Mathematics and the Alberta High School Curriculum

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From time to time, the teaching of mathematics changes. Since about 1980, the Alberta high school syllabus has undergone a certain reform, and while some of the reasons for such change seem sound, others are more obscure and questionable. The adoption of the metric system created a necessity for an update. The easy access to hand-held calculators required a different emphasis in the area of logarithms. Such traditional topics as geometry were to be treated from a different perspective because of developments in mathematics that had filtered down to the secondary school level. The inclusion of nontraditional areas, such as statistics and the minor topic of exponential growth and decay, have raised eyebrows. In this article, an attempt will be made to identify, by subject area, a few of the anomalies and difficulties that occur in our curriculum and textbooks.

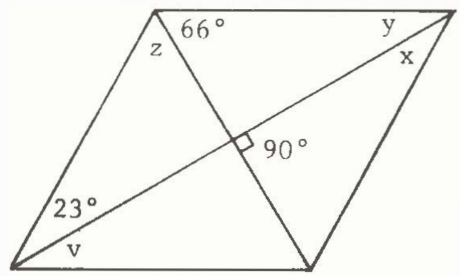
Geometry as a High School Subject

For the high school curriculum, the question of what part of geometry we present is a rather existential problem. Our present Grade 10 texts treat it very casually and with little sense of purpose, made worse by the fact that, in many places, the textbooks contain grave errors.

In the *Holt Mathematics 4* text (Hanwell, Bye, and Griffiths, p. 230), the following exercise occurs:

> Consider a parallelogram with three angles given and calculate the angles x, y, z, and v.

fig. 1



The unaware reader obtains results that correspond with the answers given in the back of the textbook. However, the exercise is completely ludicrous. Pictures and numbers have collided in a strange way. (A proper answer would be: This is a rhombus in which the diagonals are perpendicular and the diagonals also bisect the angles of the rhombus. Hence, the answer in the textbook is incorrect, and so is the "given" part.)

In order to see some of the difficulties encountered when deciding on which part of geometry should be presented in the school curriculum, we have to consider the development of geometry from a historical perspective and the more recent outlook on mathematics itself.

Once a proposition in mathematics has been settled, it becomes generally accepted. The acceptance is based on what we call proof. Over time, the significance of the proposition may change as it becomes part of a larger body of knowledge, but its quality stays the same. Since the time of Euclid, the validity of propositions in elementary geometry has been based on an axiomatic system, a collection of statements accepted as true. From these initial statements, a large collection of propositions is deduced by agreeing upon certain rules of inference. In 1931, Kurt Goedel proved that there exist axiomatic systems from which certain propositions belonging to the system can neither be proved nor disproved.

An illustration of Goedel's contention, which is even presentable in the classroom, is Goldbach's conjecture. The conjecture states that every even natural number greater than two is expressable as the sum of two primes, where primes are natural numbers divisible by one and themselves only. Up to now, no even number has been found that is not the sum of two primes. The conjecture may be true, but may not be derivable from the axioms of arithmetic. The same may apply to what is known as Fermat's last theorem. This theorem states that there are no natural numbers a, b, and c such that $a^n + b^n = c^n$ for "n" greater or equal to three and "n" a natural number. These conjectures have the charm that they can serve as illustrations in the relatively simple setting of elementary mathematics, and that, even today, these draw considerable interest from mathematicians.

The closer scrutiny of the axiomatic system was largely caused by the development of different types of geometry. In Riemannian geometry, for example, Euclid's axiom that through a point P in the plane not on line "l" a line can be drawn parallel to "l" is denied. Of course, philosophical questions arise regarding the plausibility of these geometries.

The classical belief that the properties of Euclidean geometry are valid for the world in which we live has been undermined, as it becomes evident that other geometries are equally valid. In an article entitled "Elementary Geometry, Then and Now," I.M. Yaglom (Davis, Gruenbaum, and Scherk, p. 165) speaks about geometries that draw considerable attention in this half of the twentieth century and makes a comparison to developments in the previous century. He says:

> In contrast to discrete geometry, combinatorial geometry so far has no serious practical applications; in this respect, it resembles "classical" elementary geometry, which considered properties of triangles and circles, which beautiful though they were, were scientifically blind alleys - leading nowhere, giving nothing to science at large. Still "nineteeth-century elementary geomtery" was closely bound up with what might be called the "scientific atmosphere" of those years. . . .

There are two pedagogical consequences to be drawn from Yaglom's argument. Certain aspects of geometry are culturally bound and do not necessarily lend themselves to so-called practical applications. The present curriculum seems to be preoccupied with these applications. Secondly, since Euclidean geometry is not the only valid system, we have to conclude that one of the significant objectives is to teach our students the method of a deductive system. The deductive character of a system is more easily established in Euclidean geometry than in any other part of high school mathematics. (For the 13-23-33 sequence of mathematics courses, a different perspective should prevail.)

Exponential Growth and Decay

Euclidean geometry has been, traditionally, part of the secondary school curriculum. This cannot be said of the particular minor topic presented in both approved texts for Grade 12. In order to see what is going on, we will have to go through a more or less technical explanation with omission of mathematical techniques. In the *FMT Senior* text (Dottori, Knill, and Stewart, p. 153), the exponential growth rate is explained on an intuitive basis. Since bacteria multiply by splitting, the population increases by a power of two. Without much ado, the growth function is declared to be an exponential function with base two for any increasing biological population whatsoever. It could include mice. The model in the textbook is quite reasonable as long as the bacteria are declared immortal. Such a representation violates the laws of nature.

A correct way to derive the appropriate formula for the growth rate would be by means of a simple differential equation, which is beyond the scope of high school mathematics. The proper formulation of the problem lies in the assumption that a biological population has a growth rate that is proportional to its size. In this formulation of the problem, the mortality rate is included in the hypothesis. A simple technique of elementary calculus yields the correct result. In this derivation, the base two of the textbook can be shown not to be unique. Thus, a mice population increase no longer creates a hazard for the formula.

For decay of radioactive materials, the rate of decay is again assumed to be proportional to the original mass of the material. Again, the proper formula is derived by the same differential equation. However, the textbook explanation requires the observer to watch the material for 25 years to obtain half the mass, and another 25 years to again halve the mass. After some mysterious reasoning, an exponential function emerges with the not unique base two. In *Calculus*, Volume I, Tom Apostol (p. 229) says:

> Actually, the physical laws we use here are only approximations to reality, and their motivation properly belongs to the sciences from which the various problems emanate.

The opinion has been voiced that high school courses should contain practical applications. However, some sobering thoughts come to mind if one considers the examples cited here.

1. The problem of exponential growth and decay requires mathematical techniques that are not available to the high school student.
2. If a student were to try out the methods from the textbook on a science project, it would be doomed to failure. It would also require estimation of the constants in the formula that demands the method of least squares, which is also beyond the secondary school level.
3. It seems that so-called applications borrowed from mathematical literature past the high school level lead to disastrous results.

The final conclusion has to be that this topic should be abandoned unless somebody can come up with a proof that is presentable at the high school level.

Statistics in High School

The field of statistics has grown enormously in this century and the results are being felt in almost every aspect of life. Who can imagine a political election without a poll? By its overwhelming presence, statistics has also found its way into the high school curriculum. In Grade 12, we study something about the normal distribution which, in two dimensions, is graphically represented by a bell-shaped curve. Assumptions about this distribution are, as a rule, verified by hypothesis testing. However, in high school, the experiment is absent, and so we are told that all necessary assumptions hold in order to simplify the case. Suddenly, the conclusion is drawn that we have obtained a "standardized normal distribution."

About 15 percent of the questions on the departmental exams are based on this topic. The value of this type of mental exercise is highly questionable. At present, the student has been taught to manipulate some formulae that appear out of the blue yonder.

It may be necessary to look at the historical development of statistics in order to come up with a suitable secondary program. At the moment, we only deal with the normal distribution. The danger is that we give students the impression that this is the only distribution there is, which is not true. It is also very hard to explain that mean and standard deviation have the same meaning as the first two moments of a mass in physics. Interrelationships are not established. In *Mathematics and Logic - Retrospects and Prospects*, Marc Kac and Stanilaw Ulam (p. 50) say:

> The theory (or calculus) of probability has its logical and historical beginnings in the simple problems of counting.

Indeed, it is simpler to present, in the classroom, the phenomena of tossing coins and dice than to give sound reasons for the continuous normal distribution. Since there is no long tradition in the teaching of statistics at the secondary level in any country, we are treading on very thin ice. It seems safer to go back to its original beginning and show something about the essence of its method than to show off with impressive-looking results. The normal distribution is a powerful tool in statistics, but the ability to see the full scope of its impact belongs to the professional statistician.

Conclusion

There is a great need for rethinking parts of the mathematics program. I.M. Yaglom (Davis, Greenbaum, and Scherk), in his article "Elementary Geometry, Then and Now," speaks about leading mathematicians who have written texts for secondary students. One of these is A.N. Kolmogorov, the Russian mathematician, who has written a text that is used by all secondary students in Russia. He speaks also about the French mathematician Jean Dieudonne, who wants to see geometry reduced to linear algebra and who has written a text for this purpose. Our school system cannot directly take over these ideas, but they can form a subject for study and comparison. If we want proper programs for our secondary schools, then we cannot leave the writing of textbooks to the book publishers and the forces of the marketplace.

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REFERENCES

Adler, Claire F. Modern Geometry. 2d ed. New York: McGraw-Hill, 1967.

Apostol, Tom M. Calculus. Vol. 1. New York: Blaisdell Publishing Company, 1964.

Davis, Chandler; Branko Gruenbaum; and F.A. Scherk, eds. The Geometric Vein: The Coxeter Festschrift. New York: Springer Verlag, 1981.

Dottori, Dino; George Knill; and James Stewart. FMT Senior. McGraw-Hill Ryerson Limited, 1979.

Ebos, Frank, and Bob Tuck. Math Is/6. Scarborough, Ontario: Nelson Canada, 1982.

Hanwell, Alfred P.; Marshall P. Bye; and Thomas J. Griffiths. Holt Mathematics 4. 2d ed. Holt, Rinehart and Winston of Canada, Limited, 1980.

Hoel, Paul G. Introduction to Mathematical Statistics. 3d ed. New York: John Wiley and Sons, Inc., 1966.

Kac, Marc, and Stanilaw M. Ulam. Mathematics and Logic - Retrospects and Prospects. New York and Toronto: The New American Library, 1968.

Nagel, Ernest, and James R. Newman. Goedel's Proof. New York: New York University Press, 1968.

Newman, James R. The World Of Mathematics, Vol. 3. New York: Simon and Schuster, 1966.

Scharlau, Winfried; and Hans Opolka. From Fermat to Minkowski. New York: Springer Verlag, 1984.

Wansink, Dr. Joh H. Didactische Orientatie voor Wiskundeleraren. 2 vols. Groningen, Holland: J.B. Wolters, 1966.