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EDITORIAL

Technology and Mathematics

This issue of Delta-K examines some of the questions about the use of technology and the teaching of mathematics in Alberta schools. The reader may immediately associate the word "technology" with the microcomputer. However, before examining the microcomputer, we should first determine whether all classrooms in Alberta have access to and encourage the use of the pocket calculator?

Comments

Dale Burnett explores the use of Logo in developing repeated patterns and designs. **Joan Worth** encourages the use of calculators in the elementary schools. Practical examples of the potential use are provided. **Marshall Bye** and **Bob Michie** present the arguments that the microcomputer must be used to extend mathematics and that teaching the computer is not the responsibility of the mathematics teacher. **Ron Cammaert** encourages the use of the calculator and computer in high school mathematics. The use of Lotus 1-2-3 Spreadsheet is illustrated in solving simultaneous equations. **George Cathcart** discusses the use of Logo in upper elementary and junior high schools. The activities involving programs are presented. **Charles Binder** discusses a calculus application of the computer in determining the area under a curve. **A. Craig Loewen** discusses how a computer program was used to develop individualized work sheets for upper elementary students. The program is given. **Mary Jo Maas'** lesson plan illustrates how the calculator may be used with upper elementary and junior high school students to develop problem solving skills. The Student Problem Corner has contributions from Oscar Schaaf, Art Jorgensen, and L.G. Hoye.

The 1986-87 School Year

Three issues of Delta-K are planned for the next school year. Submissions are invited; manuscripts should be typed, double-spaced on 8 1/2" X 11" paper. The September issue will focus on the effective teaching of mathematics. Deadline for manuscripts is June 1, 1986. The theme for the January 1987 issue is "Problem Solving in the Junior High School." The deadline for submissions is November 1, 1986. The final issue will re-examine the use of technology in teaching mathematics. The deadline is March 1, 1987.

Plan to attend the NCTM Conference in Edmonton, October 16-18, 1986. Bring another teacher with you.

Tinkertoys, Tangrams, and Tessellations

J. Dale Burnett

University of Lethbridge

First, a brief word about the title. The original title was "Logo: Examples Illustrating Repetition and Modularity--Mind-size Chunks." The underlying idea I wanted to put forward was that of the value of modularity when approaching complexity. The expression "mind-size chunks" is familiar to readers of Mindstorms (Papert, 1980). The intention was to provide a number of examples that illustrated the application of this perspective. As I began to prepare for the paper I explored examples using tiles, which can lead to a rich investigation of both tessellation and Escher-type drawings (Ranucci and Teeters, 1977) as well as the underlying principles of symmetry (Burnett, 1980; Stevens, 1980). I also wanted to explore a much more restrictive environment, that of straight lines, which I viewed as (wooden) rods. Another open-ended environment that I have played with is that of simulating a model-train track layout (Burnett, 1985). At this point I wanted to change the title to "...hand-size chunks." However, and I suspect this may be the important point of this paper, one idea quickly and inevitably leads to another. And I value this "idea generation" as an important component of public education. The rods of my earlier interest evolved into tinkertoys, and the tiles gave rise to specialized tile environments, in particular, tangrams. Including the tracks, I now had more alliteration than I needed, so a little trimming resulted in the temporary title you now see.

Each of the following sections may be viewed as an entry into a micro-world. The word that merits emphasis in the preceding sentence is not "micro-world," but "entry." I would like to make a case for providing less information, less information to students and perhaps more controversially, less information to teachers. The issue revolves around the relative emphasis given to processes versus that given to products. Although I do not view this distinction in terms of a dichotomy, I do believe that, with the best of intentions, we are inadvertently depriving individuals new to Logo of the opportunities to explore, either by emphasizing the syntax of the language and treating Logo as just another programming language, or by providing not only problems for students to solve, but examples of what good solutions look like. Paradoxically, it is the teachers who may be losing the most, since they are being deprived of the experience of what it genuinely feels like to be exploring in a domain where they are not sure of the consequences of their actions--an emotion that is potentially in common with many of their own students, regardless of the topic. In many cases these prescribed Logo activities are in direct response to teachers' requests for materials that they can use in their classrooms--a reasonable request, particularly by historical standards. However, such materials have the potential pitfall of turning Logo into another topic suitable for memorization. We, thus, appear to be aiming toward a subject-

dominated approach rather than an inquiry and problem-solving approach. The goal seems to be to become "Logo experts," rather than "hypothesis generators." The following examples are intended to scratch the surface of some topics that appear to be rich in alternatives, both in terms of the possible approaches as well as in the nature of the resulting products.

Tinkertoys

The seminal idea here was to do something simple. Originally, the idea was to make some comments on recursion. Thus, I began by asking the question, "What is recursion?" One of the fundamental ideas embedded in this concept is that of repetition. Therefore, it seemed appropriate to begin with an exploration of repetition. A question that quickly occurs is "What is it that you want to repeat?" The question has many facets: in music, in art, in poetry, in mathematics, perhaps even in history. Within mathematics, rather than play with repeating patterns of numbers, I wanted to stay within the turtle geometry environment and explore repeating patterns of designs. One common problem-solving heuristic is to think of a simpler problem. Thus, I asked myself, what is the simplest possible design that I could repeat? I settled on a straight line (I was not sure that dots [points] would prove to be interesting--but I would like to go back to this). The question then became: how can we repeat the straight line pattern? Somewhat surprisingly (to me) this turned out to be more complex than I initially suspected. One answer is to consider a series of lines radiating from the same point. This is the driving conceptual idea. One then enters the enabling issues related to (Logo) programming: how does one write a procedure that draws a straight line. This is even easier than drawing the proverbial Logo house. Thus:

```
TO LINE.1
  FD 70
END
```

Even at this point it is important to test the procedure to see that the resulting graphics picture coincides with the mind's eye. The next question is how to return the turtle to its original position (without changing its orientation). One way is to retrace the path backwards (are there other ways?). One procedure for retracing a line is:

```
TO R.LINE.1
  BK 70
END
```

This procedure should also be tested. The next step is to combine these two procedures into one procedure that draws a line and returns to the starting position.

```
TO LINE
  LINE.1 R.LINE.1
END
```

This procedure should also be tested. So far, so good. Now to return to the original task of drawing a series of radiating lines. This is a natural for the REPEAT command. Thus:

```
TO DESIGN.1
  REPEAT 6[LINE RT 11]
END
```

There is not much careful planning here. I am just beginning to become familiar with the problem space. There will be time for rigor and reflective thought later. Thus, the idea is to draw a line, turn a bit, draw another line, do this a few times, and see what happens. The next step is to increase the number of repetitions to some larger number.

```
TO DESIGN.2
  REPEAT 40[LINE RT 11]
END
```

So far we are just varying the value of the REPEAT parameter, we may now want to try altering the value of the orientation command.

```
TO DESIGN.3
  REPEAT 10[LINE RT 35]
END
```

One may now explore with various combinations of REPEATs and RTs. At this stage we have satisfied the original task of creating a pattern of radiating lines, but new questions suggest themselves. Under what conditions of REPEAT and RT does the pattern repeat itself? This seems to be a deeper question than the original question of how to generate a repeating design. What other interesting things can you do with a series of radiating lines (all of the same length)?

1. Use color
2. Make the lines invisible (except at their end points, try drawing a circle!)
3. Make a Pac-man

Once again, let's return to the original question of how to repeat a simple pattern. We have explored the idea of always returning to the same start point but altering the angle of orientation. Another way to repeat a simple pattern is to simply attach the start point of the next figure to the end point of the previous figure (without changing the orientation). What would this look like in the case of our straight line segment? A longer line. This does not appear interesting from a graphical perspective, but let's explore it from a programming perspective. Using REPEAT

```
TO DESIGN.4
  REPEAT 4[LINE.1]
END
```

A line which repeats itself suggests a procedure which calls itself. Recursion!

```
TO DESIGN .5
  LINE.1
  DESIGN.5
END
```

If recursion works here, could it also work in our previous example of radiating lines?

Returning from our slight digression into recursion, it is time to create a "construction set" of tinker toy-like pieces. The rod is easy. The "point" that contains the "holes" for the rods to fit into is likely to be somewhat more interesting. How many "holes" should there be? Another question that is likely to emerge fairly early relates to the length of the rods. Are some types of lengths better than others? Why? What is the relationship between these questions and the question on the number of holes? Once one creates a set of pieces, what types of patterns and designs can one make with it? In what ways is the set more flexible than the wooden sets? In what ways is it more restrictive? What are the merits of having both? Where is the greater sense of satisfaction--in constructing the set or in using the set to construct something else? I suspect the sense of accomplishment among the early developers of Logo is enormously satisfying. One of our tasks as educators is to create situations that also permit our students to have a similar sense of accomplishment.

Tracks

Model railway tracks are very much like tinkertoys. In both cases one attempts to combine a fixed set of pieces into some form of meaningful whole. The principal difference lies in the shape of the pieces; although they both contain straight sections, tinkertoys contain hubs as their other main shape, whereas tracks contain curved sections.

It is not too difficult to construct the two main types of track. Once again the earlier comments apply. Suppose that the student has constructed two sections, called S (Straight section) and C (Curved section). From here the modularity principle reigns supreme. Thus:

```
TO RECTANGLE
  REPEAT 2 [ REPEAT 10 [ S ]
    REPEAT 4 [ C ] ]
END
```

or equivalently,

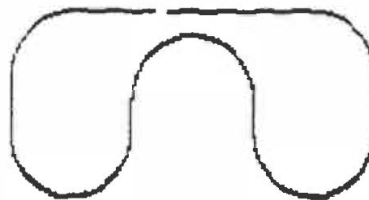
```
TO RECTANGLE
  S S S S S S S S S S C C C C
  S S S S S S S S S S C C C C
END
```

will both produce the basic train layout familiar to many children and ex-children (assuming that C produces 1/8 the circumference of a circle).



This example takes implicit advantage of the fact that all curves have the same curvature, relative to a fixed frame of reference. If one constructs both RC and LC procedures to provide for both right and left curvature, then the following slightly more interesting layout is possible.

```
TO BARBELL
  S S S S S S S
  RC RC
  S S
  RC RC RC RC
  S
  LC LC LC LC
  S
  RC RC RC RC
  S S
  RC RC
  S S S S
END
```



Why do the tracks fail to meet by just a tiny bit? In the real world of model trains, this slight gap would not likely be noticed. One would just wiggle the pieces together. However, the mathematics of the situation indicate a slight gap. It is a nice task to compute the actual length of the gap, assuming specific values for the length of the S section and for the radius of the circle corresponding to the arcs RC and LC.

In addition to the above relatively simple layouts, one can branch out into more complex track layouts involving switches. One can then get into a dynamic simulation where the switches actually work. From there it is natural to play with the idea of having a model train moving along the track. With switches and two different trains, it could become quite involved.

Tiles

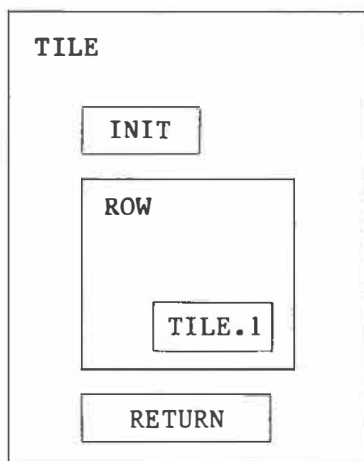
The objective here is to construct procedures for tiling a floor. The task breaks into two natural parts: one is to construct procedures for drawing tiles, the other is to construct procedures for placing them into interlocking patterns. One simple starting point is to begin with squares and attempt to create a checkerboard. From here two main points of departure suggest themselves: one is to play with other regular polygons such as triangles, pentagons, hexagons, and so forth. The other is to take a square and perform some form of deformation on it. This may lead into an exploration of Escher-type drawings (Ranucci & Teeters, 1977) while conducting a

comprehensive treatment of mathematical ideas such as symmetry and tessellation. My intent is to only open the door. Tiling the floor with more exotic patterns is left to the reader.

Creating a square tile is relatively familiar to most users of Logo.

```
TO TILE.1
  PD
  REPEAT 4[FD 50 RT 90]
  PU
END
```

There are many ways to conceptualize the task of placing tiles into an interlocking pattern. Restraining ourselves to squares and thinking of checkerboards, we may begin in the lower-left corner and repeat a row of squares, then return to the left side, move up a level and repeat another row, and so on. Thus, we can visualize the task as moving the pen to the lower left portion of the screen (using a procedure called INIT), drawing a row of squares (using a procedure called ROW), returning the pen to the proper position to begin another row (using a procedure called RETURN), and repeating these latter two steps a specified number of times. A variety of box chart conventions have been recommended for representing this top-down, modular approach to writing Logo programs. In this case we might have:



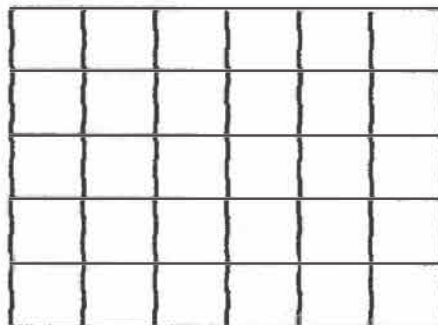
The respective procedures are all fairly straight forward.

```
TO INIT
  CS PU HT
  SETX -100 SETY -100
END

TO ROW
  REPEAT 6[TILE.1 RT 90 FD 50
  LT 90]
END

TO RETURN
  LT 90 FD 300 RT 90 FD 50
END

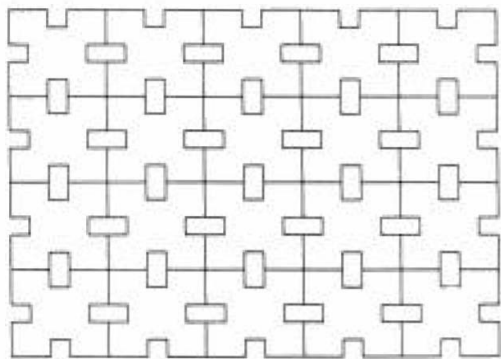
TO TILE
  INIT
  REPEAT 5[ROW RETURN]
END
```



At this point you may want to introduce variables to provide greater flexibility in determining the size of the square and the number of repetitions, both horizontally and vertically. You might also wish to play with alternate tiling strategies. Instead of tiling in a conventional horizontal manner you could experiment with procedures that begin near the centre of a given space and replicate the design in an outward, spiralling fashion. Other possibilities abound. You might also explore other types of symmetry operations (such as reflection) while experimenting with approaches to replication.

An introduction to Escher-type drawings can be achieved by realizing

that any deformation applied to the opposite sides of a square results in a shape that will tessellate. A relatively simple example involves notching the side.



Ranucci and Teeters provide a comprehensive and entertaining account of such principles, extending their treatment to other polygonal structures. Their book encourages the reader to create his or her own patterns, a philosophy consistent with much of the Logo literature.

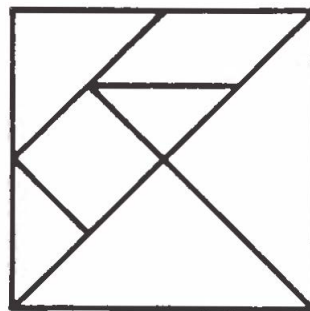
Tangrams

There are a number of books publically available on the ancient Chinese puzzle/game of tangrams. Seymour (1971) has provided an excellent book on how the puzzle can be incorporated into the mathematics curriculum. Perhaps a small digression is appropriate at this point. Seymour's book on tangrams, Ranucci and Teeters' on Escher-type drawings, Jacobs' (1970) treatment of billiard ball paths, and more recently, Abelson and diSessa's (1981) original contribution on turtle geometry all approach mathematics from a similar perspective. Instead of treating the (mathematics) curriculum as a fixed entity where we, as educators, strive to perfect a method for teaching it, they take as their starting point a rich and intrinsically interesting situation and in their exploration of it in depth, they reveal a number of

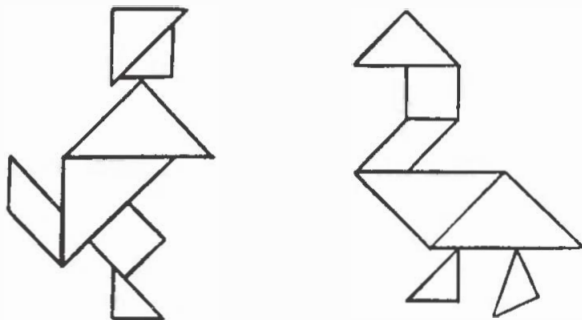
important mathematical principles. In a fundamental sense they exemplify mathematics as a process, rather than as a set of fixed facts, rules, and algorithms suitable for memorization. The current concern in education for a return to basics provides an excellent opportunity for a discussion of what these basics might be. If we state that one of our goals is to help students learn to think, then we must give them opportunities to think.

Read (1965) also provides a description of a 15-piece puzzle, for those who feel limited by the traditional tangram format. Once one is released from the initial boundary conditions of the tangram set, it is natural to consider other possibilities, and to concomitantly consider higher order principles such as: "What types of guidelines should be considered when attempting to construct other sets of pieces so that the puzzle environment is both challenging and interesting?" I would love to hear from anyone who would like to suggest what some of these principles should be. It would be interesting to see what the collective mentality of a class could provide.

From a Logo perspective, the task is to construct a tangram environment. The task has two components: one is to construct a set of procedures for drawing the seven pieces, the other is to establish a set of conventions and possibly procedures for placing the pieces on the screen.



For the newcomer to tangrams, the following two pictures are given (out of literally thousands of possibilities) of the types of shapes that are composed with the seven pieces.



There is an underlying harmony in the above tasks. As in music, there is both point and counterpoint. Like mathematics, there is both aesthetics and rigorous thought. And from psychology (or is it philosophy?), the individual--student or teacher--must construct his or her own world.

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was presented to the 1985 National Council of Teachers of Mathematics Conference which was held in Montreal.

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Let's Bring Calculators Out of the Closet

Joan Worth

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Technology is having a strong influence on elementary school mathematics programs, as indeed it should. It is no longer futuristic to talk of a technological revolution, or of a computerized society. They are here. We see evidence of calculator and computer use every day--at home, at the supermarket, at the bank, and at other commercial establishments. It has been reported that more than 75 percent of nine-year-olds either have a calculator of their own or access to a calculator at home. (Carpenter, Corbitt, Kepner, Lindquist, and Reys, 1981). To ensure that children will be able to function effectively once their formal education is completed, our instruction in school should be aimed at equipping students to cope not only with their present environment but with a changing world.

Instruction about and with calculators has been advocated for several years. In the late 1970s, numerous articles were published both in professional and popular journals and in books dealing with the pros and cons of using calculators in the elementary school. In the early 1980s, while some resistance to the use of calculators in elementary schools was still apparent, awareness of the potential instructional uses and benefits of calculators was slowly increasing as people accepted the existence of calculators in their lives and in their children's lives (Suydam, 1980).

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In April 1980, the National Council of Teachers of Mathematics (NCTM) released An Agenda for Action: Recommendations for School Mathematics of the 1980s. One of the eight recommendations dealt with computing technology: "Mathematics programs must take full advantage of the power of calculators and computers at all grade levels"(p. 8). One recommended action to accomplish this goal was as follows:

Calculators should be available for appropriate use in all mathematics classrooms, and instructional objectives should include the ability to determine sensible and appropriate uses. (p. 9)

In this province, Guidelines for the Use of Calculators Grades 1-12 was published by Alberta Education in 1981, encouraging the utilization of calculators as instructional and computational aids in all Alberta classrooms.

Given the wide accessibility and strong support for their use, what has happened in the past few years to integrate calculators into the mathematics curriculum? Not much! Calculators are not being used to advantage in many classrooms (Williams, 1983). They are, in effect, in the closet.

Factors Influencing Non-Use

What happened to slow down, if not stop, the acceptance of calculators as an important instructional aid in mathematics? Probably the most important factor was the arrival

of the microcomputer. The energies of mathematics teachers seem to have been diverted to the acquisition and use of microcomputers. In what are genuine (and laudable) efforts to further the use of technology in schools, the instructional value of calculators appears to have been forgotten or dismissed. Williams makes an interesting point when he says:

It is curious that many who questioned the educational value of the role of calculators in the mathematics education of elementary school students have no reservations about putting microcomputers in every elementary school classroom. (1983, p. 4)

Another factor contributing to the inertia in using calculators is that some of our earlier fears about them still exist. These fears which are "founded on hearsay, half-truths, and innuendos," ignore evidence from research on achievement and learning, yet form the basis of many people's beliefs and attitudes about the use of calculators (Reys, Suydam, and Lindquist, 1984, p. 11). Erroneous beliefs created by these fears need to be dispelled.

Belief 1: Students will not learn "the basics" and mathematics achievement will decline.

Over 150 studies on the effects of the use of calculators have been conducted in the last 10 years. Few other topics have received so much attention during this century. Consistently the results have indicated that "No, the calculator does not appear to affect achievement adversely" (Suydam, 1980, p. 3). In nearly all instances (97 percent), achievement scores are as high or higher when calculators are used for mathematics instruction than when they are not used for instruction, even though use of the calculator is not allowed on

the tests (Suydam, 1980). In fact, not only are there no measurable detrimental effects, there is evidence "that children in the primary grades benefit from using calculators in the study of mathematics" (Wheatley, Shumway, Coburn, Reys, Schoen, Wheatley, and White, 1979, p. 20), and "research is showing the calculator to be a powerful teaching and learning tool" (Driscoll, 1980, p. 1).

Belief 2: Students will become dependent upon calculators.

The response to this belief could be "Yes, students might become dependent upon calculators if we do not teach how to use the calculator intelligently."

Calculators are instructional tools which have certain capabilities, but which also have certain limitations. Teachers must accept the responsibility for teaching students how and when to use calculators, and thus what their limitations are. This must be done in elementary school, before attitudes and habits become ingrained. In a calculator and problem solving study, it was noted that "there was no evidence that the students became calculator dependent" (Suydam, 1979, p. 2).

Belief 3: Using calculators does not require thinking.

In a study involving 50 elementary school classrooms, Grades 2 through 6, researchers observed that "Children soon learn (much to their surprise) that the calculator does not think for them" (Wheatley et al., 1979, p. 20). Of course calculators do not think--they only follow instructions. It is the person operating the calculator who decides which keys to punch, what order to punch them in, and how to interpret the result. In the study mentioned above, both teachers and students reported

that students tried difficult mathematics problems that they would not have tackled without a calculator. In fact, a very strong argument can be made that using calculators actually increases student thinking (Reys, Suydam, and Lindquist, 1984, p. 11). Mathematics is more than computation, and using the calculator frees students from laborious and tedious computation, and forces them to dwell on the thinking that usually precedes computation.

Exposing these three beliefs as erroneous provides the most compelling reasons for using calculators with elementary school children.

Effective Uses of the Calculator

Calculators can be used as efficient and effective instructional aids in developing and practising major areas of the elementary mathematics curriculum.

Basic Facts

Clearly, a significant proportion of instruction in the early grades must continue to be based on the use of manipulative materials, which are essential for initial concept development. Equally clearly, the rapid recall of the basic facts of arithmetic must remain an important instructional goal in elementary grades. Use of a calculator has been shown to actually enhance the learning of basic facts in the primary grades (Channell, 1980).

Strong evidence also exists that students are enthusiastic about using calculators, that greater motivation for learning is accompanied by a more positive attitude toward mathematics, and that greater persistence is shown resulting in more time on task. It seems foolish not to take advantage of these "intangibles" to help all children master the basic facts of arithmetic.

Many ways of using calculators in the development and mastery of basic facts exist and can be found in the sources named in the "Student Materials" list at the end of this article. Two illustrations follow.

Beat the Calculator. One calculator with two children can provide effective practice. One child enters a basic fact in the calculator, for example, 6×9 or 5×8 , and at the same time says the fact aloud to the other child, who tries to write the answer before the first child can get the answer displayed on the calculator.

Hidden Facts. A calculator with an automatic constant addend or multiplier can provide individual practice. For example, to practise multiplying by 7, the child enters $7 \times =$ which the calculator "remembers." Any number now entered will be multiplied by 7 when $=$ is pressed. The child enters a number, for example 8, mentally does the multiplying, presses $=$ and 56 is displayed. This "hidden facts" technique can be used with any operation and any set of numbers.

Understanding and Concept Development: Operations and Place Value

Calculators emphasize when to use a certain operation and what operation to use, rather than how to perform algorithms. This emphasis can increase students' understanding of the four basic operations and of the relationships between them. The meaning and importance of place value can also be illustrated. Many examples similar to those below are contained in the sources listed in the "Student Materials" list.

Make it Big!

Put 4, 5, 6, 7 in the squares to make the the largest number possible.

$$\square \square \times \square \square$$

Which Operation?

$$8 \square 5 \square 2 = 26$$

Switch it!

Change one + to a - to get an answer of 44:

$$20 + 22 + 24 + 26$$

Wipe Out: Remove the 7

from 876543
from 0.567891

Estimation Skills

Estimation skills can be developed using a calculator, and indeed are needed in order to use a calculator effectively. Students must become alert to the reasonableness of results rather than merely accepting whatever answer the machine produces. Skill at estimation is an essential and practical daily life skill and also promotes a sense of "friendliness with numbers." Consider these examples.

Find the Number One student enters a number and pushes + then = several times. For example, enter 12 and push + then = three times. Calculator displays 48. Give the calculator to another student who is to identify the mystery number by pushing = as many times as needed. Each time = is pushed the first player gets one point. Students exchange and play again, then add their scores. The one with the highest score wins.

Target Multiplication Students are given a target area and an entry number, for example, target area 680-710, entry number 25. One student enters 25, pushes x, enters a number to try to get in the target area and pushes x again. If in the target area, that student wins. If not, the calculator is given to another student who enters a number and pushes x to get the new number in the target area. Play continues until one student "hits the target." Some experience multiplying with decimals is necessary. A similar target game may be played using divisions. For addition and subtraction target games, students are given several numbers and a target, and asked to choose two numbers whose sum or difference is closest to the target.

Pattern Searching, Exploring, and Discovering

The study of mathematics is often defined as a search for patterns. Recognizing a pattern requires a student to analyze and reflect, to notice similarities and differences, and to become aware of the distinction between essential and nonessential elements--all of which involve important aspects of problem solving. The experience of exploring and discovering patterns and relationships is enhanced by the use of the calculator because its display is easily erased and students are not discouraged by mistakes. The calculator also frees the student from the drudgery of computation where the underlying pattern can be lost in the length of the computations. The calculator only does the calculating; the student must do the thinking, observing, and testing to arrive at generalizations and discover the pattern. Pattern searching activities can also awaken

in students a fascination with numbers which helps develop a friendliness for numbers far beyond that attained by students who are merely competent "symbol manipulators." Following are examples of such activities. Again, the reader is referred to the "Student Materials" list for additional sources of activities.

Counting A calculator with a constant addend/multiplier is necessary for most counting activities. Students begin by counting by ones which should emphasize the physical link between the display and pressing the keys. Usually pushing $1 + = = =$ will make the calculator count by ones starting with 1. To begin from any other number, say 50, enter $50 + 1 = = =$. Practice in counting helps students relate the size of the number to the amount of time needed to push the buttons to count to the number. Counting both on and back by ones, twos, threes, fours, and so on, can lead to many patterns, an awareness of sequence, and preparation for all the operations. Counting on or back by tens and hundreds places an emphasis on place value, as does counting by tenths and hundredths.

Nifty Nines Students multiply each of the numbers 1 through 9 by 9, in their heads, and write the products. Then the calculator is used to multiply each of the numbers 1 through 9, by 99, by 999, and by 9999. The pattern which emerges can be tested by multiplying 99,999 without the calculator.

Seeing Double Students are told to multiply their favorite two-digit number by 101, then multiply other two-digit numbers by 101. After noting the pattern, students are to predict whether

this will happen with any two-digit number, and tell why or why not.

Problem Solving

Learning to solve problems is the main reason for the study of mathematics. The NCTM Agenda for Action (1980) recommends that "problem solving be the focus of school mathematics in the 1980s" (p. 1). The development of problem solving skills is one of the major goals listed in the Elementary Mathematics Curriculum Guide for Alberta (Alberta Education, 1982). In addition, Alberta Education has recently published a monograph entitled Let Problem Solving Be the Focus for the 1980s.

The calculator may have its greatest impact on students' problem solving abilities. Often students' principal difficulty in solving problems has been the inability to compute, and the calculator eliminates this problem. Calculators thus shift attention from the drudgery of tedious computation to strategies for problem solving. Students must be able to determine what strategies and operations to use--how to solve a problem--before they can use the calculator to get a solution.

Research indicates that students solve more problems and employ a wider variety of strategies when calculators are used (Wheatley, 1980). Utilizing a calculator as a problem solving aid allows students to deal with more realistic data than only numbers that come out evenly and decreases anxiety about computational errors. In short, calculators appear to extend students' abilities to solve problems.

Again, the "Student Materials" list at the end of this article contains many sources of problem solving activities using the calculator.

Concluding Observations

Support for the use of calculators in elementary school would seem overwhelming. We have everything to gain and nothing to lose in adopting calculators as instructional tools. Related curriculum changes are inevitable--as they should be. Even now, curriculum committees and textbook authors are including calculator activities as part of the regular program. The near future will undoubtedly see a drastic reduction in the amount of time and energy devoted to insisting that students become highly proficient in performing paper-and-pencil calculations with numbers of more than two digits. Instructional time is a precious commodity, and students who are forced to spend this time learning to behave like relatively inexpensive machines are learning the skills most subject to obsolescence! Students' time should be devoted to learning skills more likely to be needed by humans in a technological world. The near future should also see the development of tests which measure more than computational facility and which allow the use of calculators.

But significant and enduring curriculum change does not depend solely on textbooks and tests. Real curriculum change begins and ends with the classroom teacher, because only the teacher can change what actually happens in the classroom. The situation is reminiscent of a story about a small boy and a wise man:

The wise man came to the village once a year to share his wisdom. The small boy, as small boys will, decided to show the village that the wise man wasn't always right. He captured a small bird and held it in his hands. He planned to ask the wise man if the bird was alive. If the wise man said, "Yes," he would squeeze the bird and the wise man would

be wrong. If the wise man said "No," he would open his hands and the bird would fly away. When the boy finally got to the front of the line and asked the wise man if the bird was alive, the wise man responded, "It's in your hands." (Immerzeel, 1976, p. 493)

And that is the situation with calculators today--only the classroom teacher can bring calculators out of the closet. It's in your hands.

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Williams, D.E. "Remember the Calculator?"
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1. Arithmetic Teacher
November 1976 (Vol. 23 No.7)
Special Calculator Issue
1906 Association Drive
Reston, Virginia 22091
2. Beginning Calculator Math
Gerardus Vervoort and Dale Mason
Fearon Pitman Publishers
c/o Canon Canada Inc.
3245 American Drive
Mississauga, Ontario L4V 1N4
3. Calculator Activities for the Classroom
Book 1
George Immerzeel and Earl Ockenga
Creative Publications, Inc.
c/o Setsco Educational Ltd.
567 Clarke Road,
Coquitlam, B.C. V3J 3X4
4. How to Develop Problem Solving Using a
Calculator
Janet Morris
National Council of Teachers of Mathema-
tics
1906 Association Drive
Reston, Virginia 22091
5. Keystrokes: Calculator Activities for
Young Students--
Book 1: Addition and Subtraction
Book 2: Multiplication and Division
Book 3: Counting and Place Value
Book 4: Exploring New Topics
Robert Reys et al.
Creative Publications, Inc.
c/o Setsco Educational Ltd.
567 Clarke Road
Coquitlam, B.C. V3J 3X4

Mathematics Through Computers

Marshall P. Bye and Robert G. Michie

Calgary

There are two issues teachers must face regarding the use of modern technology in mathematics classes. First, they must be able to justify any additional cost, and second, they must remember that the major focus in the classroom must be on learning mathematics. These issues will be dealt with in turn.

First, what teachers do with computers must meet one or more of the following criteria:

1. There must be greater efficiency, that is, computers must save time and money.
2. There must be better results, higher scores, and greater student interest.
3. The computer must do something that should be done but is not presently being accomplished.
4. The computer must be readily and easily accessible to the teacher.

Second, teachers must focus on mathematics and the learning of mathematics. It is not the responsibility of mathematics teachers to teach computing science, computer literacy, or computer programming as a part of the mathematics curriculum. The mathematics curriculum is demanding and complete. The addition of any computer-related content will cause a decrease in mathematics content unless more time is allocated to mathematics. The teacher's responsibility is to educate students in the content of **mathematics**. If additional topics can be integrated into mathematics, it should be done. Computers should be used to reinforce and extend topics in the mathematics curriculum.

A Proposal

At present, teaching focuses on the lower cognitive skills, namely knowledge, comprehension, and application. Teaching does not adequately address the higher cognitive processes. Open search needs strengthening. The processes of analysis and synthesis are inadequately taught. Students need to strengthen skills in formalizing and generalizing.

If an innovation, the computer, is to be used in the mathematics classroom, then we must start where the classroom teacher is and move on from there. Many teachers have a knowledge of BASIC, Logo, and elementary programming--the starting points in assisting teachers to use computers in teaching mathematics.

A series of activities follows. BASIC is used in the examples. Logo could be used. The lesson outline has five parts:

1. Review of concept (skill).
2. Practice at the knowledge and comprehension levels.
3. Generalization: Students analyze specific cases and translate them into a generalized, abstract form using variables.

4. Programming. Students interpret, complete, modify, simulate, and write programs.
5. Extension. Students apply their knowledge to similar, related, or composite problems.

Activity I: Average

SETTING: The students will have a little programming experience. In class they will have just finished the conventional lesson, done some of the usual assignments (though fewer in number), and will have mastered the low level process questions.

OBJECTIVE: To have students generalize what they have learned about calculating average.

REVIEW: Very brief presentation, mostly by teacher.

PRACTICE: Assign two questions:

- a. Calculate average of 34, 67, 24, 51, 69
- b. Calculate average of 49, 32, 1, 86, 85, 39, 34

GENERALIZING: Ask students to state in writing how to calculate the average of the numbers. Then ask them to put the procedure into steps. Then complete these steps for the procedure:

```

ENTER _____
CALCULATE _____
PRINT _____

```

PROGRAMMING: Present a partially developed program for the students to complete (for those who need it) to solve PRACTICE Question "a."

```

10 PRINT "TO CALCULATE AVERAGE"
20 LET B =
30 LET C =
40 _____
50 _____
60 _____
70 LET S = B+C+ _____
80 LET A = _____ / _____
90 PRINT "AVERAGE IS ";A

```

After completing the program, ask students to check it by doing PRACTICE Question "a." (This is a good procedure in problem solving.) Next, ask the students to modify the program to do PRACTICE Question "b." Students may be asked to use the INPUT statement to do the problems.

```

10 PRINT "FIND AVERAGE"
20 LET S = 0
30 INPUT B
40 LET S = S + _____
50 IF B = 69 THEN 70

```

```

60 GOTO 30
70 LET A = S/5
80 PRINT "AVERAGE IS ";A

```

EXTENSION: This section is very important

1. Students may calculate their averages on math tests to date.
2. Students may calculate the average class attendance for the past week.
3. Students may throw a ball five times, measure the distances thrown, and calculate the average.
4. Students should calculate the average of the following set of figure skating scores:

5.5 5.7 5.8 5.2 5.9 5.5 5.8

(Remember: The highest and lowest scores are omitted then the average is calculated.)

Activity II: Circumference of a Circle

SETTING: The students will have had a little programming experience. They will have just finished the conventional lesson and done the assigned exercises as usual (though fewer in number). They will have mastered the lower level process questions.

OBJECTIVE: To have students generalize what they have learned about circumference.

REVIEW: Very brief presentation, mostly by the teacher.

PRACTICE: Four or five practice questions are assigned. (If these are done well, then move on.)

GENERALIZING: Ask the students to state in writing how to calculate the circumference of a circle given the radius--a formula in symbol form is not acceptable! Then ask for the procedure in step form or in a flowchart. The plan is generalized into these three parts:

```

ENTER _____
CALCULATE _____
PRINT _____

```

PROGRAMMING: Next, present a partially developed program for the students to complete.

```

5 REM TO CALCULATE THE CIRCUMFERENCE OF
6 REM A CIRCLE GIVEN THE RADIUS = 5 CM
10 LET R = _____
20 LET C = _____
30 PRINT "CIRCUMFERENCE OF"
40 PRINT "CIRCLE IS ";C;" CM"
50 END

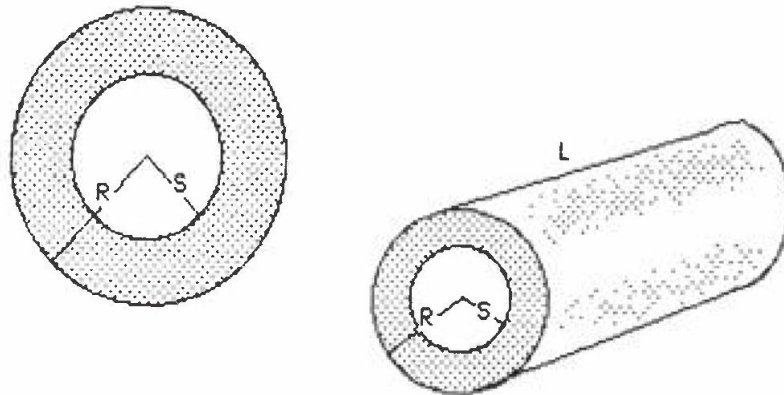
```

After completing the program, it is checked using the same question which was done in the opening.

PRACTICE: This is a good procedure in problem solving. Ask students to modify the program so that the diameter can be used directly. (Students modify lines 10 and 20 as well as line 6.) Now ask the students to modify the program using the INPUT statement since the circumference of a number of circles is to be calculated and rewriting line 10 for each is to be avoided. Students should be directed to previous exercises for which answers are available--hence, the questions are self-checking. Next, ask the students to write a program to calculate the area of a circle given the: (a) radius; (b) diameter.

EXTENSION: This section is very important. Ask students to write programs that:

- a. Calculate the area of the shaded portion.
- b. Calculate the volume of the cement drainage pipe.
- c. Calculate the mass of the pipe if 1m^3 of cement has a mass of M kg.



Activity III: Discriminant

REVIEW: Discuss briefly what the discriminant is about and where it is used. Students should be familiar with

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

PRACTICE: Determine if the equation, $3x^2 - 4x + 2 = 0$, has real solutions.

GENERALIZING: Ask questions that require the students to generalize what is done when using the discriminant, such as:

1. What do you know when $b^2 - 4ac$ is negative? Tell what you will do to solve the practice question.
2. Give the steps in your solution.
3. Draw a flowchart to show your steps.

PROGRAMMING:

```
10 LET A = _____
20 LET B = _____
30 LET C = _____
40 LET D = B ^ 2 - 4*A*C
50 IF D>=0 THEN 70
60 PRINT "NO REAL SOLUTIONS"
65 GOTO 110
70 PRINT "THERE ARE REAL SOLUTIONS"
110 END
```

- a. Complete the program
- b. Simulate the computer.
- c. Run the program and compare the answers with those in Practice 1.
- d. Run the program for
 - i. $x^2 - 3x - 88 = 0$
 - ii. $x^2 - 8x + 16 = 0$
- e. Modify the program to print the real solutions, where there are any.

```
80 LET X = (-B + SQR(D))/(2*A)
90 LET X1 = (-B - SQR(D))/(2*A)
100 PRINT "THE REAL SOLUTIONS ARE ";X;" AND ";X1
```

EXTENSION: Ask students to write programs to solve the following problems:

1. A farmer wishes to enclose a rectangular field that is already bounded on one side by a river, thus, this side will need no fence. What are the dimensions of the field and the maximum area that can be enclosed with 300 m of fence?
2. Write a program which will print, in factored form, the quadratic equation:
 $2x^2 + 11x - 21 = 0$

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Is It Time for a Truly New Mathematics?

Ron Cammaert

Alberta Education

What we teach in mathematics in our schools is not significantly different from what has been taught in schools of the past. If one were to pick up a textbook from the 1950s, the 1930s, or the 1900s, it would be more similar to our present textbooks in topics, structure, and approach than it would be different (see Figure 1).

The so-called "new math" of the 1960s was actually an attempt to make mathematics more understandable by using set theory developed in the nineteenth century. The only topics that have been added to the public school curriculum have to do with transformational geometry and some topics in statistics and probability. Most, if not all, of these concepts were developed before the beginning of this century.

As occurs elsewhere in society, changes in technology bring about changes in our social institutions. Our social institutions, however, usually change at a much slower pace. The computer is modifying how we conduct business, how we keep information, how we view the world, and how we learn. The nature of the mathematics taught in schools today and in the future may need to be different from the mathematics required for the industrial revolution (Koetke, 1985; Ralston, 1985; Frey and Heid, 1984; Usiskin, 1985). Some of the current topics may be becoming obsolete while other topics become more significant. Educators need to decide which skills are required in order to operate in the information age and consequently design a curriculum that meets the needs of students (Bork, 1985; Tall, 1984).

If we look at the impact of calculators on the mathematics curriculum as it is currently being taught, perhaps we can gain some appreciation of the scope of the change that may be coming to mathematics education. Very few high school teachers still

Figure 1

Elements of Algebra, G.A. Wentworth, 1881, p. 71.

EXERCISE XXXII.

Find the factors of:

- | | |
|--------------------------------|----------------------------------------|
| 1. $x^2 + 11x + 24$. | 11. $x^2 + 13ax + 36a^2$. |
| 2. $x^2 + 11x + 30$. | 12. $y^2 + 19py + 48p^2$. |
| 3. $y^2 + 17y + 60$. | 13. $z^2 + 29qz + 100q^2$. |
| 4. $z^2 + 13z + 12$. | 14. $a^4 + 5a^2 + 6$. |
| 5. $x^2 + 21x + 110$. | 15. $z^4 + 4z^2 + 3$. |
| 6. $y^2 + 35y + 300$. | 16. $a^2b^2 + 18ab + 32$. |
| 7. $b^2 + 23b + 102$. | 17. $x^2y^4 + 7x^4y^2 + 12$. |
| 8. $x^2 + 3x + 2$. | 18. $z^{10} + 10z^5 + 16$. |
| 9. $x^2 + 7x + 6$. | 19. $a^2 + 9ab + 20b^2$. |
| 10. $a^2 + 9ab + 8b^2$. | 20. $x^2 + 9x^2 + 20$. |
| 21. $a^2x^2 + 14abx + 33b^2$. | 24. $b^2c^2 + 18abc + 65a^2$. |
| 22. $a^2c^2 + 7acx + 10x^2$. | 25. $r^3s^2 + 23rsz + 90z^2$. |
| 23. $x^2y^2z^2 + 19xyz + 48$. | 26. $m^4n^4 + 20m^2n^2pq + 51p^2q^2$. |

125. CASE IV. To find the factors of

$$x^2 - 9x + 20.$$

The second terms of the two binomial factors must be two numbers

whose product is 20,

and

whose sum is -9.

The only two numbers whose product is 20 and whose sum is -9 are -5 and -4.

$$\therefore x^2 - 9x + 20 = (x - 5)(x - 4).$$

require students to do long calculations using logarithmic tables or slide rules. Less than 10 years ago it was uncommon to see a class of Grade 12 students working through a law of sines problem using logs and antilogs. Although some teachers will show students the square root algorithm, very few require students to complete Pythagorean theorem problems using either this method or square root tables. Teachers, particularly at the secondary level, do not spend the time that teachers once did drilling students to master basic computation skills. Instead, they allow students to use a calculator and place their focus on the concept under discussion. We find that there is a need to emphasize new skills in order that students make appropriate use of the calculator. The skills of

estimation and approximation have become more significant. Teachers are becoming more concerned with teaching the analysis of problem solving rather than the correct manipulation of numbers. It seems more important for the student to know when to divide than it is for him to be able to correctly divide by a three-digit number.

One topic that has not felt the impact of the calculator is the study of radicals. Why do we have students simplify radicals and radical expressions? The reason is that they can calculate the value of the expression more easily. Without a calculator it is difficult, if not impossible, to calculate an expression with an irrational denominator, but with a calculator the answer is relatively easy (see Figure 2).

Figure 2

Comparison of Calculation of Approximate Value of a Radical Expression

$$\begin{aligned} \frac{3\sqrt{3} - 2\sqrt{2}}{\sqrt{3} - \sqrt{2}} &= \frac{(3\sqrt{3} - 2\sqrt{2})(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})} \\ &= \frac{5 + \sqrt{6}}{3 - 2} \\ &= 5 + \sqrt{6} \\ &= 5 + 2.449 \text{ (using approximation; see p. 107, Math Is 5)} \\ &= 7.449 \end{aligned}$$

Using a calculator with two memories, the following could be calculated:

$$\begin{aligned} \frac{3\sqrt{3} - 2\sqrt{2}}{\sqrt{3} - \sqrt{2}} &= \frac{3(1.7320508) - 2(1.4142135)}{1.7320508 - 1.4142135} \\ &= \frac{2.3677254}{0.3178373} \\ &= 7.4494887 \end{aligned}$$

The greater power of the computer relative to the calculator will likely result in a much greater impact on the mathematics curriculum over the coming years (Moursund, 1985). Which topics could be affected by the introduction of computers?

The topic that usually consumes the greatest attention in the secondary mathematics curriculum is the study of algebra. One of the major objectives of this study is to be able to factor second and third degree polynomial functions in order to be able to find the routes of the function and produce a graphic representation of that function. The

underlying application is to be able to take an algebraic model of a situation and analyze the model in terms of the zeroes of the function, maximum or minimum, type of graph, and its relationship to other functions.

There may be other reasons for asking students to learn how to factor $4x^2 - 3x - 5$, but the major one is so that students can find the two routes, make a graph of the parabola, and comment on the type of function. In the typical high school program, the study of polynomials and their related functions would encompass 35 to 40 percent of the program (see Figure 3).

Figure 3

Topics and Suggested Time Allocations for High School Mathematics, Alberta Education 1982

Mathematics 10

B. Equations and graphing	16 h
E. Exponents and radicals	20 h
F. Polynomials	25 h
	<hr/>
	61 h/125 h (49%)

Mathematics 20

A. Radicals	7 h
B. Polynomials	10 h
C. Coordinate geometry	12 h
F. Quadratic functions, equations, and applications	20 h
G. Systems of equations	12 h
	<hr/>
	61 h/125 h (49%)

Mathematics 30

B. Quadratic relations (conic sections)	23 h
C. Logarithms	10 h
F. Polynomial functions	10 h
	<hr/>
	43 h/125 h (34%)

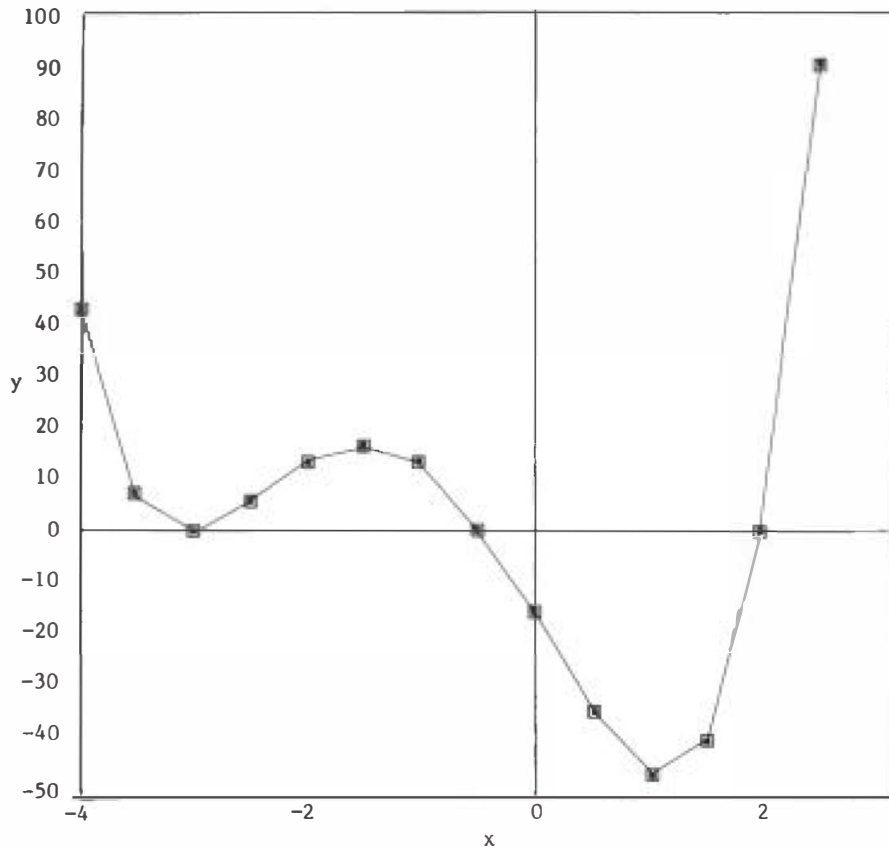
TOTAL 165 h/375 h (44%)

With the computer, using the spreadsheet program, it is possible to evaluate a polynomial of the second, third, or higher degree by substitution. Once the values for the function have been determined, a graph-plotting program can be called up to make the appropriate graph of the function. In our present curriculum, we usually teach students to solve any quadratic by factoring or

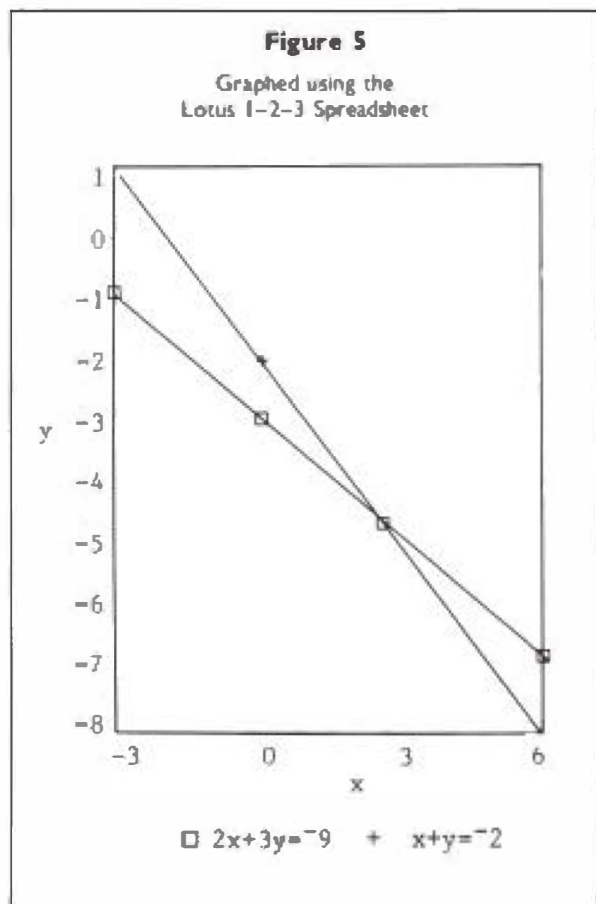
by using the quadratic formula. This allows students to solve a very limited number of higher degree functions that happen to have "nice" routes. By using the computer, all polynomial functions are subject to analysis (see Figure 4). If the function does not have "nice" routes, then successive iterations can be used to arrive at a solution which is as accurate as desired (Kimberling).

Figure 4

x	y=2x ⁴ +9x ³ -2x ² -39x-18
-4	42
-3.5	8.25
-3	0
-2.5	4.5
-2	12
-1.5	15.75
-1	12
-0.5	0
0	-18
0.5	-36.75
1	-48
1.5	-40.5
2	0
2.5	90.75



This figure was graphed using the Lotus 1-2-3-Program



Using the same program it is possible to find the solution of systems of equations by graphing each equation and observing the point of intersection (see Figure 5).

One argument for the need to study polynomials is that an understanding of this topic and logarithms is required for the study of calculus. However, the study of calculus is in decline as a result of the introduction of computers (Usiskin, 1985; Ralston, 1985). In fact, some suggest that calculus never would have been invented if computers had been available to study how functions change at any instant.

It may be neither possible nor desirable to replace all or part of the topics listed in Figure 3, but certainly teachers and curriculum developers need to be questioning the place of these topics in "modern" high school curricula.

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Logo and Measures of Central Tendency

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University of Alberta

Measures of central tendency such as the arithmetic mean, median, and mode are commonly taught in upper elementary and junior high school grades. Each of these statistics provide a different view of central tendency. The **arithmetic mean** is commonly referred to as the "average" by most students. The calculation of the mean is relatively straightforward.

The **median** is the midpoint of a set of scores; the point above and below which one-half of the scores fall. The calculation of the median is a little more involved than the calculation of the mean. Given the data, 18, 14, 12, 14, 27, 20, 30, 21, 14, it is not immediately apparent what the median is. When the data is sorted, 12, 14, 14, 14, 18, 20, 21, 27, 30, then it is obvious that 18 is the midpoint with four scores on each side. If, however, the 20 was eliminated from the list there would no longer be a specific score (data piece) that would serve as the midpoint.

12 14 14 14 ↑ 18 21 27 30

The point of balance is now midway between 14 and 18. That is, $(14 + 18)/2$.

The **mode** is the most frequently occurring value. It is probably the easiest of the three measures of central tendency to teach to children. Frequently children are asked to draw a graph (histogram) of the data. When this is done the mode is the "highest bar." The fact that some sets of data may have more than one mode adds a little difficulty to children understanding it.

Logo Procedures for Central Tendency

The activities and Logo procedures described in the remainder of this article assume that students have a reasonably good understanding of Logo, including experience with the basic list processing commands and operations. All sample procedures are written in Apple Logo (LCSI).

Writing Logo procedures to calculate the mean, median, and mode requires extensive use of some of the list processing features of Logo. Such an exercise will be a good demonstration for your students of some of the non-graphic uses of Logo.

Mean

A Logo procedure to calculate the mean is reasonably easy to write. Two steps are required: finding the sum and determining the number of scores in a set of data. Most versions of Logo have a primitive, SUM, which will calculate the sum of only two numbers. The first assignment you might give your students would be to write a procedure to calculate the sum of a list of numbers with any number of entries.

ACTIVITY 1**LOGO SUM**

Write a Logo procedure which will output (use the OUTPUT command) the sum of a list of numbers.

Elementary students or older students with minimal experience using Logo could be provided with a procedure that they could use as a primitive. You should write your own to give your students--but if you prefer you can use the following procedure.

```
TO ADDUP :ALIST
  IF :ALIST = [] [OUTPUT 0]
  OUTPUT SUM FIRST :ALIST ADDUP BUTFIRST :ALIST
END
```

Apple Logo has a COUNT primitive which returns the number of elements in a list. (If your version does not have COUNT you will have to write a short procedure to count the items in a list.) The COUNT primitive gives us the number to divide the sum by in order to obtain the mean. Assign activity 2 at this point.

ACTIVITY 2**LOGO MEAN**

Write a Logo procedure which will output (use OUTPUT) the mean of a set of numbers.

- a) Use the procedure you wrote for activity 1 (or the procedure given you by your teacher) to find the sum.
- b) Use the COUNT primitive to obtain the divisor.
- c) Use the following form for your procedure:

```
TO MEAN :ALIST
  OUTPUT _____
END
```
- d) Test your procedure using 12, 15, 20, 8, 19.

Median

To calculate the median the data should be sorted first. Only students with a good understanding of Logo will be able to write a procedure to sort data. You will likely want to give your students a sorting procedure which they can use as a primitive. Here is a procedure that sorts data into ascending order.

```
TO SORT :DATA
  IF EMPTY? :DATA [OUTPUT []]
  OUTPUT NEWLIST FIRST :DATA SORT BUTFIRST :DATA
END

TO NEWLIST :ELEMENT :DATA
  IF EMPTY? :DATA [OUTPUT (LIST :ELEMENT)]
  IF :ELEMENT < FIRST :DATA [OUTPUT FPUT :ELEMENT :DATA]
  [OUTPUT FPUT FIRST :DATA NEWLIST :ELEMENT BUTFIRST :DATA]
END
```

Next, we need to determine whether the number of entries in the data list is odd or even. This can easily be done in Logo with the REMAINDER and COUNT operations.

ACTIVITY 3

ODD/EVEN

Write a Logo statement which will determine whether the number of entries in a list is odd or even.

HINT: Experiment with REMAINDER and COUNT.

If the number of scores is odd, we have to pick out the middle score from the sorted list. If the number of scores is even, we have to find the two middle scores and compute the mean of these two scores. The Logo operations, COUNT, ITEM, and QUOTIENT, enable us to do these tasks relatively easily.

ACTIVITY 4

MIDDLE NUMBER

Write a Logo statement which will pick out the middle number from a sorted list that has an odd number of elements.

Example: Pick out the middle number, 17, in
6, 12, 17, 22, 29.

HINT: ITEM 3 :ALIST will pick out the third item
in the list called ALIST.

HINT: Experiment with QUOTIENT.

ACTIVITY 5

MIDDLE NUMBER

Write a Logo statement which will calculate the middle value for a set of data that has an even number of elements.

Example: Find the middle value, 11 in the list 5, 9, 13,
21.

ACTIVITY 6

MEDIAN

Use the statements you wrote for activities 3, 4, and 5 to write a procedure which will output (use OUTPUT) the median of any set of scores.

For activity 6, students may end up with a procedure something like this:

```
TO MEDIAN :ALIST
MAKE "SORTED.LIST SORT :ALIST
TEST REMAINDER COUNT :ALIST 2 = 0
IFFALSE [OUTPUT ITEM 1 + QUOTIENT COUNT :ALIST 2 :SORTED.LIST]
IFTRUE [OUTPUT MEAN LIST (ITEM (COUNT :ALIST)/2 :SORTED.LIST)
ITEM 1 + (COUNT :ALIST)/2 :SORTED.LIST]
END
```

Mode

While the mode may be the easiest of the three measures of central tendency to teach to children, it is probably the most difficult to write a Logo procedure for. It is recommended that you assign the mode as a challenge for your "better" programmers.

There may be a number of equally good strategies to use to get the mode. Here is one strategy that works.

1. Arrange the data into ascending order.
2. Create a list of all the unique data pieces. For example, if the sorted data was 9, 14, 14, 14, 17, 17, 20, 20, 20, then the unique list would contain 9, 14, 17, 20.
3. Count the frequency in the data list of each value in the unique list. We now have two lists in one-to-one correspondence.
Unique list: [9 14 17 20]
Frequency list: [1 3 2 3]
4. Find the maximum value in the frequency list.
5. Print all the values from the unique list which correspond to the maximum frequency.

ACTIVITY 7

UNIQUE ENTRIES

Write a Logo procedure which will create a list with all the unique elements from a set of data.

Example: 12, 18, 12, 14, 18, 21, 18

Your unique list will contain only 12, 14, 18, 21.

ACTIVITY 8

FREQUENCY COUNT

Write a Logo procedure which will count the frequency of occurrence of each value in a set of data.

Example:

Data: 12, 12, 14, 18, 18, 18, 21

Frequencies: 2, 1, 3, 1

ACTIVITY 9

MAXIMUM VALUE

Write a Logo procedure which will find the largest value in a list.

ACTIVITY 10**MODE**

Use the procedures you developed for activities 7, 8, and 9 as subprocedures in a superprocedure which will print all the modes in a set of data.

You may need to modify your procedures slightly so that they will work together.

The following superprocedure for the mode is an example of what students may end up with.

```
TO MODE :ALIST
MAKE "SORTED.LIST SORT :ALIST
MAKE "U.LIST [] (To contain the unique elements.)
MAKE "F.LIST [] (For the frequencies of each unique element.)
PRINT []
TYPE [THE MODE(S) IS (ARE):]
TYPE CHAR 32
MAKE "MAX FIND.MAX COUNT.FREQ :SORTED.LIST GET.UNIQUE :SORTED.LIST
GET.ALL.MODES :U.LIST :F.LIST
PRINT []
END
```

GET. UNIQUE takes a sorted list (ascending order) as input and outputs a list, U.LIST, of the unique elements in the list. COUNT.FREQ. then takes this U.LIST and the sorted list as inputs and outputs a list, F.LIST, of the frequencies of each data value. This list is then used as input to FIND.MAX which determines the largest frequency. Finally, GET.ALL.MODES compares each entry in the list of frequencies with the maximum. When a frequency is encountered which equals the maximum, the corresponding entry from the unique list, U.LIST, is printed.

Writing the subprocedures GET.UNIQUE, COUNT.FREQ., FIND.MAX, and GET.ALL.MODES is left as a challenge for you, the teacher, as well as for your students. Better yet, see if you can design a more elegant strategy for finding the mode using Logo.

Dr. Cathcart is a Professor in the Department of Elementary Education, University of Alberta. George has served as Delta-K editor, president of MCATA, and as a member of the executive. George invites you to compare procedures with him.

Computer Application: Area Under the Curve Probabilistic Approach

Charles Binder

Pascack Valley High School
Hillsdale, New Jersey

All of us are familiar with the typical calculus application of determining the "area under the curve." We teach this concept in our calculus classes with appropriate integration techniques, and when we cannot integrate the function in question, we revert to finding the approximate area for the interval. The most frequently used techniques are: rectangular approximations, the Trapezoidal Rule, and if, sophistication permits, Simpson's Rule.

These approximation techniques can be readily adapted to computer analysis; however, the implementation of any of them requires a rather thorough knowledge of the underlying concepts, and usually a good understanding of the limit concept.

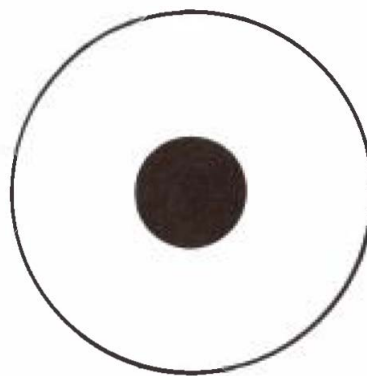
The probabilistic approach described here will work on most functions, and requires a lesser degree of sophistication in both the programming and calculus concepts. It can best be described by making an analogy to a dart game and the dart board.

Consider a game of darts, where the dart board consists of two different colored regions. Your goal is to calculate the area of one region, knowing only the entire surface area of the board. The rules are simple:

Reprinted with permission of the Association of Mathematics Teachers of New Jersey. Mr. Binder's article was first published in The New Jersey Mathematics Teacher 42, no. 2 (1985):13-17.

every dart thrown will land on the board, and its position is determined purely by random chance. Consider an unlimited number of darts where we keep track of the count of those that fall into our "targeted" region.

After many darts have been tossed we can find the ratio of darts that fall into the "targeted" region in comparison to all those thrown. The product of this ratio with the area of the entire surface will serve as our approximation.



Suppose that the area of a dart board is 81π sq. in., and of 500 darts thrown, 100 land in the "targeted" region. Thus the area of this region can be approximated by:

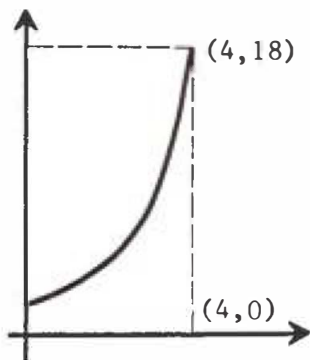
$$\frac{100}{500} * 81\pi = \frac{81\pi}{5}$$

Obviously, the more darts thrown, the better this approximation. This is where our tireless computer will come in handy.

The proper strategy for using a mathematics function and simulating this on a computer will be:

1. Sketch the curve in question, noting all boundaries.
2. Encompass this region within a rectangle of known dimensions.
3. Throw the darts and tally those that fall in the "region."
4. Compute the approximate area by taking the product of the ratio in no. 3 with the known area of the rectangle.

Now let's analyze these details with an example. Suppose our problem is to find the area of the region bounded by $y = x^2 + 2$, the x and y axes, and the line $x = 4$.



1. Sketch the curve, as shown, indicating boundary points.
2. This region can be inscribed in a rectangle whose vertices are $(0,0)$, $(4,0)$, $(4,18)$, and $(0,18)$.
3. Using the random number function with appropriate "stretching" (multiplying the random function by a value to keep the x and y values within the rectangle) parameters, we can generate the "location" of each dart by these statements: $X = 4 * \text{RND}(0)$ and $Y = 18 * \text{RND}(0)$.

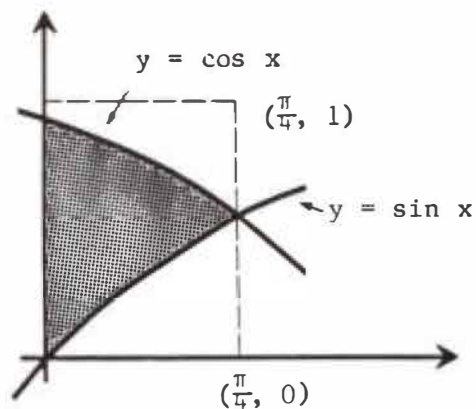
4. The tally will be accomplished by comparing the ordinate (Y) of our dart with the ordinate (for the same abscissa), on the curve:

IF $Y \leq X^2 + 2$ THEN $T = T + 1$.

If the ordinate for the dart is less than or equal to the calculated ordinate for the curve, then we will tally (T) this result. See listing no. 1 for the complete program to approximate this area.

This technique can be applied to areas of regions between two curves with only minor changes. Again, the sketch is very important, not only for the boundaries, but also to know which curve is "above" the other.

To illustrate this, let's take the area of the region bounded by $y = \sin x$ and $y = \cos x$, from 0 to $\pi/4$.



1. This sketch would clearly show that $y = \cos x$ is "above" $y = \sin x$ in this interval.
2. The rectangle would be one with vertices at $(0,0)$, $(\pi/4,0)$, $(\pi/4,1)$, and $(0,1)$, which would have an area of $\pi/4$, or approximately 0.7853.
3. The "dart" would have coordinates of:
 $X = 0.7853 * \text{RND}(0)$, $Y = \text{RND}(0)$.

4. Our check to determine whether it is in the desired region would involve a compound statement:

```
IF Y <= COS(X) AND Y >= SIN(X)
THEN T = T + 1.
```

(Refer to program listing no. 2 for the complete version of this problem.)

This probabilistic concept can easily be extended to stimulate students with other mathematical concepts. Two rather easy applications are approximating π and determining the value of $\ln x$.

The approximation of π can be determined by attempting to find the area of a unit circle. Since $A = \pi r^2$, if the radius is 1, then the area must be π . First though, let's consider finding the area of a semi-circular region, then doubling that.

The equation for the "upper" semicircle, $y = \sqrt{1 - x^2}$ can be used to easily determine the approximate area of the semicircular region. Please note that the x coordinate will have to be both "stretched and shifted" with our random function to generate the correct set of values (since the abscissas can be anywhere in the interval from -1 to 1).

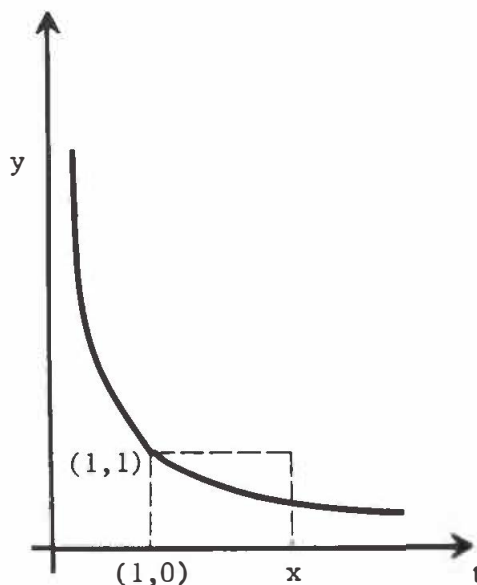
```
X = 2 * RND(0) - 1    Y = RND (0)
```

Refer to listing no. 3 for the completed program.

The value for the $\ln x$ can be determined by referring to the definition that Thomas gives in his calculus texts: $\ln x = \int_1^x 1/t (dt)$, which is an area.

Consider the case where x is greater than 1; a program can be set up dependent on a value for x entered from the keyboard by an operator of the program. The appropriate rectangle to be used will be one whose dimensions are 1 (height) by $x - 1$ (width), and this can readily be determined from our value entered from the keyboard. Refer to program listing no. 4 for the complete solution.

The program listings given on page 35 are generic enough to run on almost any computer. Slight changes may be necessary if your syntax for the random number function differs. I have used the form of this function that generates the random value between 0 and 1. Since most BASIC implementations allow five significant digits, taking into account the typical "rounding" errors, the results generated here will be improved by using more "darts" and averaging answers found on successive runs.



Probabilistic simulations can be exciting and can stimulate the students to pursue other areas. As you have seen, they do not require the student to have a sophisticated knowledge of either programming or calculus. Perhaps by examining these they will better appreciate not only the mathematical concepts, but will understand more fully how a computer can be a useful tool.

Reference

Thomas, George. Calculus and Analytical Geometry. Addison-Wesley Publishing Co., 1960.

Program Listings

```
5 REM LISTING # 1
10 REM APPROXIMATES AREA OF REGION BOUNDED BY
20 REM  $Y \pm X^2 + 2$ , THE X AND Y AXES, AND  $X = 4$ 
30 T = 0
40 FOR I = 1 TO 500
50     X = 4*RND(0)
60     Y = 18*RND(0)
70     IF  $Y \leq X^2 + 2$  THEN T = T + 1
80 NEXT I
90 A = (T/500)*72
100 PRINT"AREA IS APPROXIMATELY ";A
110 END
```

```
5 REM LISTING # 2
10 REM APPROXIMATES AREA OF REGION BETWEEN
20 REM  $Y = \sin X$  AND  $Y = \cos X$  FROM 0 TO  $1/4 \pi$ 
30 T = 0
40 FOR I = 1 TO 500
50     X = 0.7853*RND(0)
60     Y = RND(0)
70     IF  $Y \leq \cos(X)$  and  $Y \geq \sin(X)$  THEN T = T + 1
80 NEXT I
90 A = (T/500)*0.7853
100 PRINT"AREA IS APPROXIMATELY ";A
110 END
```

```
5 REM LISTING # 3
10 REM APPROXIMATES PI BY COMPUTING THE
20 REM AREA OF A UNIT CIRCLE
30 T = 0
40 FOR I = 1 TO 500
50     X = 2*RND(0) - 1
60     Y = RND(0)
70     IF  $Y \leq \sqrt{1 - (X^2)}$  THEN T = T + 1
80 NEXT I
90 A = (T/500)*2
100 P = 2 * A
110 PRINT"PI IS APPROXIMATELY ";P
120 END
```

```
5 REM LISTING # 4
10 REM APPROXIMATING THE VALUE OF LN X
20 REM X INPUT FROM KEYBOARD
30 PRINT"INPUT X"
40 INPUT X1
50 IF  $X1 \leq 1$  GOTO 30
60 FOR I = 1 TO 500
70     X = 1 + (X1 - 1)* RND(0)
80     Y = RND(0)
90     IF  $Y \leq 1/X$  THEN T = T + 1
100 NEXT I
110 A = (T/500)*(X1-1)
120 PRINT"APPROXIMATE VALUE OF LN(";X1;") IS ";A
130 END
```

Computer Assistance with Math Fact Woes

A. Craig Loewen

Rosalind School. County of Camrose

The use of computers is increasing in Alberta schools and classrooms. Teachers are left with the task of determining exactly how computers may best suit their own and their students' needs. This is an individual decision. One inexpensive, yet effective, computer application in a combined Grade 4 and 5 mathematics class is explored.

Sandra knows her multiplication and division facts perfectly. Matthew performs adequately as long as there are no factors or divisors greater than seven. Pat struggles with everything. After having worked through all the appropriate and specified exercises and activities, it became obvious that some students required remedial exercises while others would best benefit from occasional drills and exercises to ensure the retention of acquired skills. There are probably many ways of overcoming this difficulty, but the computer could be helpful. A program was written that would print computer generated mathematics fact drill sheets.

The drill sheets allow for individual differences in the students. To meet Sandra's needs, the computer could quickly print out 100 facts using all the factors 0 through 10. Just as quickly, Matthew could have a collection of questions concentrating on 8, 9, and 10 as factors. Pat could receive a series of sheets which would progressively drill her in all her weak areas. The program creates drills molded to student needs in a fraction of the time required to assemble comparable worksheets by hand or typewriter. If necessary, a single sheet can be created and duplicated for use as a class set during a timed drill or other full class exercise.

Another advantage of the program, is that unlike flashcards, it provides something tangible the students can return to the teacher. If sheets are taken home at night and returned the next day, it is easy to chart progress and areas of weakness. Flashcards seem to provide little feedback to the teacher. Identified trouble areas can be drilled again and again as each sheet the computer prints is different from the last.

The program that follows is made to be as simple as possible. It will run on any Apple II system with a printer connected in slot #1. The program asks the teacher to input minimum and maximum values for the factors. The computer does the rest. The program will print out 100 questions with factors in the range specified. When running the program, you will see something like this:

```
THIS PROGRAM GENERATES MATH WORKSHEETS
FIRST FACTOR--LOWEST VALUE: 0
FIRST FACTOR--HIGHEST VALUE: 12
SECOND FACTOR--LOWEST VALUE: 0
SECOND FACTOR--HIGHEST VALUE: 12
GENERATING WORKSHEET
```

The worksheet on the printer will have this format:

MATH FACTS DRILL
FILL IN EACH BLANK WITH THE CORRECT VALUE.

12 X 1 = _____ 0 X 10 = _____ 7 X 8 = _____ 12 X 7 = _____
10 X 10 = _____ 1 X 10 = _____ 11 X 10 = _____ 7 X 3 = _____

To make a nicer looking page for the class sets, you could beautify the sheet with clip art before duplicating.

A copy of the program follows. Before you start typing, you should know that there are other sources for similar programs. MECC has developed a package called "Mastering Math Worksheet Generator," but it is oriented specifically to the Mastering Math text series. It is available through ACCESS. Some commercial packages are available, but these can be quite expensive (\$60 to \$200), depending on the number of different types of worksheets the package is able to generate.

The Program

```
10 REM MATH DRILL SHEET GENERATOR
20 REM JANUARY 1986
30 HOME: DIM Q$(100), A$(100)
40 PRINT "THIS PROGRAM GENERATES MATH WORKSHEETS"
50 INPUT "FIRST FACTOR--LOWEST VALUE: ";FL
60 INPUT "FIRST FACTOR--HIGHEST VALUE: ";FH
70 IF FL > FH OR FH > 99 THEN PRINT "INPUT ERROR": GOTO 50
80 INPUT "SECOND FACTOR--LOWEST VALUE: ";SL
90 INPUT "SECOND FACTOR--HIGHEST VALUE: ";SH
100 IF SL > SH OR SH > 99 THEN PRINT "INPUT ERROR": GOTO 80
110 PRINT "GENERATING WORKSHEET"
120 FOR X = 1 TO 100
130 A = INT ( RND (1) * (FH - FL + 1) + FL)
140 B = INT ( RND (1) * (SH - SL + 1) + SL)
150 Q$(X) = RIGHT$ (" " + STR$ (A),3) + " X" + RIGHT$ (" " + STR$ (B),3) +
"= _____"
160 A$(X) = LEFT$ ( LEFT$ (Q$(X),12) + STR$ (A * B) + "-----", 16)
170 NEXT X
180 PR# 1
190 PRINT CHR$ (13); CHR$ (13)"MATH FACTS DRILL"
200 PRINT "FILL IN EACH BLANK WITH THE CORRECT VALUE."; CHR$ (13); CHR$(13)
210 FOR X = 1 TO 100
220 PRINT Q$(X) " ";
230 IF X / 4 = INT (X / 4) THEN PRINT CHR$ (13)
240 NEXT X
250 FOR X = 1 TO 7: PRINT CHR$ (10): NEXT
260 PRINT "MATH FACTS DRILL -- ANSWER KEY" CHR$' (13); CHR$ (13)
270 FOR X = 1 TO 100
```

```

280 PRINT A$(X) " ";
290 IF X / 4 = INT (X / 4) THEN PRINT CHR$( 13)
300 NEXT X
310 END

```

In order to make worksheets for the other three operations you will have to change these lines:

ADDITION:

```

50 INPUT "FIRST ADDEND--LOWEST VALUE: ";FL
60 INPUT "FIRST ADDEND--HIGHEST VALUE: ";FH
80 INPUT "SECOND ADDEND--LOWEST VALUE: ";SL
90 INPUT "SECOND ADDEND--HIGHEST VALUE: ";SH
150 Q$(X) = RIGHT$( " " + STR$( A),3) + " + " + RIGHT$( " " + STR$( B),3) +
" = ----"
160 A$(X) = LEFT$( ( LEFT$( Q$(X),12) + STR$( A + B) + "----",16)

```

SUBTRACTION:

```

50 INPUT "DIFFERENCE--LOWEST VALUE: ";FL
60 INPUT "DIFFERENCE--HIGHEST VALUE: ";FH
80 INPUT "SUBTRAHEND--LOWEST VALUE: ";SL
90 INPUT "SUBTRAHEND--HIGHEST VALUE: ";SH
150 Q$(X) = RIGHT$( " " + STR$( A + B),3) + " -" + RIGHT$( " " + STR$( B),3) +
" = ----"
160 A$(X) = LEFT$( ( LEFT$( Q$(X),12) + STR$( A) + "----",16)

```

DIVISION:

```

50 INPUT "QUOTIENT--LOWEST VALUE: ";FL
60 INPUT "QUOTIENT--HIGHEST VALUE: ";FH
80 INPUT "DIVISOR--LOWEST VALUE: ";SL
90 INPUT "DIVISOR--HIGHEST VALUE: ";SH
150 Q$(X) = RIGHT$( " " + STR$( A * B ),3) + " %" + RIGHT$( " " + STR$( B),3)
+ " = ----"
160 A$(X) = LEFT$( ( LEFT$( Q$(X),12) + STR$( A) + "----",16)

```

Now, simply by running any of these four programs, the teacher can create individualized worksheets or class drill sheets without the previously required time and work. In a split elementary mathematics class, time is valuable, but so is the students' learning. With this program, neither has to be sacrificed.

A. Craig Loewen teaches mathematics and computing science courses at Rosalind School in the County of Camrose. Craig was the County of Camrose nominee for the Alberta School Trustees Edwin Parr Award in 1985.

Hone on the Range

Mary Jo Maas

University of Lethbridge

Objectives

1. To practise mental arithmetic skills using decimals.
2. To round decimals.
3. To investigate multiplication by decimals greater than or less than 1.

Level

Grades 5 - 8

Materials

One calculator for every two students.

Problem Solving Skills

1. Making reasonable estimates.
2. Recognizing limits and/or eliminating possibilities.
3. Guessing and checking.

Teacher Instructions

1. Have students pair off and ensure that each group has a calculator.
2. Explain the rules:
 - a. As a pair, decide on a range, such as between 730 and 760. The objective is to get the answer to a multiplication problem to be within the range.
 - b. Designate each player as either A or B.
 - c. Player A enters a two-digit number on the calculator and pushes the X key.
 - d. Player B enters a number and pushes the X key, trying to get the answer to be within the range.
 - e. If Player B's answer is not within the range, then player A enters a number and pushes the X key, trying to get the answer to be within the range.
 - f. Play continues until one of the players does get an answer within the range.
3. Play one game with the class, with the teacher being Player A and the class Player B, to ensure that the rules are understood.

4. Suggest other possible ranges: 600 to 605, 850 to 855, 175 - 180, 335 - 340
Superhard: 199 - 200, 3000 - 3001.

Variations

Instead of using the X key, use the divide key. Pick a low range, like 40 - 45. Start with a large three-digit number, like 638.

Have enough calculators for each student. Have students play the same range of numbers and see how many steps it takes them to "hone in on the range."

Reference

Schaaf, Oscar. Problem Solving in Mathematics for Grade 7. Eugene, Oregon: Lane Education Service District, 1981.

Mary Jo Maas was a teacher at G.R. Davis School in Fort Macleod. She is currently on secondment to the Faculty of Education at the University of Lethbridge where she is teaching curriculum and instruction courses in mathematics and supervising student teachers. Jo is the secretary of MCATA.

STUDENT PROBLEM CORNER

Students are encouraged to examine the problems presented below.
Send your explanation or solution to:

The Editor
Delta-K
c/o 2510 - 22 Avenue S
Lethbridge, Alberta
T1K 1J5

Delta-K will publish the names of students who successfully solve the problems.

Make a Star

A Problem Solving Lesson for Secondary Geometry

Oscar Schaaf
University of Oregon

Comments and Suggestions

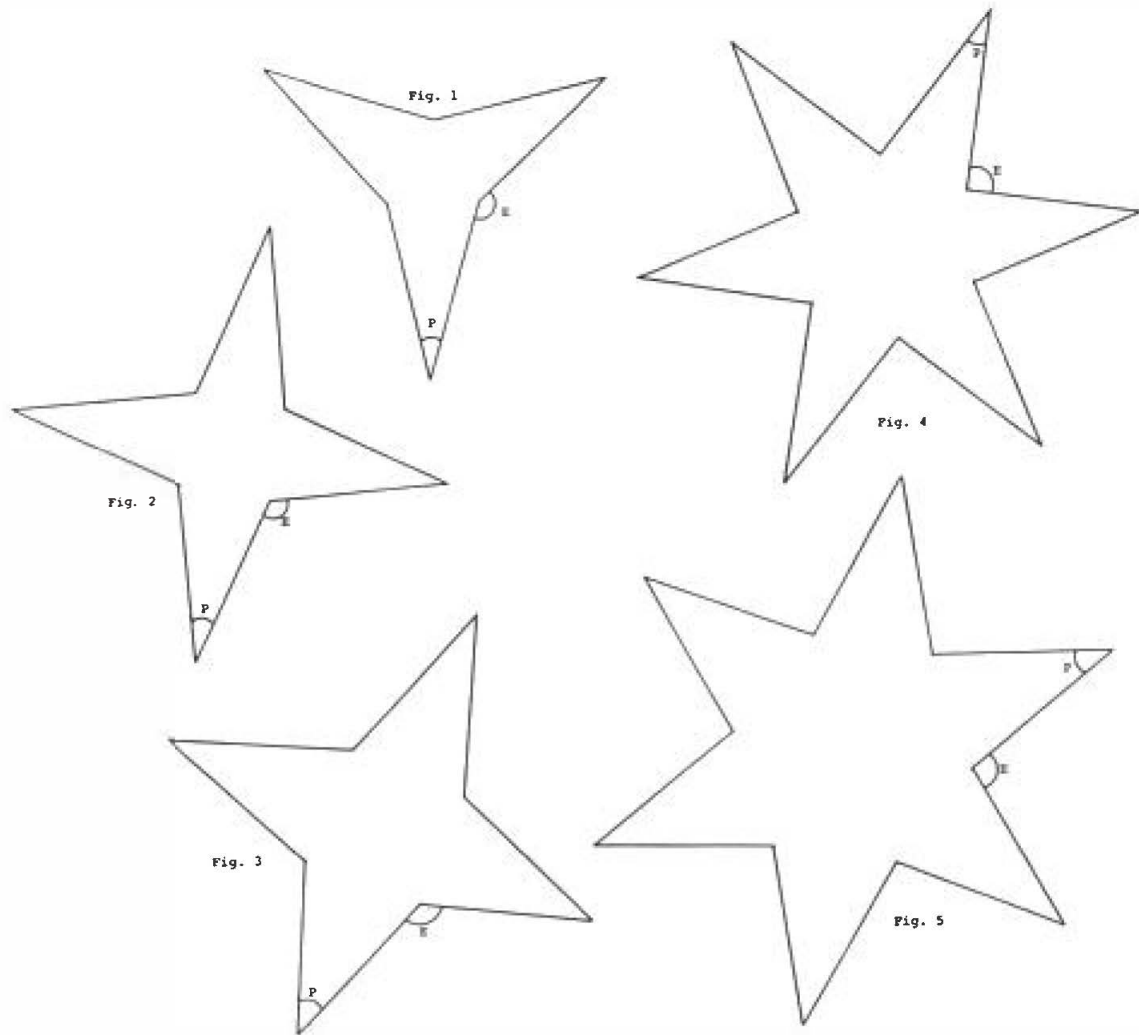
Geometry students should understand before starting this "Make a Star" lesson that the goal of the lesson is to become familiar with a process frequently used in discovering a relationship. This lesson, with the exception of problem 6, could be used in pre-high school geometry classes. The lesson does require a certain degree of skill in using a protractor. Models of stars are provided.

To the Student

A method was used to make the three-, four-, and six-pointed stars shown on page 42. The procedure depends upon the measures (m) for angles E and P, and the number (n) of points in the star. Your task is to discover the method by collecting data from the stars, recording the data in the table below, searching for a relationship (conjecture) between E, P, and n , and then testing the conjecture by drawing stars.

1. Collect the data from the stars and record in the table.

	n	mE	mP
Fig.1			
Fig.2			
Fig.3			
Fig.4			
Fig.5			



2. Study the table for patterns. Predict the measure of E if $n = 4$ and $P = 20^\circ$. Check your prediction by trying to draw the star.
3. Predict the measure of E if $n = 6$ and $P = 20^\circ$. Check your prediction by making a drawing.
4. Draw a five-pointed star using the same procedure.
5. Describe the procedure you used in making the stars or write the formula with the variables n , E , and P .
6. Prove that the relationship is true for any star drawn in this way.

Oscar Schaaf is Professor Emeritus at the College of Education, University of Oregon. He has been a speaker at many NCTM meetings including those held in Alberta. Dr. Schaaf was director of the Lane County Mathematics Project which focused on problem solving.

The Road to Four Villages

L.G. Hoyer

University of Lethbridge

Four villages are situated at the vertices of a square of sides which are one mile long. The inhabitants wish to connect the villages with a system of roads, but have only enough material to make $3 + 1$ mile(s) of road. How do they proceed?

Reference

Coxeter, H.S.M. Introduction to Geometry, 2nd Edition. (Toronto: J. Wiley & Sons, Inc., 1969), p. 392.

Professor Hoyer is an Associate Professor of Mathematics and Associate Dean of the Faculty of Arts and Science, University of Lethbridge. The problem can be solved by using algebra or calculus, or by developing a computer program.

The Altar Window

Arthur Jorgensen

Jamaica Ministry of Education

Painters are at work painting and decorating the inner walls of a church. Somewhat above the altar there is a circular window. For decoration, the painters have been asked to draw two vertical lines tangent to the circle and of the same height as the circular window. They were then to add half circles above and below, closing the figure. This area between the lines and the window is to be covered with gold. For every square inch, x amount of gold is needed. How much gold will be needed to cover this space (give the diameter of the circle); or what is the area between the circle and the lines?

Dr. Jorgensen, former editor of the MCATA Newsletter and a member of the MCATA executive, forwarded this problem from Jamaica where he is currently on an 18 month position developing elementary mathematics curricula.

National Council of Teachers of Mathematics

October 16 - 18, 1986
Westin Hotel, Edmonton

This Regional Meeting of the National Council of Teachers of Mathematics is being co-sponsored by MCATA and will feature:

- * over 100 sessions and workshops with speakers from across Canada and the U.S.
- * drop-in "Make It--Take It Workshop" open all day Friday and Saturday
- * commercial exhibits
- * special sessions for administrators

THURSDAY, OCTOBER 16:

7:30 p.m. - 9 p.m. Opening General Session
- "Effective Mathematics Teachers Make Every Minute Count,"
David Johnson, Nicolet High School, Milwaukee, Wisconsin

9 p.m. - 12 p.m. Reception and Cash Bar

FRIDAY OCTOBER 17:

8:30 a.m. - 5 p.m. Sessions and Workshops

7 p.m. - 1 a.m. Gold Rush '86
- Casino, cash bar, light supper, music, auction (\$21.00)

SATURDAY, OCTOBER 18:

7 a.m. - 8:30 a.m. Klondike Breakfast (\$9.00)
"A Sure Cure for Teacher Obsolescence," Gordon Elhard,
Calgary Board of Education

8:30 a.m. - 3 p.m. Sessions and Workshops

REGISTRATION FEES

--NCTM Members	\$33.00
--Nonmembers	74.00
--One-Day Registration (on-site only)	42.00
--elementary teachers from schools having an institutional membership in NCTM (advance registration only; no limit to members)	33.00
--registrants (one only) from organizations and institutions having institutional membership in NCTM (advance registration only)	33.00

Note: MCATA members are not automatically members of NCTM. MCATA members will receive programs containing registration forms and hotel reservation forms in June.

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