Mathematics Through Computers

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There are two issues teachers must face regarding the use of modern technology in mathematics classes. First, they must be able to justify any additional cost, and second, they must remember that the major focus in the classroom must be on learning mathematics. These issues will be dealt with in turn.

First, what teachers do with computers must meet one or more of the following criteria:

1. There must be greater efficiency, that is, computers must save time and money.
2. There must be better results, higher scores, and greater student interest.
3. The computer must do something that should be done but is not presently being accomplished.
4. The computer must be readily and easily accessible to the teacher.

Second, teachers must focus on mathematics and the learning of mathematics. It is not the responsibility of mathematics teachers to teach computing science, computer literacy, or computer programming as a part of the mathematics curriculum. The mathematics curriculum is demanding and complete. The addition of any computer-related content will cause a decrease in mathematics content unless more time is allocated to mathematics. The teacher's responsibility is to educate students in the **content of mathematics**. If additional topics can be integrated into mathematics, it should be done. Computers should be used to reinforce and extend topics in the mathematics curriculum.

A Proposal

At present, teaching focuses on the lower cognitive skills, namely knowledge, comprehension, and application. Teaching does not adequately address the higher cognitive processes. Open search needs strengthening. The processes of analysis and synthesis are inadequately taught. Students need to strengthen skills in formalizing and generalizing.

If an innovation, the computer, is to be used in the mathematics classroom, then we must start where the classroom teacher is and move on from there. Many teachers have a knowledge of BASIC, Logo, and elementary programming--the starting points in assisting teachers to use computers in teaching mathematics.

A series of activities follows. BASIC is used in the examples. Logo could be used. The lesson outline has five parts:

1. Review of concept (skill).
2. Practice at the knowledge and comprehension levels.
3. Generalization: Students analyze specific cases and translate them into a generalized, abstract form using variables.

4. Programming. Students interprete, complete, modify, simulate, and write programs.

5. Extension. Students apply their knowledge to similar, related, or composite problems.

Activity I: Average

SETTING: The students will have a little programming experience. In class they will have just finished the conventional lesson, done some of the usual assignments (though fewer in number), and will have mastered the low level process questions.

OBJECTIVE: To have students generalize what they have learned about calculating average.

REVIEW: Very brief presentation, mostly by teacher.

PRACTICE: Assign two questions:

a. Calculate average of 34, 67, 24, 51, 69
b. Calculate average of 49, 32, 1, 86, 85, 39, 34

GENERALIZING: Ask students to state in writing how to calculate the average of the numbers. Then ask them to put the procedure into steps. Then complete these steps for the procedure:

ENTER ____________
CALCULATE ________
PRINT ____________

PROGRAMMING: Present a partially developed program for the students to complete (for those who need it) to solve PRACTICE Question "a."

```
10 PRINT "TO CALCULATE AVERAGE"
20 LET B =
30 LET C =
40 ______________
50 ______________
60 ______________
70 LET S = B+C+____
80 LET A = ___/___
90 PRINT "AVERAGE IS ";A
```

After completing the program, ask students to check it by doing PRACTICE Question "a." (This is a good procedure in problem solving.) Next, ask the students to modify the program to do PRACTICE Question "b." Students may be asked to use the INPUT statement to do the problems.

```
10 PRINT "FIND AVERAGE"
20 LET S = 0
30 INPUT B
40 LET S = S + ____
50 IF B = 69 THEN 70
```

```
60 GOTO 30
70 LET A = S/5
80 PRINT "AVERAGE IS ";A
```

EXTENSION: This section is very important

1. Students may calculate their averages on math tests to date.
2. Students may calculate the average class attendance for the past week.
3. Students may throw a ball five times, measure the distances thrown, and calculate the average.
4. Students should calculate the average of the following set of figure skating scores:

   5.5 5.7 5.8 5.2 5.9 5.5 5.8

   (Remember: The highest and lowest scores are omitted then the average is calculated.)

Activity II: Circumference of a Circle

SETTING: The students will have had a little programming experience. They will have just finished the conventional lesson and done the assigned exercises as usual (though fewer in number). They will have mastered the lower level process questions.

OBJECTIVE: To have students generalize what they have learned about circumference.

REVIEW: Very brief presentation, mostly by the teacher.

PRACTICE: Four or five practice questions are assigned. (If these are done well, then move on.)

GENERALIZING: Ask the students to state in writing how to calculate the circumference of a circle given the radius--a formula in symbol form is not acceptable! Then ask for the procedure in step form or in a flowchart. The plan is generalized into these three parts:

ENTER ______________
CALCULATE___________
PRINT ______________

PROGRAMMING: Next, present a partially developed program for the students to complete.

```
5 REM TO CALCULATE THE CIRCUMFERENCE OF
6 REM A CIRCLE GIVEN THE RADIUS = 5 CM
10 LET R = ____________________________
20 LET C = ____________________________
30 PRINT "CIRCUMFERENCE OF"
40 PRINT "CIRCLE IS ";C;" CM"
50 END
```

After completing the program, it is checked using the same question which was done in the opening.

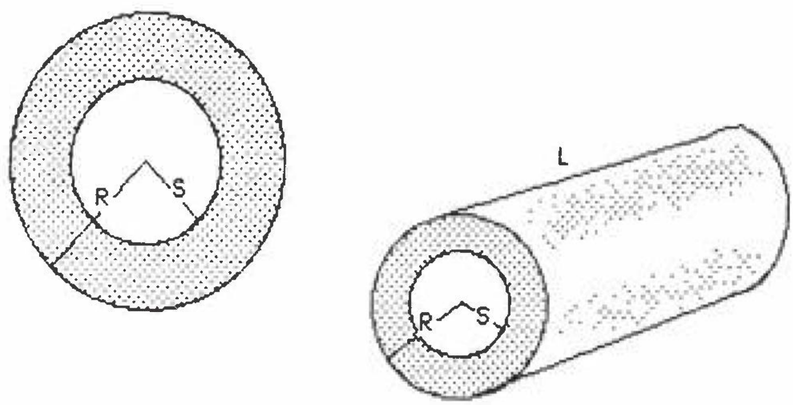
PRACTICE: This is a good procedure in problem solving. Ask students to modify the program so that the diameter can be used directly. (Students modify lines 10 and 20 as well as line 6.) Now ask the students to modify the program using the INPUT statement since the cirumference of a number of circles is to be calculated and rewriting line 10 for each is to be avoided. Students should be directed to previous exercises for which answers are available--hence, the questions are self-checking. Next, ask the students to write a program to calculate the area of a circle given the: (a) radius; (b) diameter.

EXTENSION: This section is very important. Ask students to write programs that:

a. Calculate the area of the shaded portion.

b. Calculate the volume of the cement drainage pipe.

c. Calculate the mass of the pipe if $1 m^3$ of cement has a mass of M kg.



Activity III: Discriminant

REVIEW: Discuss briefly what the discriminant is about and where it is used. Students should be familiar with

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

PRACTICE: Determine if the equation, $3x^2 - 4x + 2 = 0$, has real solutions.

GENERALIZING: Ask questions that require the students to generalize what is done when using the discriminant, such as:

1. What do you know when $b^2 - 4ac$ is negative? Tell what you will do to solve the practice question.
2. Give the steps in your solution.
3. Draw a flowchart to show your steps.

PROGRAMMING:

```
10 LET A = __________
20 LET B = __________
30 LET C = __________
40 LET D = B ^ 2 - 4*A*C
50 IF D>=0 THEN 70
60 PRINT "NO REAL SOLUTIONS"
65 GOTO 110
70 PRINT "THERE ARE REAL SOLUTIONS"
110 END
```

a. Complete the program

b. Simulate the computer.

c. Run the program and compare the answers with those in Practice 1.

d. Run the program for
   i. $x^2 - 3x - 88 = 0$
   ii. $x^2 - 8x + 16 = 0$

e. Modify the program to print the real solutions, where there are any.

```
80 LET X = (-B + SQR(D)/(2*A)
90 LET X1 = (-B - SQR(D)/(2*A)
100 PRINT "THE REAL SOLUTIONS ARE ";X;" AND ";X1
```

EXTENSION: Ask students to write programs to solve the following problems:

1. A farmer wishes to enclose a rectangular field that is already bounded on one side by a river, thus, this side will need no fence. What are the dimensions of the field and the maximun area that can be enclosed with 300 m of fence?

2. Write a program which will print, in factored form, the quadratic equation:

$$2x^2 + 11x - 21 = 0$$

Marshall P. Bye has served as president of MCATA. He has written texts used in Alberta during 1984-85 and was involved in a research project for the province and the Calgary Board of Education. In addition he has taught mathematics methods courses at the University of Calgary. Robert Michie is president of MCATA. He teaches mathematics at James Fowler High School in Calgary where he is the Assistant Principal.