# Tinkertoys, Tangrams, and Tessellations

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First, a brief word about the title. The original title was "Logo: Examples Illustrating Repetition and Modularity--Mind-size Chunks." The underlying idea I wanted to put forward was that of the value of modularity when approaching complexity. The expression "mind-size chunks" is familiar to readers of Mindstorms (Papert, 1980). The intention was to provide a number of examples that illustrated the application of this perspective. As I began to prepare for the paper I explored examples using tiles, which can lead to a rich investigation of both tessellation and Escher-type drawings (Ranucci and Teeters, 1977) as well as the underlying principles of symmetry (Burnett, 1980; Stevens, 1980). I also wanted to explore a much more restrictive environment, that of straight lines, which I viewed as (wooden) rods. Another open-ended environment that I have played with is that of simulating a model-train track layout (Burnett, 1985). At this point I wanted to change the title to "...hand-size chunks." However, and I suspect this may be the important point of this paper, one idea quickly and inevitably leads to another. And I value this "idea generation" as an important component of public education. The rods of my earlier interest evolved into tinkertoys, and the tiles gave rise to specialized tile environments, in particular, tangrams. Including the tracks, I now had more alliteration than I needed, so a little trimming resulted in the temporary title you now see.

Each of the following sections may be viewed as an entry into a micro-world. The word that merits emphasis in the preceding sentence is not "micro-world," but "entry." I would like to to make a case for providing less information, less information to students and perhaps more controversially, less information to teachers. The issue revolves around the relative emphasis given to processes versus that given to products. Although I do not view this distinction in terms of a dichotomy, I do believe that, with the best of intentions, we are inadvertently depriving individuals new to Logo of the opportunities to explore, either by emphasizing the syntax of the language and treating Logo as just another programming language, or by providing not only problems for students to solve, but examples of what good solutions look like. Paradoxically, it is the teachers who may be losing the most, since they are being deprived of the experience of what it genuinely feels like to be exploring in a domain where they are not sure of the consequences of their actions--an emotion that is potentially in common with many of their own students, regardless of the topic. In many cases these prescribed Logo activities are in direct response to teachers' requests for materials that they can use in their classrooms--a reasonable request, particularly by historical standards. However, such materials have the potential pitfall of turning Logo into another topic suitable for memorization. We, thus, appear to be aiming toward a subjectdominated approach rather than an inquiry and problem-solving approach. The goal seems to be to become "Logo experts," rather than "hypothesis generators." The following examples are intended to scratch the surface of some topics that appear to be rich in alternatives, both in terms of the possible approaches as well as in the nature of the resulting products.

## Tinkertoys

The seminal idea here was to do something simple. Originally, the idea was to make some comments on recursion. Thus, I began by asking the question, "What is recursion?" One of the fundamental ideas embedded in this concept is that of repetition. Therefore, it seemed appropriate to begin with an exploration of repetition. A question that quickly occurs is "What is it that you want to repeat?" The question has many facets: in music, in art, in poetry, in mathematics, perhaps even in history. Within mathematics, rather than play with repeating patterns of numbers, I wanted to stay within the turtle geometry environment and explore repeating patterns of designs. One common problem-solving heuristic is to think of a simpler problem. Thus, I asked myself, what is the simplest possible design that I could repeat? I settled on a straight line (I was not sure that dots [points] would prove to be interesting--but I would like to go back to this). The question then became: how can we repeat the straight line pattern? Somewhat surprisingly (to me) this turned out to be more complex than I initially suspected. One answer is to consider a series of lines radiating from the same point. This is the driving conceptual idea. One then enters the enabling issues related to (Logo) programming: how does one write a procedure that draws a straight line. This is even easier than drawing the proverbial Logo house. Thus:

```
TO LINE.1
   FD 70
 END
```

Even at this point it is important to test the procedure to see that the resulting graphics picture coincides with the mind's eye. The next question is how to return the turtle to its original position (without changing its orientation). One way is to retrace the path backwards (are there other ways?). One procedure for retracing a line is:

```
TO R.LINE.1
  BK 70
END
```

This procedure should also be tested. The next step is to combine these two procedures into one procedure that draws a line and returns to the starting position.

```
TO LINE
  LINE.1 R.LINE.1
END
```

This procedure should also be tested. So far, so good. Now to return to the original task of drawing a series of radiating lines. This is a natural for the REPEAT command. Thus:

```
TO DESIGN.1
  REPEAT 6[LINE RT 11]
END
```

There is not much careful planning here. I am just beginning to become familiar with the problem space. There will be time for rigor and reflective thought later. Thus, the idea is to draw a line, turn a bit, draw another line, do this a few times, and see what happens. The next step is to increase the number of repetitions to some larger number.

```
TO DESIGN.2
  REPEAT 40[LINE RT 11]
END
```

So far we are just varying the value of the REPEAT parameter, we may now want to try altering the value of the orientation command.

```
TO DESIGN.3
  REPEAT 10[LINE RT 35]
END
```

One may now explore with various combinations of REPEATs and RTs. At this stage we have satisfied the original task of creating a pattern of radiating lines, but new questions suggest themselves. Under what conditions of REPEAT and RT does the pattern repeat itself? This seems to be a deeper question than the original question of how to generate a repeating design. What other interesting things can you do with a series of radiating lines (all of the same length)?

1. Use color
2. Make the lines invisible (except at their end points, try drawing a circle!)
3. Make a Pac-man

Once again, let's return to the original question of how to repeat a simple pattern. We have explored the idea of always returning to the same start point but altering the angle of orientation. Another way to repeat a simple pattern is to simply attach the start point of the next figure to the end point of the previous figure (without changing the orientation). What would this look like in the case of our straight line segment? A longer line. This does not appear interesting from a graphical perspective, but let's explore it from a programming perspective. Using REPEAT

```
TO DESIGN.4
  REPEAT 4[LINE.1]
END
```

A line which repeats itself suggests a procedure which calls itself. Recursion!

```
TO DESIGN .5
  LINE.1
  DESIGN.5
END
```

If recursion works here, could it also work in our previous example of radiating lines?

Returning from our slight digression into recursion, it is time to create a "construction set" of tinker toy-like pieces. The rod is easy. The "point" that contains the "holes" for the rods to fit into is likely to be somewhat more interesting. How many "holes" should there be? Another question that is likely to emerge fairly early relates to the length of the rods. Are some types of lengths better than others? Why? What is the relationship between these questions and the question on the number of holes? Once one creates a set of pieces, what types of patterns and designs can one make with it? In what ways is the set more flexible than the wooden sets? In what ways is it more restrictive? What are the merits of having both? Where is the greater sense of satisfaction--in constructing the set or in using the set to construct something else? I suspect the sense of accomplishment among the early developers of Logo is enormously satisfying. One of our tasks as educators is to create situations that also permit our students to have a similar sense of accomplishment.

## Tracks

Model railway tracks are very much like tinkertoys. In both cases one attempts to combine a fixed set of pieces into some form of meaningful whole. The principal difference lies in the shape of the pieces; although they both contain straight sections, tinkertoys contain hubs as their other main shape, whereas tracks contain curved sections.

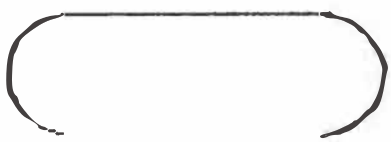
It is not too difficult to construct the two main types of track. Once again the earlier comments apply. Suppose that the student has constructed two sections, called S (Straight section) and C (Curved section). From here the modularity principle reigns supreme. Thus:

```
TO RECTANGLE
  REPEAT 2 [ REPEAT 10 [ S ]
  REPEAT 4 [ C ] ]
END
```

or equivalently,

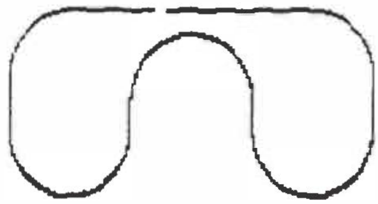
```
TO RECTANGLE
  S S S S S S S S S S C C C C
  S S S S S S S S S S C C C C
END
```

will both produce the basic train layout familiar to many children and ex-children (assuming that C produces 1/8 the circumference of a circle).



This example takes implicit advantage of the fact that all curves have the same curvature, relative to a fixed frame of reference. If one constructs both RC and LC procedures to provide for both right and left curvature, then the following slightly more interesting layout is possible.

```
TO BARBELL
  S S S S S S S
  RC RC
  S S
  RC RC RC RC
  S
  LC LC LC LC
  S
  RC RC RC RC
  S S
  RC RC
  S S S S
END
```



Why do the tracks fail to meet by just a tiny bit? In the real world of model trains, this slight gap would not likely be noticed. One would just wiggle the pieces together. However, the mathematics of the situation indicate a slight gap. It is a nice task to compute the actual length of the gap, assuming specific values for the length of the S section and for the radius of the circle corresponding to the arcs RC and LC.

In addition to the above relatively simple layouts, one can branch out into more complex track layouts involving switches. One can then get into a dynamic simulation where the switches actually work. From there it is natural to play with the idea of having a model train moving along the track. With switches and two different trains, it could become quite involved.

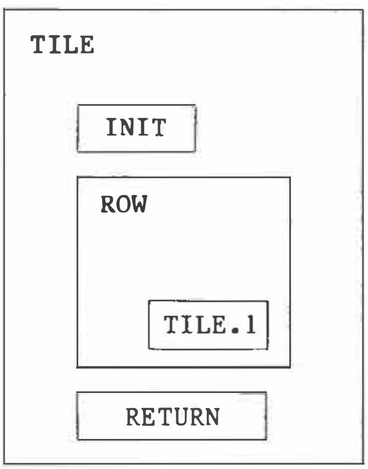
## Tiles

The objective here is to construct procedures for tiling a floor. The task breaks into two natural parts: one is to construct procedures for drawing tiles, the other is to construct procedures for placing them into interlocking patterns. One simple starting point is to begin with squares and attempt to create a checkerboard. From here two main points of departure suggest themselves: one is to play with other regular polygons such as triangles, pentagons, hexagons, and so forth. The other is to take a square and perform some form of deformation on it. This may lead into an exploration of Escher-type drawings (Ranucci & Teeters, 1977) while conducting a comprehensive treatment of mathematical ideas such as symmetry and tessellation. My intent is to only open the door. Tiling the floor with more exotic patterns is left to the reader.

Creating a square tile is relatively familiar to most users of Logo.

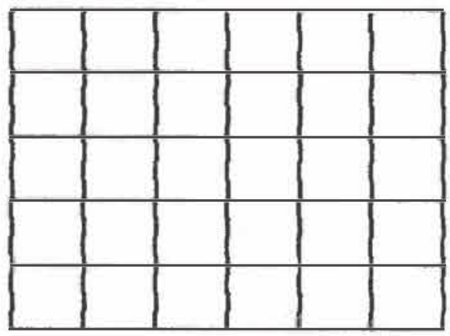
```
TO TILE.1
  PD
  REPEAT 4[FD 50 RT 90]
  PU
END
```

There are many ways to conceptualize the task of placing tiles into an interlocking pattern. Restraining ourselves to squares and thinking of checkerboards, we may begin in the lower-left corner and repeat a row of squares, then return to the left side, move up a level and repeat another row, and so on. Thus, we can visualize the task as moving the pen to the lower left portion of the screen (using a procedure called INIT), drawing a row of squares (using a procedure called ROW), returning the pen to the proper position to begin another row (using a procedure called RETURN), and repeating these latter two steps a specified number of times. A variety of box chart conventions have been recommended for representing this top-down, modular approach to writing Logo programs. In this case we might have:



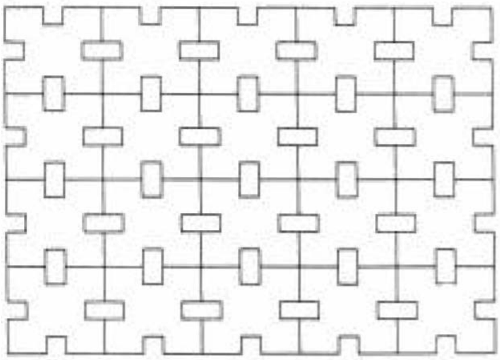

The respective procedures are all fairly straight forward.

```
TO INIT
  CS PU HT
  SETX -100 SETY -100
END

TO ROW
  REPEAT 6[TILE.1 RT 90 FD 50
  LT 90]
END

TO RETURN
  LT 90 FD 300 RT 90 FD 50
END

TO TILE
  INIT
  REPEAT 5[ROW RETURN]
END
```



At this point you may want to introduce variables to provide greater flexibility in determining the size of the square and the number of repetitions, both horizontally and vertically. You might also wish to play with alternate tiling strategies. Instead of tiling in a conventional horizontal manner you could experiment with procedures that begin near the centre of a given space and replicate the design in an outward, spiralling fashion. Other possibilities abound. You might also explore other types of symmetry operations (such as reflection) while experimenting with approaches to replication.

An introduction to Escher-type drawings can be achieved by realizing that any deformation applied to the opposite sides of a square results in a shape that will tessellate. A relatively simple example involves notching the side.



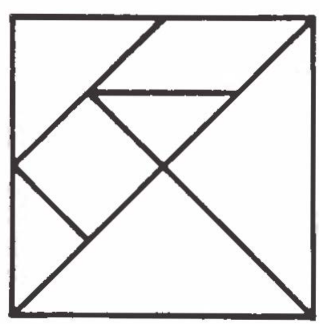
Ranucci and Teeters provide a comprehensive and entertaining account of such principles, extending their treatment to other polygonal structures. Their book encourages the reader to create his or her own patterns, a philosophy consistent with much of the Logo literature.

## Tangrams

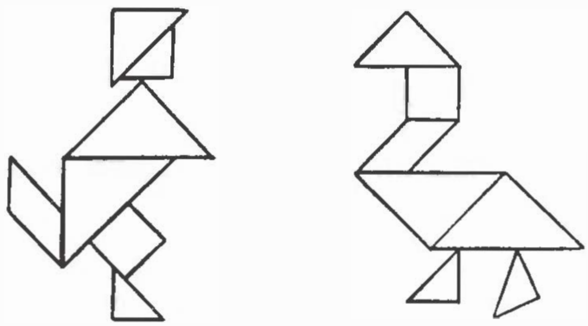
There are a number of books publically available on the ancient Chinese puzzle/game of tangrams. Seymour (1971) has provided an excellent book on how the puzzle can be incorporated into the mathematics curriculum. Perhaps a small digression is appropriate at this point. Seymour's book on tangrams, Ranucci and Teeters' on Escher-type drawings, Jacobs' (1970) treatment of billiard ball paths, and more recently, Abelson and diSessa's (1981) original contribution on turtle geometry all approach mathematics from a similar perspective. Instead of treating the (mathematics) curriculum as a fixed entity where we, as educators, strive to perfect a method for teaching it, they take as their starting point a rich and intrinsically interesting situation and in their exploration of it in depth, they reveal a number of important mathematical principles. In a fundamental sense they exemplify mathematics as a process, rather than as a set of fixed facts, rules, and algorithms suitable for memorization. The current concern in education for a return to basics provides an excellent opportunity for a discussion of what these basics might be. If we state that one of our goals is to help students learn to think, then we must give them opportunities to think.

Read (1965) also provides a description of a 15-piece puzzle, for those who feel limited by the traditional tangram format. Once one is released from the initial boundary conditions of the tangram set, it is natural to consider other possibilities, and to concomitantly consider higher order principles such as: "What types of guidelines should be considered when attempting to construct other sets of pieces so that the puzzle environment is both challenging and interesting?" I would love to hear from anyone who would like to suggest what some of these principles should be. It would be interesting to see what the collective mentality of a class could provide.

From a Logo perspective, the task is to construct a tangram environment. The task has two components: one is to construct a set of procedures for drawing the seven pieces, the other is to establish a set of conventions and possibly procedures for placing the pieces on the screen.



For the newcomer to tangrams, the following two pictures are given (out of literally thousands of possibilities) of the types of shapes that are composed with the seven pieces.



There is an underlying harmony in the above tasks. As in music, there is both point and counterpoint. Like mathematics, there is both aesthetics and rigorous thought. And from psychology (or is it philosophy?), the individual--student or teacher--must construct his or her own world.

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