solve ever-increasing complicated problems.

However, the extent to which an innovation meets local needs, as perceived by school personnel, is related to successful implementation (Fullan and Pomfert, 1977). The uniqueness of the local environment, the need for local fiscal control, the need for increased local public participation, and recent developments in management theories are cited as reasons for local school involvement.

Inservice orientation was designed to persuade teachers of the need for change. Personnel from the University of Alberta attempted to make teachers aware of the model adopted by Alberta Education. References were made to materials that teachers could access, and some strategies for problem solving were given. The major emphasis was to have teachers become committed to pursue the idea on their own. Little, if any, follow-up was planned or occurred.

Generally, curriculum development plans receive more attention from Alberta Education than do implementation plans (Alberta Education, 1980). Provincial responsibility for curriculum implementation in the past decade or so has mainly been with regional office consultants in the Program Delivery Division. It is the perception of Alberta Education that each school jurisdiction should have its own local implementation plan for new or revised curricula in keeping with the intent of the provincial thrust (Alberta Education, 1985). In the final analysis, it is the classroom teacher who bears the majority of the responsibility for curriculum implementation, with support coming from system supervisors, professional association resources, or consultative assistance from the universities in Alberta or from Alberta Education.

There were no formal plans made for evaluation, nor for monitoring of the process. Some jurisdictions monitored the implementation on an informal basis, and regional office personnel evaluated the process as part of their school evaluation program.

The unit of change, as perceived by Alberta Education and school jurisdictions, was the school system. There is considerable research evidence to support the view that the individual school is the unit of change that is most successful in bringing about curricular improvement. A major finding of the studies conducted by John I. Goodlad and the Rand Change Agency Study indicate the need for local involvement and the reality of local control of education despite the influence of forces operating at the state and national levels (Goodlad, 1975; Berman and McLaughlin, 1975). Unless conditions for change exist at the school building and in the individual class, no change will occur (Neal, Bailey, and Ross, 1981).

Alberta Education includes teachers in provincial committees when curriculum changes are being planned. Teacher responses to the questionnaire indicated the desire to develop materials at the local level. No such action occurred in this zone. Investigators in the Rand study found that successful change resulted when mutual adaptation occurred, that is, when both the innovative practice and the local school organization were changed (Berman and McLaughlin, 1975).

Responses from teachers indicated that having representatives from each school system attend a training session, so that there would be "experts" in each jurisdiction, failed to serve its purpose. The day-long session was not seen as intense enough to allow most people to sufficiently develop the knowledge base and training techniques necessary for them to feel comfortable with this role in their jurisdictions. The people selected to attend the workshop had varying degrees of experience with the model, varying degrees of ability to conduct inservice orientations, and varying degrees of commitment to the model. Some teachers were not aware of what their role would be upon returning to their school system. The selected personnel were given no training on how to provide coaching within their systems, and there was little or no provision for this in most jurisdictions.

When Lippitt and his associates surveyed teachers to determine what they believed were the forces facilitating innovation of teaching practice, they found that the availability of help from consultants was considered very important (Lippitt, 1967). Teachers indicated that the innovator needs to work through the new ideas with the teachers to solve problems at the practical level, rather than simply conduct a one-shot information session (Tanner and Tanner, 1980).

The commitment of central office personnel was one of the most significant variables in determining the success of curriculum implementation in a system. Where curriculum projects have been successful, one of the most significant elements is that the personnel involved were deeply committed to the project. Activities to inform principals were not part of the implementation plan. Principals became aware only if they happened to attend in-system presentations, were informally contacted by regional office personnel, undertook professional reading on their own, or were apprised at the system administrators' forum.

Curriculum development must have the support and backing of school administration (Zenger and Zenger, 1984). In developing commitment, the first stage is to make certain all those affected understand the change and the reasons for the change. As indicated earlier, a little more than half of the teachers reported attending one inservice session. Central office personnel developed their understanding in a different setting, and principals may or may not have received any information regarding the change.

If teachers are to fully implement the problem solving model within their teaching, they must shift from a content orientation to a process one. It is likely that this disparity between the values and objectives of teachers and the planned innovation would cause difficulty in developing commitment in teachers. Problem solving reflects a "discipline" approach to curriculum rather than a "subject" orientation. One of the criticisms of "discipline" organization is that insufficient inservice assistance is given. If the values and goals in a particular change project match those of project participants, then commitment is more likely to occur (Leithwood and Fullen, 1984; Neill, 1982; Kienappel, 1984).

The fact that the innovation was not seen as major by the province could also account for the difficulty in developing the commitment of school personnel, since they have been charged with a large number of pressing and major changes in the school environment. No more than one or two areas of the curriculum should be studied or changed at one time (Zenger and Zenger, 1984).

Summary

At this time, the innovation has not been internalized by a majority of teachers in southern Alberta. However, the innovation has been picked up by some teachers, and one needs to keep in mind that the implementation process is still continuing. The provincial mathematics achievement tests at Grades 3 and 6 will reflect this emphasis in the curriculum, and some educators feel that this will help teachers develop the awareness required. In order to enhance the likelihood of successful implementation of a curriculum change, it may be necessary for educators to use some of the results of current research and modify the implementation model now used.

Ron Cammaert is the mathematics consultant for Alberta Education, Lethbridge Regional Office. Mr. Cammaert is past president of MCATA, having served as president for two years. He served as principal of Barnwell School prior to joining the Department of Education.

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Seven-Link Chain Problem

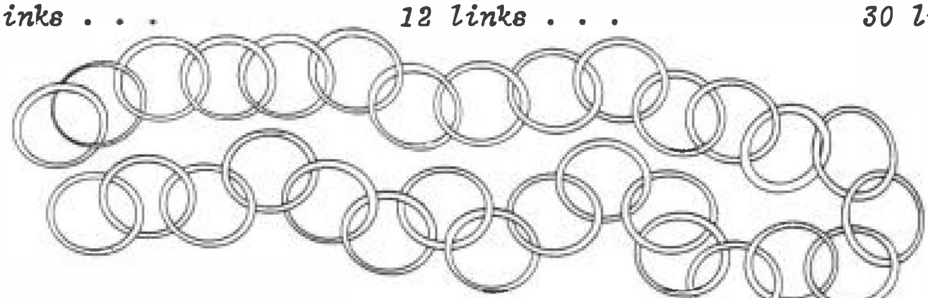
(continued from page 4)

Variation #1

The prince's brother escapes with a section of the gold chain and also finds haven at the home of another peasant. The same conditions (one link per day, with no prepayment or arrears) are negotiated.

The prince states that he needs to cut *two links only* to meet the conditions. What is the *maximum* length of the chain (measured in links) that this second prince had when he escaped?

7 links . . . *12 links . . .* *30 links?*



Variation #2

The oldest prince escapes with a section of the gold chain that is 63 links long. A third peasant offers a haven to this prince, and again the same conditions are negotiated.

The prince and peasant agree that the conditions may be met by cutting *three* links. Which links were cut? What is the length of the longest segment?

Problem Solving: Mathematics and Social Studies

A. Harold Skolrood and M. Jo Maas
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Effective parallels can be drawn between the intellectual processes inherent in the social inquiry process in social studies and problem solving in mathematics. Both require knowledge of content and specific problem solving processes, and both lead to finding a solution to a problem. Both emphasize an "inquiry" approach which involves students in thinking about possible solutions to problems.

Inquiry may be viewed as a process in which students are actively involved in seeking knowledge. It is a systematic process for thinking about a problem or social issue and consists of a number of intellectual processes such as definition of a problem, gathering data and organizing, analyzing, and evaluating it in terms of relevance to solution of the problem, making inferences and generalizations from the data. All of these skills are applied in inductive and deductive reasoning in the solution of a mathematical problem and in making a decision about an issue in social studies. John Dewey has been credited with taking scientific method of the pure sciences and adapting the process involved to problem solving in the social sciences. Dewey describes the inquiry process in terms of:

> active, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it and the further conclusion to which it tends.[1]

Learning through inquiry is a process of formulating and testing ideas. To this end, a number of inquiry or problem solving models have been developed, but in general, the majority of them seem to apply to both mathematics and social studies:

Problem Solving in Mathematics	Inquiry in Social Studies
Understand the problem.	Identify and focus on the problem.
Plan the solution.	Formulate research questions.
Implement the plan.	Data processing: gather, organize, analyze, evaluate, and synthesize.
Solve the problem.	Make a decision.

[1]John Dewey, How We Think: A Restatement of the Relation of Reflective Thinking to the Education Process, revised edition, p. 8. Chicago: Regnery, 1971.

Current educational literature has reemphasized the need to teach children to think more effectively. Efforts have been made to identify subsets of thinking skills inherent in the thinking process, along with some teaching strategies to accomplish this desirable goal. Many of the skills required in learning one subject are transferrable to the learning of another subject, and the teacher has to be alert to opportunities to help students effect the transfer of learning that should take place. Sound pedagogy demands this type of reinforced learning.

Too often the skills in one subject have been taught as discrete entities, with the students being left to discover, often incidentally, the similarity of skill between subjects. There is probably little that is more rewarding for the teacher than to see a student suddenly have a "revelation" in discovering that what he is learning in one subject has some carry-over into another subject. Reinforcement of specific skills, in this case, becomes automatic. Interesting parallels can be made between the skills needed in problem solving in mathematics and those needed in the social inquiry process in social studies.

	Problem Solving Process in Mathematics	Social Inquiry Process in Social Studies
1.	**Understand the problem.** (What is the question?) Students are to think about the problem before attempting the skills. SKILLS: - identifying key words - using manipulatives - interpreting pictures - restating the problem in your own words - asking relevant questions - identifying wanted, given, and needed information - identifying extraneous information - considering alternative interpretations	Identify and focus on the issue. SKILLS: - interpret and ask - coherent in issue - vocabulary needed - definition of terms - interpretation of intent of question - paraphrasing to clarify meaning - clues to type of data required
2.	**Develop a plan.** (Strategies for solving the problem are considered.) SKILLS: - collecting and organizing data (charts and graphs) - acting it out - using manipulatives - identifying and applying relationships - making diagrams and models - using a simpler problem - using logic or reason - constructing flow charts	**Formulate research questions.** SKILLS: - hypothesize possible solutions - formulate research questions to guide information gathering - select sources of information

3. Carry out the plan. (Carry out the plan developed in Step 2.) SKILLS: - collecting and organizing data (charts and graphs) - acting it out - using manipulatives - identifying and applying relationships - making diagrams and models - using a simpler problem - using logic or reason - constructing flow charts	Data processing. Gather print, visuals, interviews, and surveys. ORGANIZE: note-taking, outlining, paraphrasing, tabulating, mapping, charting, and graphing ANALYZE: categorize, look for relationships, discriminate relevant and irrelevant data - detecting bias, subjectivity, and objectivity SYNTHESIZE: relate cause and effect, formulate generalizations, and summarize
4. Solve the problem. (Encourage students to assess the effectiveness of the solution process.) SKILLS: - stating an answer to the problem - restating the problem with the answer - determining the reasonableness of the answer - explaining the answer - reviewing the solution process - considering the possibility of other answers - looking for alternative ways to solve the problem - making and solving similar problems - generalizing solutions	**Resolve the issue.** SKILLS: - formulate alternative solutions - analyze values underlying each alternative - evaluate alternatives and make decision on the issue

An examination of the above comparison between problem solving in mathematics and the social inquiry process in social studies suggests that teachers be cognizant of the commonality of problem solving skills that exist between two very diverse subjects. Since learning skills are intellectual in nature and competent acquisition of these skills is cumulative, teachers need to explore every opportunity to reinforce their attainment. Students will discover that "thinking" in mathematics is much like "thinking" in social studies and other subjects. Integration of skills from one subject area to another will not be left entirely to chance.

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Enlightened Problem Solving: A Parable

Dr. Daiyo Sawada
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The Lord said, "Let there be light," and so there was, and it was good. The mathematician said, "Let x be the unknown quantity," and so it was. Many problems were solved, and it was good. The NCTM said, "Let problem solving be the focus," and indeed it was, and it was good. In fact, there was a great flurry of activity and enthusiasm and commitment, and this was very good.

It came to pass that the ideas of a great scholar named Polya were carried far and wide, eventually to be accepted and enshrined as "The Four Commandments," which all disciples preached and sometimes practiced. Children, both young and old, were encouraged and guided to practice the Commandments, for it was believed that, in this way, genuine problem solvers would come about.

One day, a young lad, a most successful problem solver to be sure, realized that it was the Fourth Commandment, often called "looking back," that enabled him to continue to grow and to harvest the fruits of his past activity, for it was the Fourth Commandment that indeed commanded him to harvest, although he had not realized this before, since he thought it was simply a commandment to review, a commandment that his teachers routinely practised in many other settings almost to an excess.

He continued to solve problems, for the environment was rich in problems. He devoted more and more of his time to the Fourth Commandment, looking back at his work with anticipation of seeing something he had overlooked, and each time was rewarded handsomely, so perceptive had he become. He had, indeed, become a master of looking back, of reflecting upon his activities, sometimes to the exclusion of other activities.

"Have you ever wondered what happens to a problem on its way to a solution?"

His teacher noticed that he was no longer the first one to ask for new problems. He had taken to only solving the "problem of the week." He did this masterfully and with keen insight, sometimes programming general solutions on the microcomputer when this was appropriate, but his teacher was worried about his lack of productivity. Noticing that his teacher was concerned, and knowing his teacher was truly a believer in the Four Commandments, he sat down beside her one day and asked, "Have you ever wondered what happens to a problem on its way to a solution?" Somewhat taken aback, but not totally surprised, she replied, "Well not really. The FC (Four Commandments) don't really lead us to ask such a question, do they?"

"No, I guess they don't," he said, "and I suppose that's one reason why I haven't felt that it was appropriate to ask."

Sensing that he was still somewhat hesitant to ask, she said, "Now that the question's been raised, I find it intriguing. What led you to it?"

"I'm not really sure, but in following the Fourth Commandment, it came to me one day that all these problems that you have provided for us are really not that different at all."

"You mean, for example, that the chickens and pigs problem is similar to the motorcycles and cars problem?" she asked.

"That's part of it, I guess, but only a small part of it. It's much bigger than two problems being alike. It's all problems being alike. Like there's more to it than the problems. In fact, the problems themselves aren't very important. Even the mathematics in the problems isn't very important!" His voice quivered.

This was the first time she had ever had a student who had dared to probe beyond the FC. "I'm fascinated. Tell me more."

"I'm having trouble understanding it myself, but I'll try. Actually, it is more than just the question of what happens to a problem on its way to a solution - much more, in fact. Even so, I think the best place to start is with the question that got me started. That question is this: What is an answer? For some time now, I have been asking myself this question each time I came up with an answer to a problem. Now I ask myself this question before I come up with an answer, and believe me, it really helps me to understand both the problem and what I'm doing to solve it."

"That sounds fascinating, but I'm not quite sure I understand," she said. "Could you give me an example?"

"I'll try. Remember last week when you asked me to help Marvin with his multiplication facts? I think I learned more than he did. He couldn't seem to remember anything and didn't want to. I said it was important. He said it wasn't. I tried to explain why it was, but to be honest, I really didn't even convince myself that it was important. That bothered me. I remember one thing very well. When I asked him what is 2x8, he said 8+8. I was just about to tell him he was wrong again, but I didn't. I just looked at him and he looked at me. He wasn't wrong! 2x8 = 8+8. In fact, there were a whole lot of correct answers: 4x4, 20-4, 4 squared, 10+6. At that moment, it dawned on me that I could keep on giving correct answers forever, and all of them would be different, yet correct. Then why do we insist that 16 is *the* correct answer? I asked Marvin, but he didn't know. Marvin asked me. As I stumbled my way to an explanation, I eventually stated that 16 is *the* answer simply because it is the simplest way to write the number 2x8. Marvin wouldn't let me off the hook with that. He wanted to know what was so simple about 16. It didn't seem so simple to him. Perhaps it was simple only to simple minded persons! I told him then that I really didn't know. He said that's why he hated math. Then it dawned on me: 16 is simple because it is the answer according to tens and ones. We always group by tens and ones. If we always write a number according to tens and ones, it becomes simple. We get used to it. We expect it. We have gotten so used to expecting the answer to be in tens and ones that we have forgotten that we expected it in that way."

"We have gotten so used to expecting the answer to be in tens and ones that we have forgotten that we expected it in that way."

"I think you're really onto something here," she said. "What you're saying is that basic facts like 2x8 = ? are really problems, but that we have gotten so used to experiencing them as facts that we overlook what constitutes an answer to these problems. I think you may be right. But I think I may have interrupted your story."

"Not really, but there is more. When I told Marvin that 16 was *the* answer because it was the name for 2x8 in tens and ones, he shrugged his shoulders as if to say, 'So what?' I started to get excited because I thought I saw a way to get him into this. Even though you have said that the math balance is not a very good device, I got it off the shelf and put two weights on the eighth peg on the left side, and told Marvin that that was two groups of eight or 2x8. He had no trouble with that. I then put four weights on the fourth peg on the right-hand side, and said that was 4x4. The balance was level, so we knew that 4x4 was a correct answer. I asked Marvin to put on some other correct answers. He put two weights on the eighth peg of the right-hand side, and it worked. I wouldn't have done that, but 2x8 certainly is a correct answer to 2x8. 'But what is *the* answer, Marvin?' Without hesitating, he put a weight on the tenth peg and a weight on the sixth peg, and nearly glared at me. We must have done 40 or 50 basic facts on the balance that morning, and I don't think it entered Marvin's mind that he should be bored."

"That explains Marvin's behavior. This week he hasn't been nearly as disruptive. But what has this got to do with problem solving?"

"That's exactly what I asked myself when I did the Fourth Commandment after working with Marvin. I asked myself why Marvin pursued so many examples on the math balance. He didn't get bored, but I certainly did. I tried to look back at what we were doing. First, we had some information about a number (such as 2x8), but the information was not in the form that we wanted it. What we wanted was the same information to be in the form of tens and ones. So next, we put the initial information (2x8) on the math balance, and then arranged it in tens and ones on the other side. And finally, we read off the number as 16. And Marvin knew that was the answer because it was in the form we wanted."

"This is interesting. You're telling me that problem solving is simply transforming information. That we begin with some information as given in the problem. Then, knowing that the answer is simply asking for a particular form for the initial information, we transform the information (using the math balance in this case) until we get it into the desired form. Have I got it?"

"I think you've got it exactly. What was really bothering me earlier was that I didn't realize that for basic facts, there was a desirable form for the answer. It now seems perfectly clear to me that if I didn't know the desired form for the information, finding an answer would make no sense whatsoever. And I think that was Marvin's case."

"I think you're onto something important here, and I hate to dampen your enthusiasm, but perhaps other problems do not fit your scheme," she said.

"You may be right. All I can say is that I've tried it with all the problems you gave as problems of the week, and they all fit the scheme. The pigs and chickens problem is a good example. Remember: 18 animals; 50 legs. How many chickens; how many pigs? I'll never forget when you asked us to imagine that we were commanders-in-chief of the barnyard and all animals would obey us. 'All animals on your back.' We see 50 legs up. 'Each animal put down two legs.' Now we see 2x18, or 36 legs, go down, leaving 50-36, or 14 legs, remaining up. We also see that only pigs have legs up and that each pig has two legs up. So, with 14 legs up and two per pig, there must be 14/2 or seven pigs. There must then be 18-7, or 11 chickens. We had the initial information: 18 animals (pigs and chickens) and 50 legs. We knew the final form for the information: so many chickens, so many pigs. So we began transforming the initial information, putting it into various forms, each hopefully getting us closer to the desired form. When we saw the 14 legs up with two per pig, we knew we had the information nearly the way we wanted it. At this point, the problem was all but solved (we nearly had the information in final form). Seeing the seven pigs and the 18-7=11 chickens was then seeing the answer, the final form for the information."

"Yes, that does fit nicely. But don't we also have problems in which we don't know the form for the answer? In fact, sometimes we don't really know what an answer looks like, and the problem really is finding out what an answer might be. For example, when a mathematician is trying to solve an unsolved problem, he or she may not know the exact form for the answer, making it very difficult to transform the initial information until it is in final form." She paused almost as if she were trying to answer her own question. "But yet, in such a case, the problem is to find an appropriate form for an answer and to defend the appropriateness of any form we may create."

The lad did not choose to argue with his teacher. Somehow he felt that what she had just said indeed did fit what he had done when he thought he had come up with what, for him, were new forms for problem solving. His scheme was an answer to the open-ended problem, "What happens to a problem on its way to an answer?" Whether or not his scheme was an appropriate form for the answer, only time would tell. In the meantime, Marvin sure seemed to be a changed person, and so was his teacher.

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Content Reading Skills in Mathematics

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Words and symbols are parts of the language of mathematics. If students cannot read mathematics with understanding, they will be handicapped whether they are reading to carry out a task, for information or enjoyment, or to further their academic knowledge.

Reading mathematics is not easy. To be able to read mathematics, children must acquire some special skills. While every teacher teaches reading, the mathematics teacher has the special responsibility of teaching children to read mathematics. The reader of mathematics must:

1. possess a specialized vocabulary.
2. know the meaning and various uses of special symbols.
3. follow notational agreements and abbreviations.
4. be aware of the sequence of steps recorded when computations are displayed.

General Guidelines for Reading Mathematics

Reading SLOWLY for DETAIL

Mathematics should be read slowly and carefully - usually more than once using paper and pencil. Often, students are not aware of this. As children develop their ability to read, they are encouraged to read as rapidly as they can. When reading stories, they seldom reread. In reading mathematics, however, speed is only a minor concern. Comprehension of every detail is so important that rereading is a highly recommended procedure.

Why is it necessary to read slowly and carefully and reread passages of mathematics? The answer lies in the compactness of the symbolism, the precise meaning of most items, and the need for the reader to reason and recall relevant and previously learned information. Reasons for using this technique should be discussed with children. They must be convinced of the need to reread.

Reading QUICKLY for an OVERVIEW

In various situations, it can be advisable to have children practice a reading technique called skimming. Skimming is rapid reading to obtain a general impression.

Skimming to preview. Skimming can be useful in previewing a chapter in a mathematics textbook. It can be done by examining the chapter title, the table of contents, and page titles. This should be done with the teacher, who points out the important topics coming up.

Previewing a printed mathematics lesson alerts students to the main ideas of the lesson, to new terms, new phrases, or new principles. These ideas are usually highlighted. Previewing may also help recall related knowledge or encourage a review of forgotten material.

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Skimming to review. Skimming may also be used to look back over a unit of study. Under the guidance of the teacher, children can rapidly read through section headings and page titles of material, noting the topics they have studied.

Skimming to understand the organizational pattern of a book. Skimming may be useful in helping children recognize the organizational pattern of their textbook. Most mathematics textbooks follow some organizational pattern. Ideas or algorithms are usually presented in one or two ways and are followed by exercises and problems.

In general, teachers should have children skim material before they begin slow and careful reading. These two general guidelines may appear somewhat contradictory as one advises rapid reading; the other a deliberate approach. However, there is no conflict when these guidelines are applied. Students should be instructed and helped to preview material rapidly and then to read and reread slowly and carefully, making sure they understand every line.

Students will not do these things naturally. It is your responsibility to help them see the wisdom of these guidelines and to provide practice in following them.

Reading Skills in Mathematics

There are several specific skills which are necessary in order to read mathematics efficiently. In some cases, these are extensions of the skills taught in language arts; in other cases, the skills are quite different. Teachers who are planning to help children learn to read the best book need to incorporate these skills into their instruction.

Knowledge of Vocabulary

Mathematics has its own vocabulary. While many mathematical terms are borrowed from everyday English, a reader of mathematics must be skilled in knowing both the *mathematical and everyday meanings of words.* Words have many meanings. Often there is a difference between the everyday meaning and the mathematical meaning of a word. See Table 1 on the following page for examples.

In order to teach the meaning of a word, a teacher should provide:

1. Examples of objects with reasons to which the term refers, examples of objects that have the property denoted by the term, or examples of actions the term signifies.
2. Nonexamples, explaining why the object is not an example of the term you are teaching.
3. Characterizations or conditions of the concept contained in the term or object.
4. Comparisons or contrasts of a word with related words.

Practice is essential in building a vocabulary. In teaching reading and in practice sessions, include discussions and exercises on using:

1. root words
2. prefixes
3. different phrases having the same meaning.

Some examples of *root words* are: Add serves as a root, or stem, for at least six mathematical terms (add, addition, adding, addend, additive, and adds). Measure, measuring, and measurement all share a common root, as do multiple, multiplier, multiply, and multiplication.

Roots can be important clues to the meaning of a word. Students are not always aware of this. A national study of seventh and eighth grade students revealed that 98 percent knew the meaning of sum, but only 17 percent knew the meaning of summation; 92 percent reported knowing the meaning

TABLE 1.
A List of Words in School Mathematics Having an "Everyday Meaning" and a "Mathematical Meaning"

acute	commute	lateral (area)	ray
add	compass	law	real (number)
alternate (interior angle)	complement	leg	right (angle)
altitude	concave	less	(square) root
angle(s)	cone	like (fractions)	round (off)
array	convex	lowest (terms)	row
associate	correspond	major (arc)	ruler
axes	count	map	scale (drawing)
balance	cross (product)	mean	second
bar	curve	minor (arc)	set
base	degree	mixed (number)	sign
between	distance	natural (number)	similar (figures)
borrowing	distribute	negative	simple (closed curve)
boundary	divide	odd	simple (form of fraction)
braces	element	opposite	solution
cancel	even	origin	space
cardinal	exterior	perfect (number)	square
carrying	face	place	term
casting (out nines)	factor	plane	twin (primes)
check	foot	plot	union
chord	greater	point	unit
clock (arithmetic)	intercept	power	volume
closed	interior	prime	yard
column	intersect	product	
common (denominator)	intersection	property	
	invert	radical	
	irrational	rational (number)	

of equal, but only 24 percent knew the meaning of equate.

Prefixes, too, play a role in the development of vocabulary. In school mathematics, the following prefixes are common:

bi-	binary, bisect, bisection
ex-	exterior, extract, extreme, expand, exponent
in-	interior, inscribe, incentre, internal, intersect
in-	infinite, inequality
mid-	middle, midpoint
non-	nonsimple, nonnegative, nonterminating, nonmetric, nonlinear
poly-	polygon, polyhedron
re-	rename, replace, regroup
trans-	transversal, transform, transitive, translate, translation
tri-	triple, trisect, tripod
un-	unequal, undefined, unknown, unlimited, unlike

Prefixes behave much like roots and can be incorporated into practice sessions involving root words.

In addition to helping children with roots and prefixes, you should also alert them to different forms of the same terms or phrases. The following set of terms or phrases - associative, associative principle, associative law, associativity - mean essentially the same thing. Similar sets of terms and phrases can be constructed for commutative, distributive, and transitive.

Knowledge of Symbols

Symbols are the shorthand of mathematics. Children learn the meaning of symbols and their pronunciation by means of explanation, demonstration, advice, and usage. Many mathematical symbols are used in more than one way. See Tables 2 and 3 for different uses and pronunciations. Since the use of symbols is extensive and complex, students need a great deal of practice using and recognizing them.

TABLE 2.
A List of Symbols Used in School Mathematics

0 1 2 3 4 5 6 7 8 9 10

\+ − × ÷ · :

< > ≤ ≥ = ≠

% ° $ *d* ' "

I II III IV V VI VII VIII IX X

Δ π → ↔

TABLE 3.
Different Uses and Pronunciations of Symbols

Different uses of "3":

3, 34, 342, 2^3, p_3, 25.3

Different uses of "−":

−4, $\frac{3}{2}$, 18 − 12, $-\frac{4}{2}$, $\overline{AB}$ $.2463\overline{4}$

Different uses of ".":

.333 . . ., $\pi \doteq 3\frac{1}{7}$, 3 · 5

Note also how 3, −, and · can all be used together:

3 · 3, −3 · 3, 3 − 3, −3.3, $\frac{1}{3}$, b^3

Special Reading Problems

There are several reasons why computations are difficult to read. The first has to do with eye movement. See Table 4 for examples. Eye movement in reading a computation is seldom left to right. Children have to learn many different eye movement sequences - practically one for each different algorithm. Teachers must demonstrate the order for an algorithm and get students to practice following it.

Reading Graphs and Tables

Learning how to read a graph or table is an essential skill in reading mathematics and in reading periodicals, newspapers, and other printed material. Although tables and graphs can take many forms, reading them is easier if done in two steps. First, have the students skim the table or graph for the topic. Have them identify all the categories reported. Let them obtain a general impression. Then, ask for a detailed study.

Reading Word Problems

Word problems, whether they are simple applications of previously learned computational procedures or challenging *mind benders*, are an important part of a mathematics program. Problems must be read and understood before they can be worked. Children should follow the two general guidelines discussed earlier. Once they can do this, then they can move on to the algorithm needed to solve the problem. Discussion of George Poly's four phases of problem solving can be a useful exercise for setting up and solving the problem.

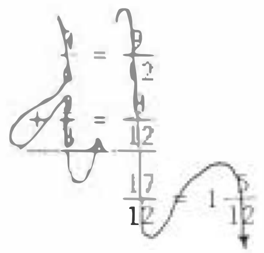
Diagnosing Mathematics Reading Problems

One way to measure a child's ability to read mathematics is to test his

TABLE 4.
Reading Computational Exercises

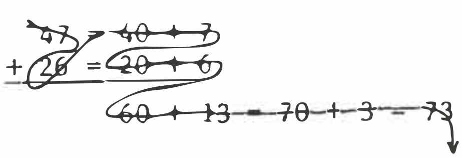
$$\frac{3}{4} = \frac{9}{12}$$
$$+\ \frac{2}{3} = \frac{8}{12}$$
$$\frac{17}{12} = 1\frac{5}{12}$$

COULD BE READ:



$$47 = 40 + 7$$
$$+\ 26 = 20 + 6$$
$$60 + 13 = 70 + 3 = 73$$

COULD BE READ:



or her knowledge of the mathematical terms and symbols he or she reads. Most textbooks or teachers' guides contain vocabulary lists for this purpose at the end of each unit of study. Another method is to periodically complete a reading ability checklist on each child. This can be done by observing a child read a selected passage and respond to directions in the passage, and by talking with the child about that material that has been read. The following sample items might serve as a guide to compiling the checklist.

Basic Techniques:

1. Usually skims a passage and obtains a valid general impression of the main topic.
2. Is able to skim a passage and obtain a valid general impression of the main topic.
3. Reads at an appropriate pace for the difficulty of the material and his/her ability to understand.
4. Usually rereads material to ensure understanding.
5. Is aware of the need for skimming, careful reading, and rereading.

General Skills:

1. Realizes terms can have mathematical and nonmathematical meanings.
2. Can give a valid explanation of the mathematical term being read.
3. Can use the meaning of the roots and prefixes previously studied to explain the meaning of a term.
4. Can pronounce terms being read.
5. Can pronounce names given to symbols being used.
6. Can relate words and symbols to a picture in a given situation.

Special Skills:

1. Computations
   (a) Can read an algorithm in the proper order.
   (b) Can relate a given algorithm to a previously understood and more detailed algorithm or to the manipulation of real or pictorial objects.
2. Graphs
   (a) Can skim a graph to obtain the topic, main categories, elements in each category, and the general relationship expressed in the graph.
   (b) Can extract details from a graph.
3. Word Problems
   (a) Skims a problem to obtain a general impression, then reads and rereads the problem.

(b) Can restate a given problem in own words and identify the question to be answered.

Reading as a Teaching Technique

A primary reason why children have difficulty reading mathematics is that reading is rarely taught or utilized in mathematics classes. Children quickly learn that the most efficient way to study mathematics is to listen carefully, watch the teacher work sample problems, then go directly to the problems in the text and work through them in the same way. They see no need to develop reading skills until it is too late. If we are to change this, we must make reading the mathematics text a frequent part of in-class and out-of-class activities. We must strive to teach children skills they need to read mathematics, and we must make sure these skills are used.

Some suggested activities are:

1. Have students read the mathematics book aloud in class. Pick a page from the textbook that gives a clear explanation of the topic you want to teach. Have the students skim and state their general impression. Then have the students take turns reading portions of the page aloud. When graphs, tables, examples, pictures, or other explanatory material are referred to in the text, examine them carefully. Encourage the reader and the other students to pay attention by asking questions.
2. Have stuents read the text silently and begin working the assignment. While the students are reading and beginning to do the assignment, you should be available to answer questions. Use this time to ascertain which students have difficulty reading silently and what their specific problems are.
3. Give a homework assignment in which students learn by reading how to do the work. This procedure should be used only after students are doing well on the assignments. Make sure the students understand that they read first and work the problems afterward.

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A Grade 5 Classroom: Organizing for Problem Solving

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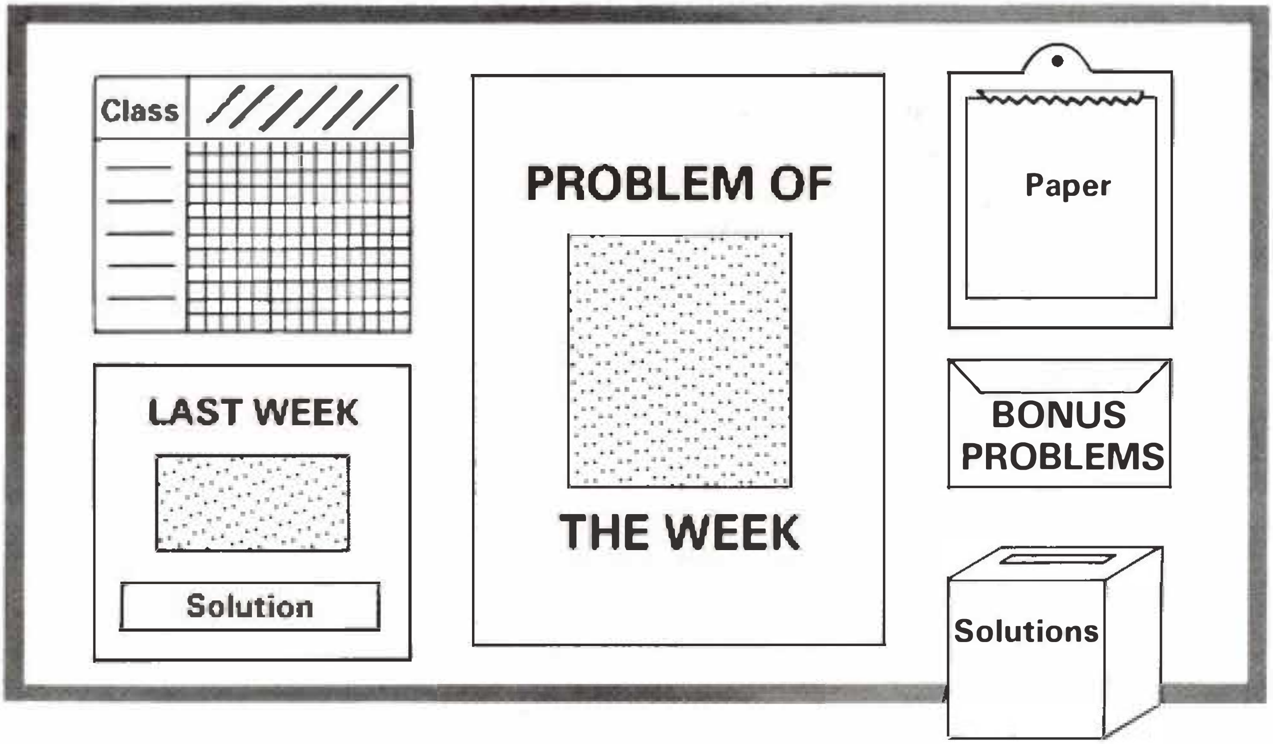
How do teachers get started in teaching students to solve problems? How can the busy teacher keep the problem solving model before the students? One method that has proven successful is outlined below.

A good beginning is based on establishing a positive classroom atmosphere from day one. Students must feel comfortable, receive positive reinforcement, and have opportunities to be successful. Routines and expectations must be established early. Content and initial problem solving experiences should be relatively easy in order to develop confidence and establish a positive attitude. Bonus activities can be included to challenge the high achievers.

Begin the year by "baiting" students with a selection of highly motivating problems and activities (about two or three a week). They can be placed on the overhead, and all should be short attention-getting problems that students enjoy. After a few of these, most students begin asking, "When can we do more problems?"

FIGURE 1.



It is at this point that "Problem of the Week" corner is initiated. This is most successful when students have been "captured" by the initial problems and, in turn, request more on a regular basis. It becomes their corner and, with a little guidance, they build the entire program in the form of a contest.

A new problem is posted each Monday. Students are allowed to enter a solution that day, but are only permitted one entry. The entries are checked that evening, and correct entries receive five points. On day two (Tuesday), everyone else can enter again. The correct solutions on this day are worth four points. On Wednesday, three points can be awarded; on Thursday, two points; and on Friday, one point. After 10 weeks, prizes and certificates are awarded, and a new round begins.

Although optional, about two-thirds of the class participates regularly. Select problems to complement class work that week or to demonstrate or practice a particular strategy.

The first two or three problems are relatively easy, and most students earn three or more points. A good selection of bonus problems is necessary to challenge those who have solved the problem of the week on the first days. Having a panel of students bring in, make up, or select bonus problems is worthwhile.

Once the problem of the week program has been established, group students into teams of four, and present the general framework for problem solving (Polya's model). Since there are four steps involved, it is relatively easy to direct students to create a wall display similar to that shown in Figure 2.

Introduce problems that are solved by using certain strategies. Attach these strategies to the appropriate step. As new strategies are discovered,

FIGURE 2.

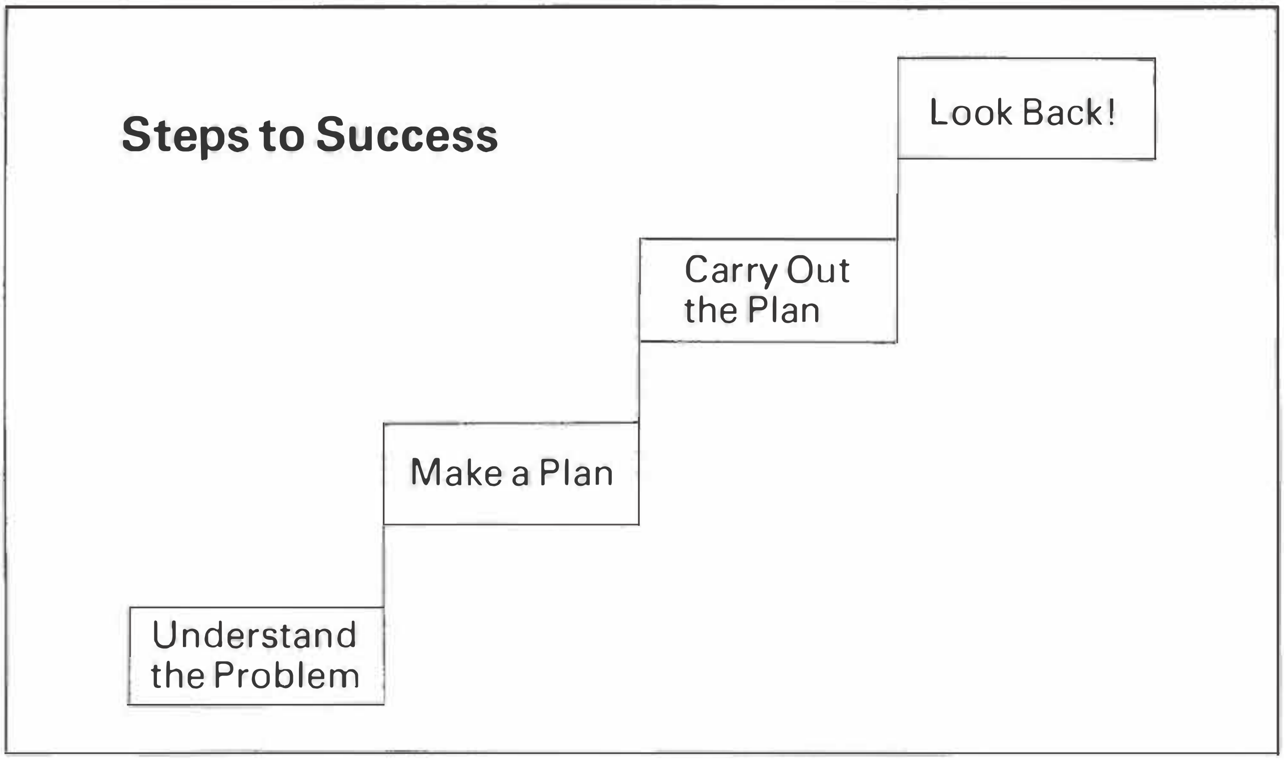
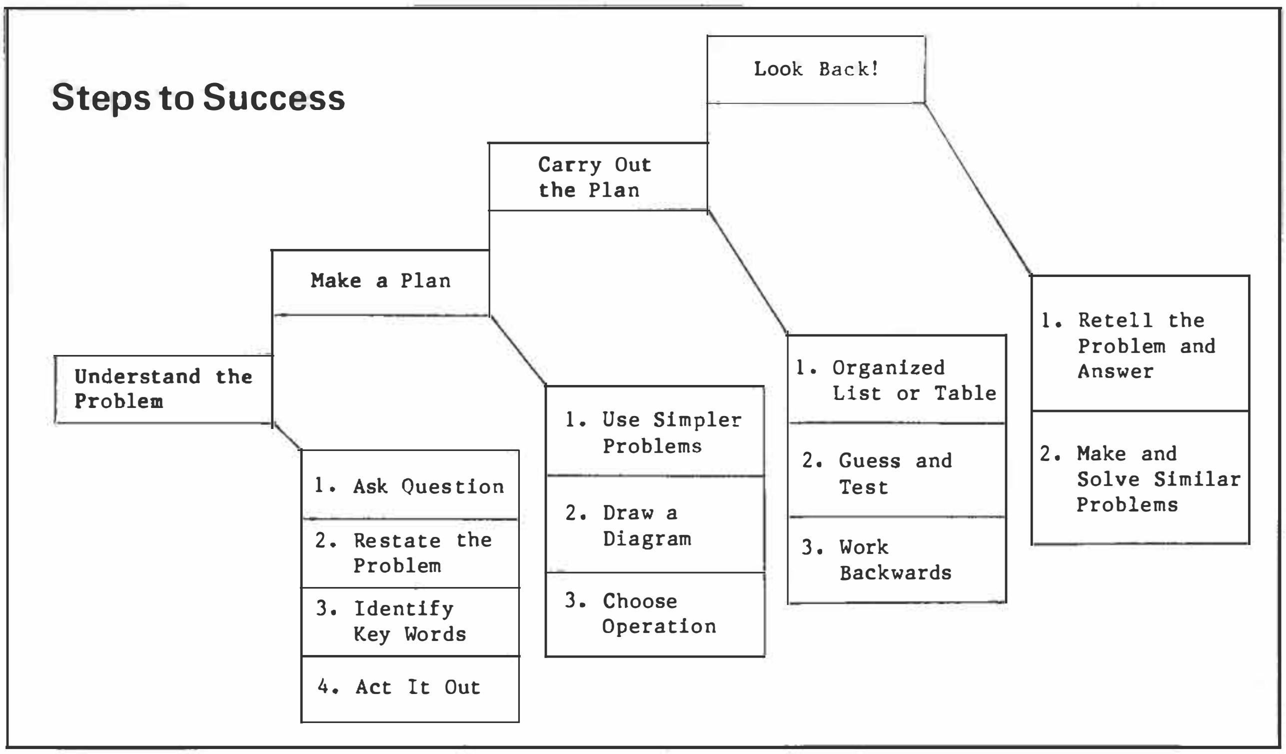


FIGURE 3.



they are added, and the lists grow throughout the year. Posting them serves as a reminder and reinforces the strategies learned.

Try to provide problem solving lessons that teach specific strategies on a regular basis during the school year. The emphasis is placed on the strategies themselves rather than the final answer. Providing a variety of meaningful and interesting problems in an atmosphere of success and having students work in small groups are integral to having a successful program. Getting your students involved in the problems, encouraging them to create their own, and maintaining your enthusiasm also helps to maintain that successful program.

Joe Krywolt teaches Grade 5 in Park Meadows Elementary School in Lethbridge. Currently, Mr. Krywolt serves as director of the Mathematics Council of The Alberta Teachers' Association.

Drill, Review, and Practice in a Problem Solving Setting

M. Jo Maas
University of Lethbridge

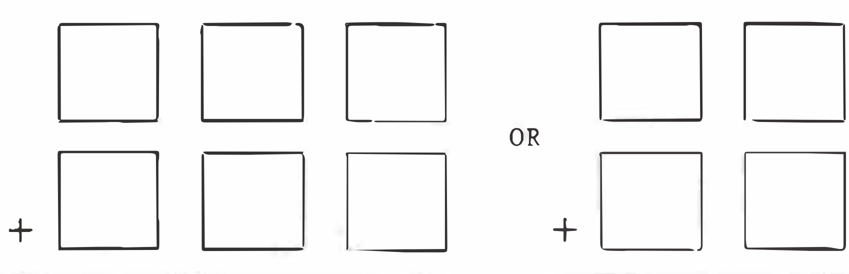
One activity that I have used in my mathematics class is "Digit Draw." This can be found on pages 57 through 60 in the Grade 7 book of the Lane County problem solving material, published by Dale Seymour.

I especially like this activity because it can be adapted to suit the various skill levels of students. It is an excellent way to give students drill and practice in whole numbers, decimals, and place value. The activity can be used at any time, but is useful at the beginning of the school year for review and/or drill and practice.

Division II students can play the game in the following manner to reinforce place value and addition skills.

Basic Activity

Students copy the following arrangement of boxes:



If three digits are too many, make it two two-digit numbers. Explain that you will draw out six cards (from a set of 10 cards numbered zero to nine), one at a time. The students place the digit in one of the boxes. The digit cannot be moved once it is placed in a box. Be sure all students have written the digit in before drawing the next card. The object is for students to make the largest possible sum that they can.

After the six cards are drawn, have students find their sum, and then see who has the largest. The basic activity may be modified. Note that all four basic processes could be used. Further draws may be made without replacing the digit or after the digit is replaced. The following examples show several ways in which this basic activity can be modified.

Application 1

After everyone has found their sum, ask students how they could use the six digits drawn in an arrangement that produces the largest sum possible.

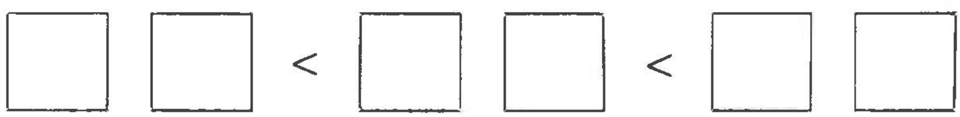
You can also direct the class to find the smallest sum, or the sum closest to a chosen number (for example, 600).

This activity can be adapted for multiplication, subtraction, division, and place value.

Application 2

An example of how to give students drill and practice in sequencing and place value follows.

Provide the following arrangement of six boxes, or have students draw their own:

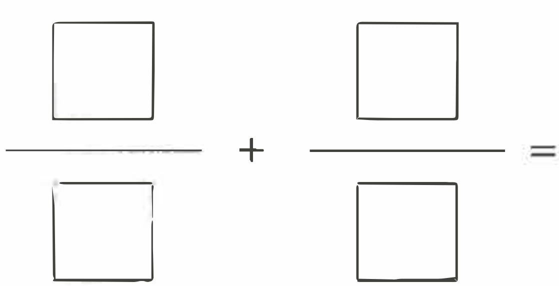


Draw six cards, one at a time, and have the students record the numeral in the boxes so that a true mathematical statement is made.

You may wish to modify this activity to four boxes for two two-digit numbers.

Application 3

A fraction drill and practice can be done easily by using the following arrangement:



Have the students find the largest fractional answer or the answer closest to a number you have chosen. The sign can also be changed to give practice in any of the other operations.

The activity may be applied or varied by excluding or replacing the card after each draw. Some teachers have replaced the 10 cards with a 10-sided die or tumbler, marked from zero to nine. Using this tumbler allows students to work with a partner and play some of the drill and practice games that follow. The activities may be undertaken with a draw of a digit.

Have students place the following arrangement of seven boxes on their page:

$$\begin{array}{cccc} \square & \square & \square & \square \\ + & \square & \square & \square \\ \hline \end{array}$$

Divide the class into pairs of students, and give each pair a tumbler. The object of the game is to make the largest sum, with the following conditions:

Each student takes a turn rolling the tumbler. The student who rolls the tumbler has the option of using the digit for his/her own sum or giving it to his/her partner.

For example, if Student A rolls a two on the first roll, she would realize that this wouldn't give her the largest sum, so she writes it in the thousands box on Student B's page. Then Student B rolls and must decide if the number he rolls will help him or hinder Student A. When all students have filled up their seven boxes, each student adds up his/her numbers, and the winner is determined.

Application 4

The following exercise using the tumbler involves several operations. Have students set up the following mathematical sentence:

$$\square \times \square + \square + \square - \square = \underline{\qquad}$$

Tell students that you will roll the tumbler or draw a digit and that they are to place it in any of the boxes. The objective is to make the largest (or smallest) answer.

Application 5

Still another game can be played to give students further practice in place value and addition.

Have students copy the following table:

	100s	10s	1s
A	____	____	____
B	____	____	____
C	____	____	____
D	____	____	____
E	____	____	____
F	____	____	____
G	____	____	____
TOTAL			

The objective is to be the closest to 500, or between 275 and 300, or some similar total. The teacher rolls the tumbler. Students must decide whether to place it in the hundreds, tens, or ones column. Suppose a two is rolled. It can be 200, 20, or two. On the next roll, a five comes up. It can be a 500, 50, or five. After seven rolls, the students total up their charts to see if they have met the objective.

Summary

Digit Draw, in all of its various forms, using cards or a tumbler, also fosters problem-solving strategies such as eliminating possibilities and breaking the problem into manageable parts. One method used to help students develop problem-solving skills, while doing this activity, is to have them explain to the class how they found their answer.

For example, in

□ □ □ + □ □ □ = closest to 500,

the student might say that when the number eight or nine was drawn, she knew that it wasn't any help to place it in the hundreds position, but it might be useful in the tens or ones place.

In this way, the class is exposed to a variety of strategies, and students can then choose their favorite method to try next time.

The adaptability of this activity for different grade levels, problem solving strategies, and review of various concepts makes Digit Draw an excellent activity.

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REFERENCE

Schaaf, Oscar. Problem Solving in Mathematics for Grade 7. Eugene, Oregon: Lane Education Service District, 1981.

Seven-Link Chain Problem

(continued from page 14)

Once you have solved the basic problem on page 4 and the two variations on page 14, arrange the data into an organized list. Analyze the data to determine relationships. Some sample questions could include:

- How does the shortest link that is more than one unit long compare to the number of individual links?
- How do the links that measure more than one link increase?
- Can a formula be derived for the length of the longest link?

A partial chart is provided below, and the basic problem is solved:

		Lengths of Multiple-Link Segments				
Chain Length	Unit Tally	First	Second	Third	-----	n.
7 links	1	2	4			
?	1.1	?	?	?		
63 links	1.1.1	?	?	?	?	

Fraction and Decimal Numeration Suggestions for Curriculum Revision

John Van de Walle
Virginia Commonwealth University

During the last five years, significant studies have produced convincing evidence that we are doing a rather poor job of connecting conceptual meanings or understandings with the symbolization of fractions and decimals in the fourth through seventh grades. Peck and Jencks (1981) found that out of 20 sixth graders, nine could draw pictures of simple fractions like 1/4, 1/3, 1/5, and 3/4. Only five of the nine could use these ideas to compare fractions correctly, and only two of the five could use a conceptual approach to add fractions such as 2/3 and 1/4. Peck and Jencks maintain that this small sample is representative of hundreds of children they interviewed.

In another study, less than 30 percent of entering freshman at City University of New York could correctly select the smallest of a list of five decimal numbers, while much higher percentages were able to compute accurately with decimals (Grossman, 1983). The Second National Assessment of Educational Progress reported that less than 30 percent of 13-year-old children selected the correct decimal equivalent for simple fractions such as 5/8, and 38 percent selected .5 as an equivalent to 1/5 (Carpenter, et al., 1981). These and many other researchers and reviewers have concluded that significantly more time must be spent on conceptual development of both fractions and decimals.

Hiebert points out in an excellent review of current research (1984) that a major flaw in children's mathematical learning is a failure to make connections between understanding and symbols. According to Hiebert, connections are possible at three sites: meaning of symbols, understanding of procedural rules, and consideration of the reasonableness of solutions (a sort of real world understanding linked to symbolic manipulation). The first and third sites have received the least amount of attention in curriculum development. He contends that "if students can be assisted in developing rich meanings for the symbols and in recognizing that solutions to written problems should make sense, their struggle to link form and understanding, to learn mathematics in a meaningful way, would be greatly enhanced."

Perhaps the most detailed and extensive investigation into what children know about fractions and how they learn fraction concepts is being done in a series of studies by Post, Wachsmuth, Lesh, and Behr (1985). Their teaching experiments and interviews with children have revealed that the concept of fraction is a complex and slowly evolved construct. Their work, however, is beginning to lead the way for curriculum reform in the area of fraction concept development. Many of the ideas presented as suggestions in this paper are derived from their studies.

Less research seems to have been done on the use of models such as place value materials to develop decimal concepts or on ways to help children connect fraction and decimal concepts. The suggestions in the activities section of this paper are based on extension of the fraction work by Post, et al., and on my own experience with fifth grade children earlier this year.

The current curriculum simply does not permit teachers the time to do required concept development.

The overriding suggestion is that curriculum revisions must be made to create a significant period of time for students in the fourth through sixth grades to work with a variety of both fraction models and place value models. This would allow them to develop not only the concepts of rational numbers, but also the connections that these concepts have with the symbolism of fractions and decimals. This time, which seems essential from both a developmental standpoint and from results of empirical research, can be created by postponing all rules for fraction and decimal symbol manipulation until at least the latter half of the sixth grade. The current curriculum simply does not permit teachers the time to do required concept development.

In a fifth grade class in which I was working this year, students at one point had not begun the study of decimals and were well into multiplication and division algorithms for fractions. Yet only one of the seven children I interviewed had a firm concept of fraction. None of those I talked with had even a vague idea of the meaning of decimal numerals, and none could give any conceptual explanation of the fraction algorithms with which they were becoming quite adept. About three weeks later, the study of decimals had progressed to the multiplication algorithm. However, in a class discussion which I conducted, there was virtually no indication that students understood decimals as representations of simple fractions.

To change this relatively absurd situation, the curriculum must shift from an emphasis on symbolic rules, which appear meaningless to children. Also, activities must be developed so that teachers will know what to do with the various models for fractions and decimals.

The activities suggested in the following pages are offered as ideas. The objective of these activities is to develop an understanding of rational number concepts and to connect these ideas to the symbolism of fractions and decimals. Special attention is given to the effort of relating the symbolism of fractions to that of decimals. While many of the activities are based on recent research, many remain to be tested and further developed. The intent is to get more physical models and activities with models into the classroom. Whenever possible, connection of an oral form of the concept is connected with the models prior to symbolization. Symbolic rules for equivalent fractions are not discussed, although some activities guide students to develop these ideas in an intuitive manner.

Another focus of these activities is to utilize a variety of models whenever appropriate. Evidence suggests that a well-formed mathematical concept becomes model free. Dependence on a single model has a tendency to leave a concept tied inflexibly to the model. One observes this quite readily in children who seem to have their concept of fraction firmly tied to circular regions.

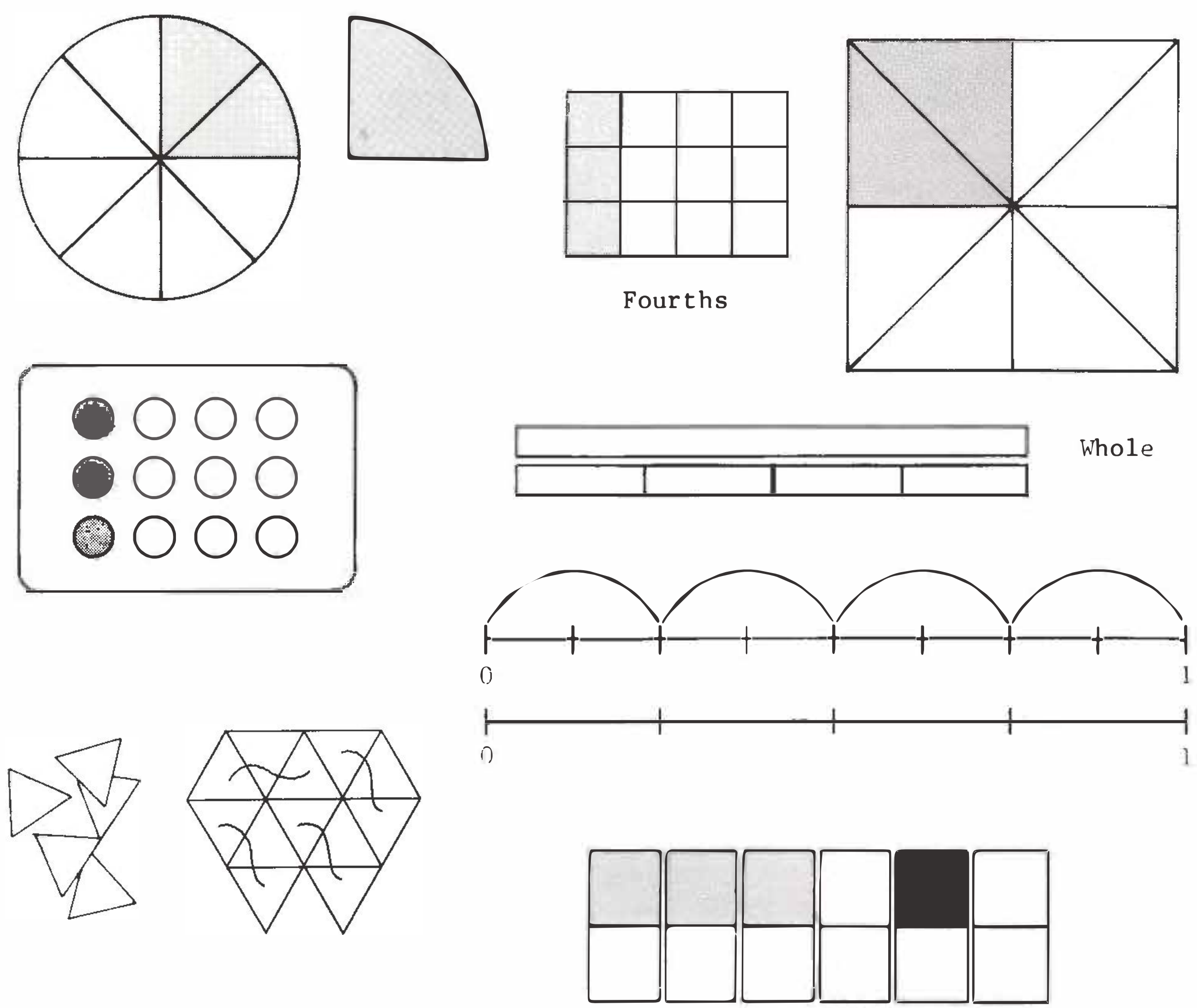
Suggested Activities

OBJECTIVE:
To make meaningful transformations between models and symbols.

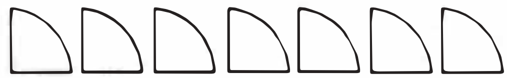
1. Develop fraction "words" (halves, thirds, fourths, fifths, and so on) in connection with assorted models. For each fraction word, there are two factors that are essential: the correct number of parts must be presented, and all parts must be the same size.

   Introduce this notion with as many models as possible:

   - circular pies
   - squares and rectangles subdivided into smaller parts for shading
   - sets of two-color or two-sided counters
   - colored fraction strips or Cuisenaire rods
   - number lines, subdivided into various subunits
   - regions made from posterboard squares or equilateral triangles

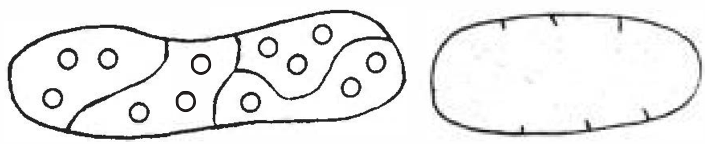


2. Orally, count unit fractions with each model.



"One-fourth, two-fourths . . . seven-fourths."

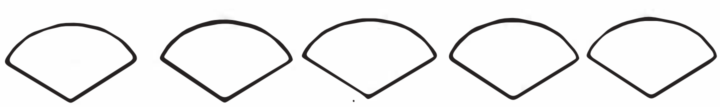
"One-third, two-thirds . . . five-thirds."



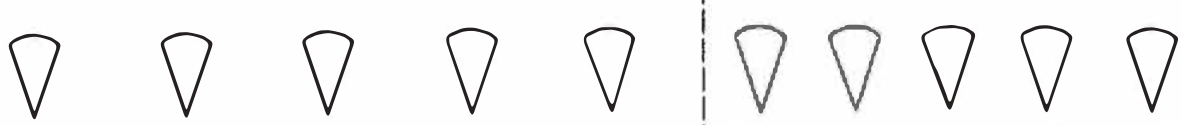
"One-fourth, two-fourths . . . six-fourths."

3. When children can identify fractional words and can count them accurately, introduce the written form for fractions as the counting is done. Use as many models as possible.

It is significant to count several different fractional parts in a parallel fashion. This helps to develop the difficult inverse relationship between number of parts to make a whole, and size of the parts. In comparison of 3/4 with 5/12, for example, children need to see that while five is more parts than three, the fourths are much larger. This type of reasoning is slow to develop and requires a significant amount of varied experience.



1/3 ... 2/3 ... 3/3 ... 4/3 ... 5/3
(ONE)



1/10 ... 2/10 ... 3/10 ... 4/10 ... 5/10 | 6/10 7/10 8/10 9/10 10/10
(ONE)

4. Use models to go from either a given unit or whole to a given fraction and vice versa. A progressively difficult series of five question types is suggested.

(a) **Given a unit fraction, find the whole.**

EXAMPLE: If the red (2) strip is 1/5, what strip is the whole?
Answer: Orange (10).

EXAMPLE: If three counters makes 1/5, how many beans in a whole set?
Answer: 15.

(b) **Given a unit fraction, find a nonunit fraction.**

EXAMPLE: If the light green (3) strip is 1/4, what strip is 3/4?
Answer: Blue (9).

EXAMPLE: If four counters are 1/2 of a set, how many counters in 3/2 of a set?
Answer: 12.

(c) **Given a nonunit fraction, find the unit fraction.**

EXAMPLE: If the dark green (6) strip is 3/4, what strip is 1/4?
Answer: Red (2).

EXAMPLE: If 12 counters are 3/4 of a set, how many counters in 1/4 of a set?
Answer: 4.

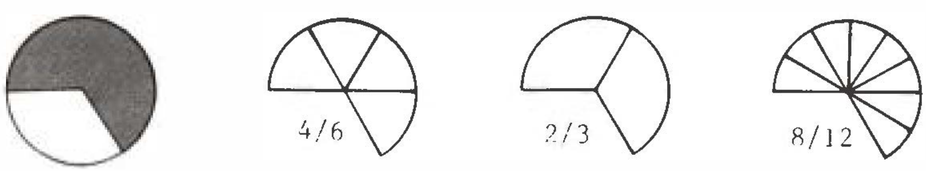
(d) **Given a nonunit fraction, find the whole.**

(e) **Given the whole, find a nonunit fraction.**

NOTE: The above questions cannot be done with pie pieces, nor any model in which the whole is a fixed size.

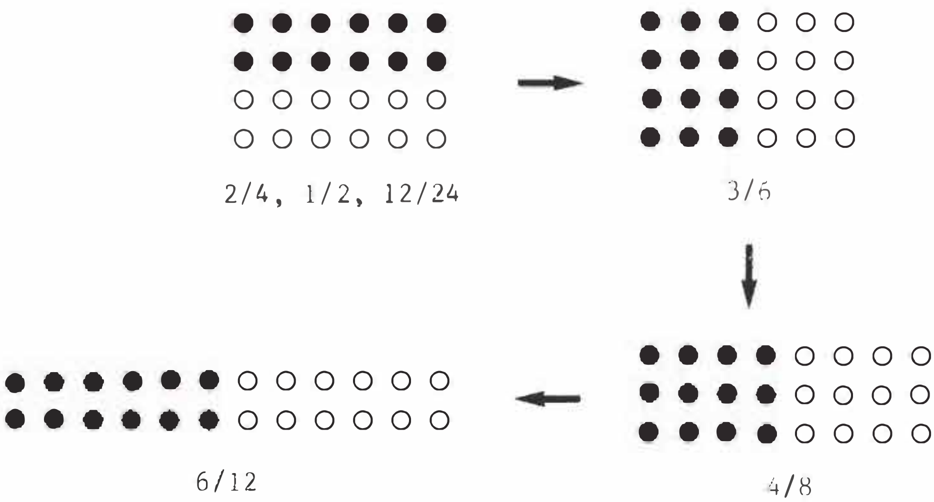
OBJECTIVE:
Develop the basic concept of equivalence. That is, multiple fraction names can be used to represent the same amount.

1. Provide drawings or cutouts of a fractional part of a region, and have children find fraction names for the region by covering it in as many ways as possible using pieces of the same size. Children should write all results.



Later, extend the exercise to include pieces not in the set of manipulatives. For example, "What if all of our 1/8 pieces were cut in half. What could we call this part?"

2. Provide sets of counters or drawings of counters in two different colors. What fraction is each color? By rearranging the pieces, children can find different fraction names.



OBJECTIVE:
To make comparisons between two or more fractions.

1. Find a way to show two fractions of the same unit or whole at the same time. (This activity is only useful with strips or counters, or with other models in which the size of the whole can vary.

2. Fraction pairs should be given to children in written form. Children use models to determine which is greater. Many children will "know" (either correctly or incorrectly) which is greater without recourse to models. These children should use models to confirm or check their thoughts. Fraction pairs can be given in three different categories:

   (a) like denominators and unlike numerators
   (b) like numerators and unlike denominators
   (c) both numerator and denominator different

   Select fraction pairs so that it is possible within the available model to form equivalent fractions with like denominators.

The last two activities provide experience with all of the concepts that are necessary for creating a general equivalent fraction concept. However, no algorithm nor symbolic rule is developed (such as multiplying or dividing top and bottom number by a constant), nor should such a rule be introduced. Experience indicates that as soon as such a symbolic rule exists, children will mindlessly use the rule and will ignore the conceptual referent no matter how poorly their ideas are formed. The next activity will challenge children even further toward development of their own symbolic rule for equivalent fractions. No rule should be provided.

OBJECTIVE:

To investigate in a symbolic mode the concept of equivalent fractions.

Provide models for children to use in completing equations of the type shown. Notice that there are four different types of exercises. Each type should be worked on within the context of several different models.

$\frac{6}{8} = \frac{\square}{4}$ $\frac{2}{3} = \frac{\square}{9}$ $\frac{3}{5} = \frac{9}{\square}$ $\frac{8}{12} = \frac{2}{\square}$

OBJECTIVE:

To help with the connection between fraction and decimal notation.

The models described here permit fractions with denominators of 10 and 100 to be shown easily. Some models even show thousandths.

- Circular pie pieces which include tenths and fifths. (If not used earlier, they should be included prior to work with decimals, since children's connections with fractional parts of pies is very strong.)
- Larger interlocking pie pieces marked around the edge in tenths and hundredths.
- Squares drawn on paper cut into tenths, hundredths, and thousandths. (Models are given on page 43.)
- Metre sticks with decimetre strips and centimetre squares of tagboard.
- Place value pieces, such as centimetre strips and squares.

These materials can all be used as fraction models, and many of the activities described earlier should be done with these. It is important to use some of the same models for fractions and decimals so that connections between the two can be developed.

OBJECTIVE:

To extend previously learned fraction concepts to fractions with denominators of 10, 100, and 1000.

1. Using all of the models above, go from fraction to model and from model to fraction. That is, given a model, name the fraction and vice versa.

2. Emphasize fraction equivalences between tenths, hundredths, and thousandths. Show by means of the models that 23/100 is the same as 2/10 and 3/100. (Do not use any reference to common denominators or symbolic addition rules. Use only models.)

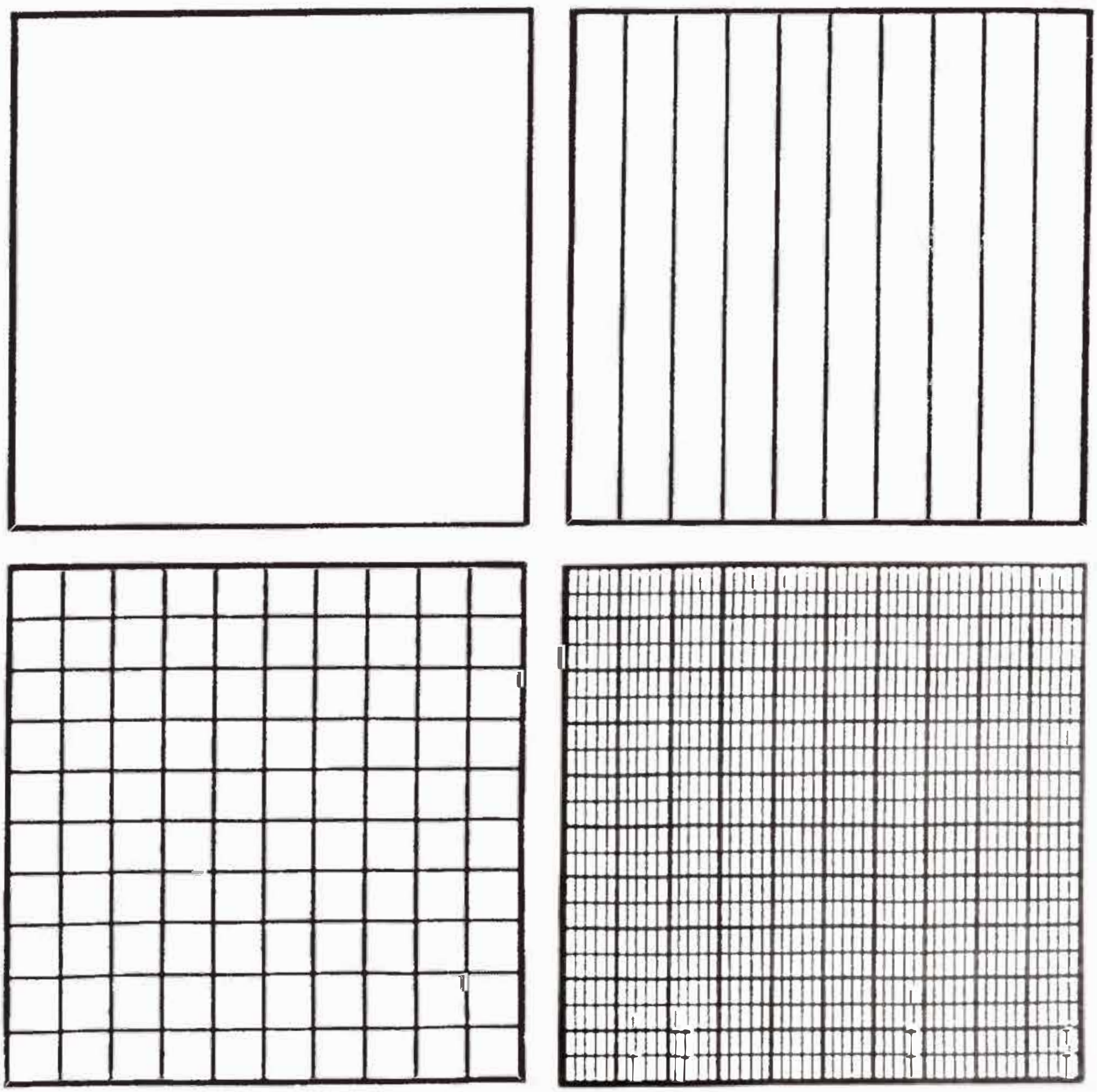
   The following examples are to be done with children first in an oral mode using models.

27/100 = □/10 and □/100 and 4/10 and 3/100 = □/100

6/10 = □/100 and 40/100 = □/10

Use similar examples with thousandths when appropriate models are available. Children apparently do not automatically extend these ideas to thousandths without working with models.

Decimal Squares



OBJECTIVE:
To develop equivalences between simple and familiar fractions and fractions with 10, 100, and 1000 as denominators.

1. Using the decimal fraction models, have children find equivalent names for these simple fractions:

   - halves, fourths, fifths (requires only tenths and hundredths)
   - eighths (requires thousandths)
   - thirds (requires a discussion of continuous subdivision by tens)

2. Provide common and familiar models (counters, pies, and strips) for fractions in the above families, and ask students to show the same amount using one of the decimal models.

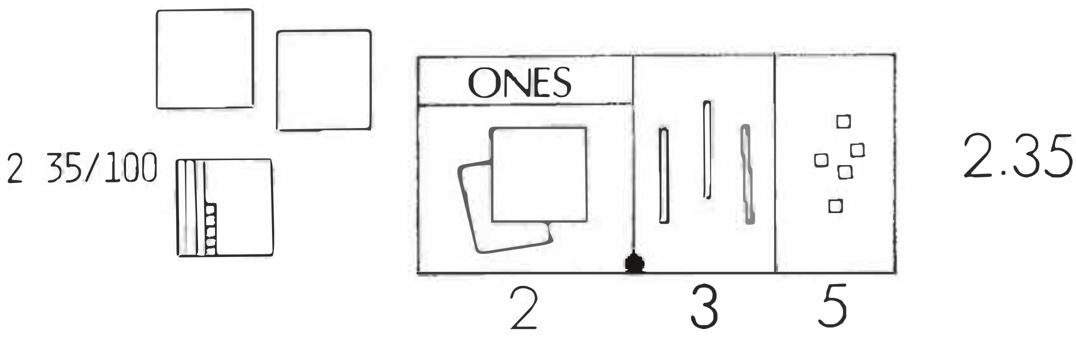
OBJECTIVE:
To introduce decimal symbolism.

The following activities are based on children's familiarity with the whole number place value system. In particular, children should know that 10 of anything in one position is the same as one in the position to the left. Ten ones makes one 10, 10 tens makes a hundred, and so on. Likewise, one in any position can be exchanged for 10 in the position to the right.

With this understanding, decimals may be introduced as another convention for writing fractions. That is, we can show children a way to write simple fractions using "regular" numbers. Decimal numeration in this scheme comes after an understanding of fractions and even fractions with "decimal" denominators (10, 100, 1000 . . .). The reverse approach is essentially to assume that children can somehow extend the whole number decimal concept to place values less than one. Later, we try to explain that these are really just fractions. The latter way seems developmentally awkward, if not backward.

1. Using centimetre strips and squares, agree that the 10 by 10 square will represent one. Discuss briefly how a ten would be made of 10 of these squares (a 10 by 100 strip), and a hundred made of 100 of these squares (a 100 by 100 square). Discuss where each would go on a place value chart. If you wanted to place the strips (that the children already know to be 1/10), where would they go? Clearly, to the right of the ones (by pattern or progression). Now ask students to show, using their place value pieces, a number such as 3 7/10. Place the pieces on the place value chart, and write down what the chart shows. Enter the decimal point, because without the decimal, 3 7/10 looks like 37. The decimal is needed to indicate which digit is the ones digit.

   With this introduction, have children first model with the strips and squares various fractions and mixed numbers that are tenths and hundredths. The pieces are then moved to a place value chart and the decimal number written.



Model as a fraction. Put on place value chart. Write decimal.

2. Use other fraction models subdivided in tenths and hundredths (from the list provided earlier), and have children translate fractions illustrated with these models to decimal numeration. To do this, they first show the same fraction using strips and squares, and then move these pieces to a place value chart.

3. Give children fractions with the familiar fraction models (pies, fraction strips, sets of counters), and have them show the same fraction with a decimal fraction model.

4. Starting with nondecimal fractions from the list of familiar fractions in the previous set of activities (halves, fourths, fifths, eighths, and thirds), have students translate these to decimal numerals. This is done by modeling the fraction with strips and squares, translating it to a place value chart, and then writing the decimal numeral.

5. Give children decimal numerals, and have them show these with a decimal fraction model (the reverse of activity #2 above).

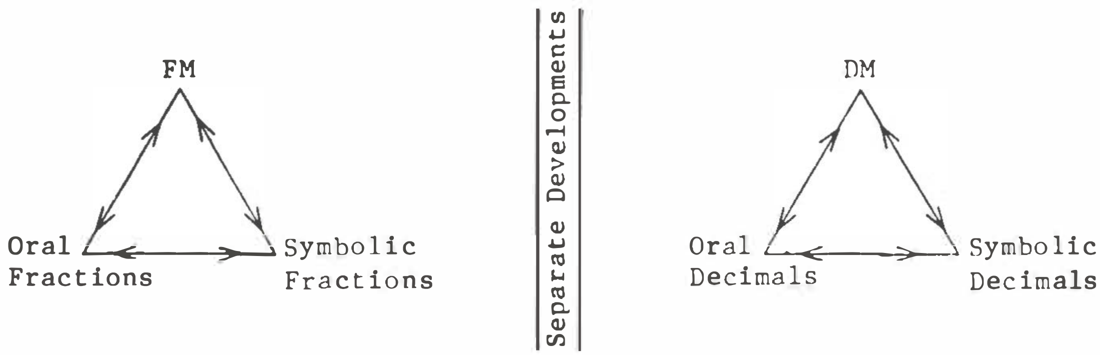
6. Give children decimal numerals, and have them show these using regular fraction models (the reverse of activity #3 above).

7. Have children move back and forth between decimal and fraction equivalents using assorted models to verify their reasoning.

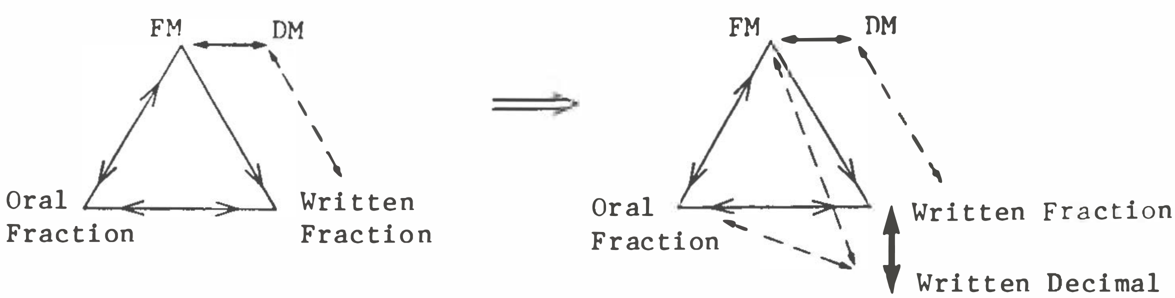
8. Use decimal models to illustrate decimals that are not "nice" fractions, but which are close to more familiar fractions. For example, which of these fractions is .23 close to: 1/2, 1/3, 1/4, or 1/5? Similar approximation exercises should be done with a wide assortment of one, two, and three-place decimal numbers.

Discussion

The traditional curriculum has kept the subjects of fractions and decimals fairly separated. Even in the best developments, oral and written names for fractions are connected to a very special set of fraction models (FM). Similarly, decimal numeration in both oral and written form is related to a very different set of models for decimals (DM). Later, children are taught decimal-fraction equivalents. This is frequently done through an unfamiliar meaning of fractions, division. Bottom numbers are divided into top numbers, and decimal equivalents somewhat magically appear. The meanings that children give to decimals in this approach is based largely on their ability to extend the whole number system and understand the words tenths, hundredths, and thousandths. Evidence suggests that this understanding of decimals is weak, and that there is very little understanding of the relationships between fractions and decimals.



The objective proposed throughout the sequence of introductory activities for decimals is essentially to firmly develop fraction concepts with standard fraction models. These concepts must be developed to the point that the notion of fraction becomes model free. That is, children will demonstrate an ability to translate fraction concepts from one model to another, and to use any model to illustrate meaning behind symbolic activities. When this is done, models that readily illustrate tenths, hundredths . . . (DM) are introduced for familiarity within the already established concept scheme. Next, the symbolic scheme for decimal numeration is introduced and connected with the concepts via the already familiar model. Decimals are, in this development, simply a new way of writing about ideas children already understand.



Notice that oral fractions and oral decimals are essentially indistinguishable. A new oral language does not need to be developed.

Clearly, this is not a complete development of decimals. Comparison of decimals is a conceptual skill that is important, but was not addressed here specifically. Nor was a general conversion scheme between fractions and decimals (if that is, in fact, desired). The ideas here are presented for trial introduction and discussion as this complex area of numeration is researched further in the classroom.

Dr. John Van de Walle is a professor of education at Virginia Commonwealth University, Richmond, Virginia. Dr. Van de Walle's article was presented to the NCTM annual meeting in San Antonio, Texas, in April 1985.

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Reconciling Differing Anti-Derivatives

David R. Duncan and Bonnie H. Litwiller
University of Northern Iowa

Three students, Cindy, Ginger, and Roy, took a calculus test. One of the questions was to evaluate the indefinite integral $\int(\sin x \cdot \cos x)dx$.

Cindy first made the substitution $u = \sin x$, yielding $du = \cos x\, dx$ and $dx = \frac{du}{\cos x}$. Then, $\int(\sin x \cdot \cos x)dx = \int(u \cos x)\frac{du}{\cos x} = \int u\, du$

$$= \frac{u^2}{2} + C$$

$$= \frac{\sin^2 x}{2} + C\ .$$

Ginger first made another substitution, namely $u = \cos x$. Then, $du = -\sin x\, dx$ and $dx = \frac{-du}{\sin x}$. $\int(\sin x \cdot \cos x)dx = \int(\sin x\, du)(\frac{-du}{\sin x})$

$$= -\int u(du)$$

$$= \frac{-u^2}{2} + C$$

$$= \frac{-\cos^2 x}{2} + C\ .$$

In his work, Roy recalled that $\sin 2x = 2(\sin x)(\cos x)$, and thus $\sin x \cdot \cos x = \frac{\sin 2x}{2}$.

$$\int(\sin x \cdot \cos x)dx = \int(\frac{\sin 2x}{2})dx$$

$$= \frac{1}{2}\int(\sin 2x)dx\ .$$

Next, Roy substituted $u = 2x$, yielding $du = 2dx$ and $dx = \frac{du}{2}$. Now, by substitution, the integral is written $= \frac{1}{2}\int(\sin u)\frac{du}{2}$

$$= \frac{1}{4}\int \sin u\, du$$

$$= -\frac{1}{4}\cos u + C$$

$$= -\frac{1}{4}\cos 2x + C\ .$$

Are these three answers, which differ in appearance, all equivalent? Recall that the constant added to the results of the integration is intended to reflect the fact that an infinite number of anti-derivatives (all differing by constants) may arise from the same integration problem.

To see the meaning of this concept in the integral problem $\int(\sin x \cdot \cos x)dx$, rewrite Ginger's and Roy's results in forms resembling Cindy's.

GINGER:
$$\frac{-\cos^2 x}{2} = \frac{-(1 - \sin^2 x)}{2}$$
$$= \frac{\sin^2 x - 1}{2}$$
$$= \frac{\sin^2 x}{2} - \frac{1}{2}$$
$$= \text{Cindy's answer} - \frac{1}{2}.$$

ROY:
$$-\frac{1}{4}\cos 2x = -\frac{1}{4}(1 - 2\sin^2 x)$$
$$= \frac{1}{2}\sin^2 x - \frac{1}{4}$$
$$= \frac{\sin^2 x}{2} - \frac{1}{4}$$
$$= \text{Cindy's answer} - \frac{1}{4}.$$

It is now apparent that these three answers do, in fact, differ by a constant. This shows that a calculus teacher must be alert in grading tests!

Challenges for the Reader:

1. Integrate $\int(\tan x \cdot \sec^2 x)dx$ in several ways and show that the answers differ by a constant.

2. Find other examples of this type of problem.

Dr. Duncan and Dr. Litwiller are professors, Department of Mathematics and Computer Science, University of Northern Iowa, Cedar Falls, Iowa. Dr. Duncan is chairman of the department. In addition to their teaching responsibilities within the department, both offer methods courses to elementary and secondary preservice teachers.

STUDENT PROBLEM CORNER

Students are encouraged to examine the problems presented below. Send your explanation or solution to:

The Editor
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c/o 2510 - 22 Avenue S
Lethbridge, Alberta
T1K 1J5

Delta-K will publish the names of students who successfully solve the problems.

Finding the Sum of Circled Numbers: Magic and Mathematics

John B. Percevault
Editor

NOTE: The following problem is suitable for students at the junior high school level.

Many tricks of magic have a mathematical basis. The magician uses knowledge of algebra to astound the audience.

Discuss the following directions and the result with your friends to determine the algebraic basis the magician used to determine the sum of the circled numbers.

Instructions

1. The magician hands a calendar and a pair of scissors to a member of the audience and asks that person to cut a four-by-four array of numbers from one month of the calendar.

   The magician states that he is not to see the four-by-four array.

2. The array is then passed to a second member of the audience, who has previously been given a "magic" marker. The magician asks the second person to circle one number only, and then to cross out the numbers in the *row* and *column* in which the circled number appears.

   While these instructions are being completed, the magician retrieves the rest of the calendar page, crumples it, and tosses the remains into a waste basket.

3. The array is now passed to a third person, and the instructions given in item #2 above are repeated. No number that has a line through it may be circled.

   Again, the magician repeats that he is not to see the circled numbers or those that are crossed out. He turns his back to the audience.

4. Now the array is passed to a fourth person, and directions given in item #2 are repeated once again.

   The magician writes something on an acetate sheet that is to be used on an overhead projector.

5. The array is now passed to a fifth member of the audience, who verifies that only one number remains which is not circled and does not have a line through it. This number is then circled.

6. The array is passed to a sixth person, who determines the sum of the four circled numbers.

   The magician moves to the overhead projector.

7. As the sixth person states the sum, the magician turns on the overhead projector. The sum given orally is the same as the numeral which appears on the screen.

Your Challenge

How did the magician know the sum?

A clue is provided. The piece of paper that was crumpled and discarded is reproduced below.

F E B R U A R Y 1 9 8 6

SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
						1
2					7	8
9			THIS IS THE FOUR-BY-FOUR ARRAY		14	15
16					21	22
23					28	

Variation of a Problem

Luigi Esposito
Student, University of Lethbridge

NOTF: The following problem is suitable for students at the upper elementary and junior high school levels.

One hundred (100) cows were in a farmer's pasture. Suddenly, a spaceship appeared, zapped the herd with a space ray, and changed some of the cows into gigantic spiders. A battle ensued, with cows killing spiders and spiders killing cows. All the bodies of the spiders and cows were disintegrated. The legs were left intact. The last animal self-destructed, leaving only the legs.

In the morning, the farmer found nothing but cows' or spiders' legs. He collected 500 legs.

How many cows were turned into giant spiders?

HELP!

We need articles for *The Canadian Mathematics Teacher*. This is your publication, and it is only as good as the members want to make it. Please let us share activities or methods you have found to be successful in your class. Provide the ideas, and we will put them in final form for publication. Handwritten copies are acceptable. Send your ideas to your *Delta-K* editor.

Let's hear from you!

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