Is It Time for a Truly New Mathematics?

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What we teach in mathematics in our schools is not significantly different from what has been taught in schools of the past. If one were to pick up a textbook from the 1950s, the 1930s, or the 1900s, it would be more similar to our present textbooks in topics, structure, and approach than it would be different (see Figure 1).

Figure 1

Elements of Algebra, G.A. Wentworth, 1881, p. 71.

EXERCISE XXXII.

Find the factors of:

1. $x^2+11x+24$.
2. $x^2+11x+30$.
3. $y^2+17y+60$.
4. $z^2+13z+12$.
5. $x^2+21x+110$.
6. $y^2+35y+300$.
7. $b^2+23b+102$.
8. x^2+3x+2.
9. x^2+7x+6.
10. $a^2+9ab+8b^2$.
11. $x^2+13ax+36a^2$.
12. $y^2+19py+48p^2$.
13. $z^2+29qz+100q^2$.
14. a^4+5a^2+6.
15. z^6+4z^3+3.
16. $a^2b^2+18ab+32$.
17. $x^8y^4+7x^4y^2+12$.
18. $z^{10}+10z^5+16$.
19. $a^2+9ab+20b^2$.
20. x^6+9x^3+20.
21. $a^2x^2+14abx+33b^2$.
22. $a^2c^2+7acx+10x^2$.
23. $x^2y^2z^2+19xyz+48$.
24. $b^2c^2+18abc+65a^2$.
25. $r^2s^2+23rsz+90z^2$.
26. $m^4n^4+20m^2n^2pq+51p^2q^2$.

125. CASE IV. To find the factors of

$$x^2-9x+20.$$

The second terms of the two binomial factors must be two numbers

whose *product* is 20,
and whose *sum* is -9.

The only two numbers whose product is 20 and whose sum is -9 are -5 and -4.

$$\therefore x^2-9x+20=(x-5)(x-4).$$

The so-called "new math" of the 1960s was actually an attempt to make mathematics more understandable by using set theory developed in the nineteenth century. The only topics that have been added to the public school curriculum have to do with transformational geometry and some topics in statistics and probability. Most, if not all, of these concepts were developed before the beginning of this century.

As occurs elsewhere in society, changes in technology bring about changes in our social institutions. Our social institutions, however, usually change at a much slower pace. The computer is modifying how we conduct business, how we keep information, how we view the world, and how we learn. The nature of the mathematics taught in schools today and in the future may need to be different from the mathematics required for the industrial revolution (Koetke, 1985; Ralston, 1985; Frey and Heid, 1984; Usiskin, 1985). Some of the current topics may be becoming obsolete while other topics become more significant. Educators need to decide which skills are required in order to operate in the information age and consequently design a curriculum that meets the needs of students (Bork, 1985; Tall, 1984).

If we look at the impact of calculators on the mathematics curriculum as it is currently being taught, perhaps we can gain some appreciation of the scope of the change that may be coming to mathematics education. Very few high school teachers still require students to do long calculations using logarithmic tables or slide rules. Less than 10 years ago it was uncommon to see a class of Grade 12 students working through a law of sines problem using logs and antilogs. Although some teachers will show students the square root algorithm, very few require students to complete Pythagorean theorem problems using either this method or square root tables. Teachers, particularly at the secondary level, do not spend the time that teachers once did drilling students to master basic computation skills. Instead, they allow students to use a calculator and place their focus on the concept under discussion. We find that there is a need to emphasize new skills in order that students make appropriate use of the calculator. The skills of estimation and approximation have become more significant. Teachers are becoming more concerned with teaching the analysis of problem solving rather than the correct manipulation of numbers. It seems more important for the student to know when to divide than it is for him to be able to correctly divide by a three-digit number.

One topic that has not felt the impact of the calculator is the study of radicals. Why do we have students simplify radicals and radical expressions? The reason is that they can calculate the value of the expression more easily. Without a calculator it is difficult, if not impossible, to calculate an expression with an irrational denominator, but with a calculator the answer is relatively easy (see Figure 2).

Figure 2

Comparison of Calculation of Approximate Value of a Radical Expression

$$\frac{3\sqrt{3} - 2\sqrt{2}}{\sqrt{3} - \sqrt{2}} \doteq \frac{(3\sqrt{3} - 2\sqrt{2})(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})}$$

$$\doteq \frac{5 + \sqrt{6}}{3 - 2}$$

$$\doteq 5 + \sqrt{6}$$

$\doteq$ 5 + 2.449 (using approximation; see p. 107, Math Is 5)

$$\doteq 7.449$$

Using a calculator with two memories, the following could be calculated:

$$\frac{3\sqrt{3} - 2\sqrt{2}}{\sqrt{3} - \sqrt{2}} \doteq \frac{3(1.7320508) - 2(1.4142135)}{1.7320508 - 1.4142135}$$

$$\doteq \frac{2.3677254}{0.3178373}$$

$$\doteq 7.4494887$$

The greater power of the computer relative to the calculator will likely result in a much greater impact on the mathematics curriculum over the coming years (Moursund, 1985). Which topics could be affected by the introduction of computers?

The topic that usually consumes the greatest attention in the secondary mathematics curriculum is the study of algebra. One of the major objectives of this study is to be able to factor second and third degree polynomial functions in order to be able to find the routes of the function and produce a graphic representation of that function. The underlying application is to be able to take an algebraic model of a situation and analyze the model in terms of the zeroes of the function, maximum or minimum, type of graph, and its relationship to other functions.

There may be other reasons for asking students to learn how to factor $4x^2 - 3x - 5$, but the major one is so that students can find the two routes, make a graph of the parabola, and comment on the type of function. In the typical high school program, the study of polynomials and their related functions would encompass 35 to 40 percent of the program (see Figure 3).

Figure 3

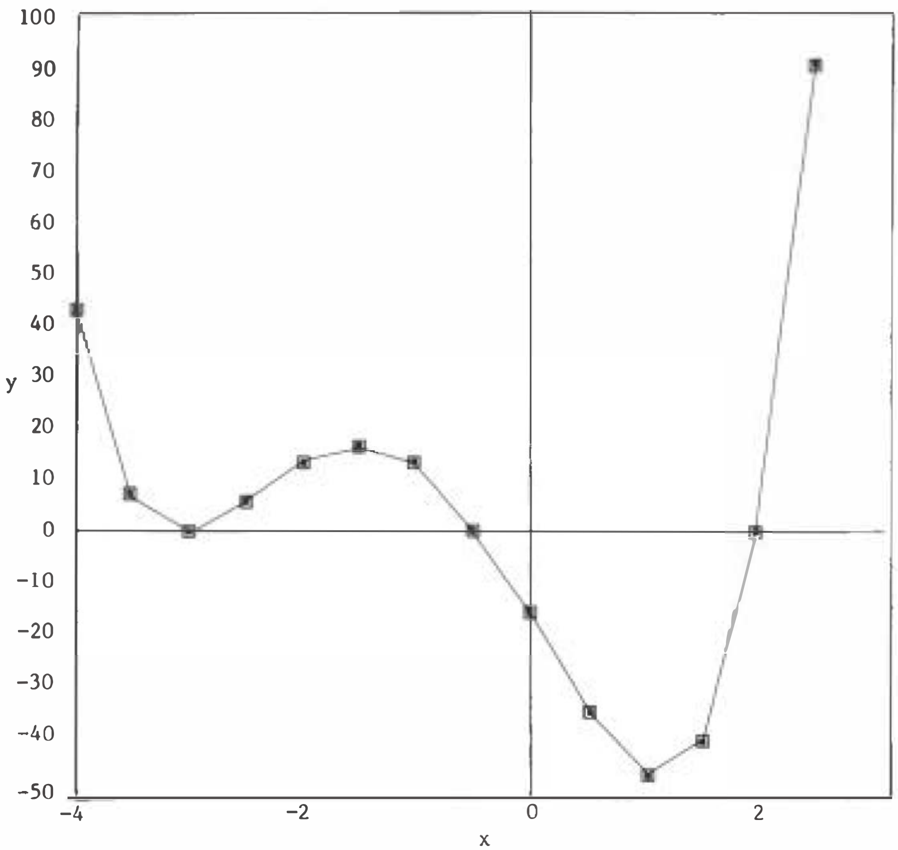
Topics and Suggested Time Allocations for High School Mathematics, Alberta Education 1982

Mathematics 10		
B.	Equations and graphing	16 h
E.	Exponents and radicals	20 h
F.	Polynomials	25 h
		61 h/125 h (49%)
Mathematics 20		
A.	Radicals	7 h
B.	Polynomials	10 h
C.	Coordinate geometry	12 h
F.	Quadratic functions, equations, and applications	20 h
G.	Systems of equations	12 h
		61 h/125 h (49%)
Mathematics 30		
B.	Quadratic relations (conic sections)	23 h
C.	Logarithms	10 h
F.	Polynomial functions	10 h
		43 h/125 h (34%)
	TOTAL	165 h/375 h (44%)

With the computer, using the spreadsheet program, it is possible to evaluate a polynomial of the second, third, or higher degree by substitution. Once the values for the function have been determined, a graph-plotting program can be called up to make the appropriate graph of the function. In our present curriculum, we usually teach students to solve any quadratic by factoring or by using the quadratic formula. This allows students to solve a very limited number of higher degree functions that happen to have "nice" routes. By using the computer, all polynomial functions are subject to analysis (see Figure 4). If the function does not have "nice" routes, then successive iterations can be used to arrive at a solution which is as accurate as desired (Kimberling).

Figure 4

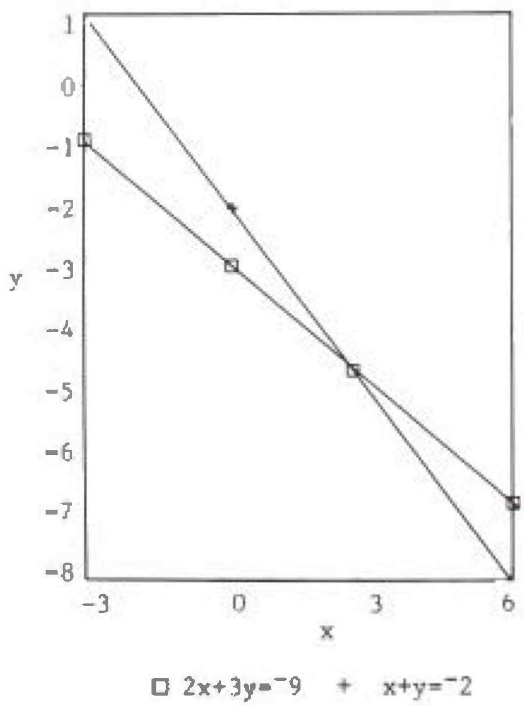
x	y=2x^4+9x^3-2x^2-39x-18
-4	42
-3.5	8.25
-3	0
-2.5	4.5
-2	12
-1.5	15.75
-1	12
-0.5	0
0	-18
0.5	-36.75
1	-48
1.5	-40.5
2	0
2.5	90.75



This figure was graphed using the Lotus 1-2-3-Program

Figure 5

Graphed using the Lotus 1-2-3 Spreadsheet



Using the same program it is possible to find the solution of systems of equations by graphing each equation and observing the point of intersection (see Figure 5).

One argument for the need to study polynomials is that an understanding of this topic and logarithms is required for the study of calculus. However, the study of calculus is in decline as a result of the introduction of computers (Usiskin, 1985; Ralston, 1985). In fact, some suggest that calculus never would have been invented if computers had been available to study how functions change at any instant.

It may be neither possible nor desirable to replace all or part of the topics listed in Figure 3, but certainly teachers and curriculum developers need to be questioning the place of these topics in "modern" high school curricula.

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References

Bork, Alfred. "Computers in Education--And Some Possible Futures." Phi Delta Kappan, December 1984, pp. 239-43.

Hansen, Viggo P., ed. National Council of Teachers of Mathematics, 1984. "Imperatives and Possibilities for New Curricula in Secondary School Mathematics." Computers in Mathematics Education by James Frey and M. Kathleen Heid.

Kimberling, Clark. "Roots: Half-Interval Search." Mathematics Teacher, February 1985, pp. 120-23.

Koetke, Walter. "Teaching Computer Science Versus Using the Computer to Enhance Mathematics Teaching." Arithmetic Teacher, March 1985, pp. 2-3.

Moursund, David. "Will Mathematics Education Rise to the Challenge of Computers?" Mathematics Teacher, December 1985, pp. 660-61.

Ralston, Anthony. "The Impact of Computer Science on School Mathematics." Mathematics Teacher, October 1985, pp. 494-95.

Ralston, Anthony. "The Really New College Mathematics and Its Impact on the High School Curriculum." The Secondary School Mathematics Curriculum. National Council of Teachers of Mathematics, 1985, pp. 29-42

Tall, David. "The Mathematics Curriculum and the Micro." Mathematics in Schools, September 1984, pp. 7-9.

Usiskin, Zalman. "We Need Another Revolution in Secondary School Mathematics." The Secondary School Mathematics Curriculum. National Council of Teachers of Mathematics, 1985, pp. 1-19.

Wentworth, G.A. Elements of Algebra. Boston: Ginn and Company, 1881.