Higher Order Thinking Skills and Mathematics Education

Minnesota Department of Education
and Minnesota Council of Teachers of Mathematics

EDITOR'S NOTE: This position paper was formulated at a conference in May 1985 at Ruttger's Bay Lodge, and was presented to the MCTM Meeting in Washington in April 1986. Permission to publish this paper was obtained from David Dye, Minnesota Department of Education, who was a member of the writing team for the Ruttger's Bay Conference.

Higher order thinking skills need greater emphasis in American schools. At least, this is one conclusion that can be drawn from the recent flurry of reports on the state of education in the United States. While this concern crosses discipline boundaries, it is clear that the curriculum of mathematics can provide a powerful medium for attacking this problem.

In response to the national outcry and in an attempt to bring a focus to the mass of information and opinion that have been printed on the issue of higher order thinking skills, the Minnesota Department of Education gathered together a group of mathematics educators in May of 1985. The participants, representing all levels of mathematics instruction in Minnesota, were asked to express concerns and provide direction for continued effort regarding this critical problem.

In order to initiate the discussion, five of the participants prepared presentations to pose questions on specific topics. These topics were:

1. Problem Solving
2. Decision Making
3. Logic
4. Analysis, Synthesis, and Evaluation
5. Understanding Concepts

A small group of participants then met with each presenter to develop a report outlining their reactions to the issues that had been raised. During this meeting, the participants agreed that the outcome of the conference should result in the preparation of: (1) a position paper on higher order thinking skills, (2) a working definition of higher order thinking skills in mathematics, and (3) a conference report.

Defining Thinking Skills

Many writers do not attempt to define "thinking skills." However, to clarify the group's understanding for purposes of discussion, the following was written as a *working definition:*

> Thinking skills are the dynamic mental processes, both intuitive and logical, used in collecting, organizing, interpreting, and applying information for the purpose of arriving at decisions and/or gaining new knowledge.

The conference participants generated a partial list of thinking skills. These thinking skills were then grouped into six main categories, five of which were the topics used to initiate the discussion of higher order thinking skills. The following

examples have been grouped with the understanding that the skills listed may not be exclusively associated with any one particular heading.

Problem Solving -
selecting strategies, comparing, contrasting, ordering, grouping, labeling, categorizing, sorting, identifying relevant and nonrelevant information, modeling, examining special cases, being flexible, and breaking a mind set

Quantitative Thinking -
estimating, sequencing, using algorithmic skills, recalling, and recognizing

Logic -
proving, using analogies, reasoning inductively, and reasoning deductively

Analysis, Synthesis, Evaluation -
asking appropriate questions, generalizing, inventing, creating, evaluating, observing, generating unifying concepts, seeing relationships, using patterns, translating, distinguishing between fact and opinion, recognizing systems, and condensing long lists

Understanding Concepts -
visualizing, designing algorithms, hypothesizing, verbalizing abstractions, and simplifying

Decision Making -
communicating, generating alternatives, elaborating, and evaluating anticipated outcomes

The chart on the following page is intended to show the relationship of some of the skills discussed in this paper. It incorporates quantitative thinking as fundamental to the process of understanding mathematics. Of course, since we are emphasizing higher order thinking skills, those on the upper levels of Bloom's taxonomy (analysis, synthesis, and evaluation) permeate this understanding.

Position Statement

Of all the skills learned during a lifetime, one of the most basic is the ability to think. Thinking subsumes all of the other basic skills associated with learning. While thinking is not used exclusively in the realm of mathematics, the study of mathematics provides many opportunities for teaching and learning thinking skills.

Thinking is inherent to human survival. The question is not of whether to teach students to think, but of identifying certain skills that can be practiced in a variety of situations and environments to make people better thinkers. We must provide activities and experiences that give an opportunity for practice and development of those skills. Just as musicians, athletes, and artists must develop and depend on fundamental skills in order for their talents to reach full potential, so must students be aware of fundamental thinking skills in order to develop their potential as thinkers and problem solvers. Skills must not only be learned, they must also be practiced - both alone and with others!

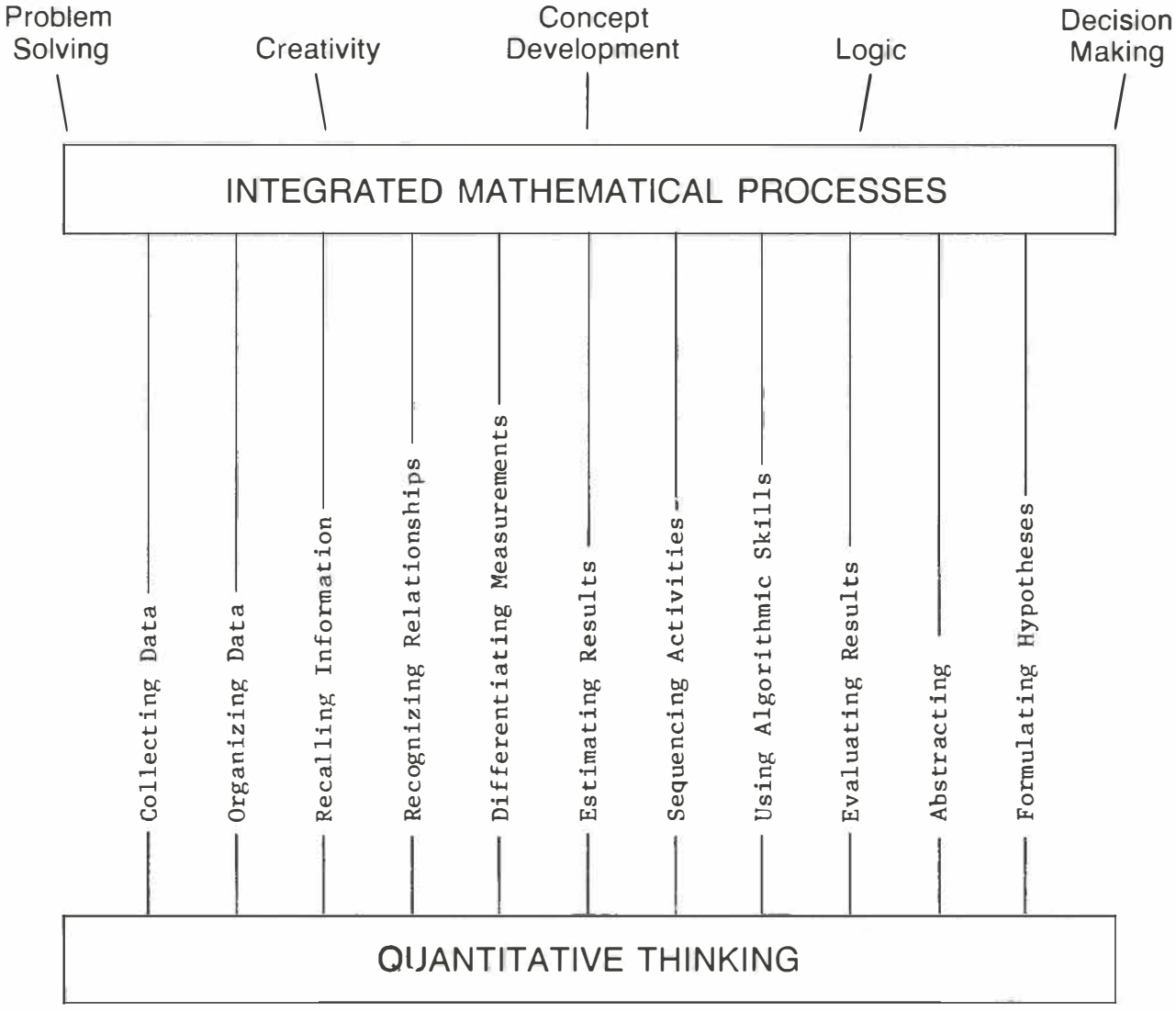
In elementary and secondary education, mathematics instruction is intended for all students, even though the expectations are different for different students. We believe that all students can be taught skills that will enable them to think better than they presently do.

The process of teaching higher order thinking skills has implications for the delivery of mathematics instruction to students. There must be a different climate in the classroom to reflect different emphases. It is important that the process whereby students are expected to memorize material, and then attempt to apply it prior to understanding the underlying concepts, be scrutinized. The participants believe - and their position is supported by a range of research -

that it is imperative for students to understand concepts at least simultaneously with, and preferably previous to, the memorization of facts and algorithms. At the same time, it is important that school personnel be aware of different learning styles among students, and make allowances for these differences.

Such changes in emphases carry clear implications for the amount of time needed for teaching mathematics. Either the time available must be increased, or deletions must be made in the present curriculum. Since it is highly unlikely that a significant amount of additional time can be extracted from an already full school day, the most obvious candidate for deletion would be the vast amount of time spent on pencil-and-paper arithmetic drill.

The proliferation of technology in schools can affect the way we teach, and an emphasis on higher order thinking skills will certainly have implications for the choice and use of that technology. Specifically, the calculator should be used to a much greater extent than it is at the present time.



Increased use of the calculator will reduce some of the time currently spent on arithmetic drill.

If we are sincere in our desire to emphasize higher order thinking skills, we must also be committed to developing suitable means for evaluating the achievement of these skills. Present testing procedures, which place a high priority on recall, recognition, and arithmetic skills, must be modified to encompass a more comprehensive approach that will assess how well the students are learning concepts and understanding processes.

Finally, there are implications for teacher training, both preservice and inservice. Teacher training programs must place more emphasis on higher order thinking skills so that teachers themselves become better problem solvers. In addition, teachers must become aware of the latest research being done in this area and its implications for their classrooms. That is, teachers must become convinced of the importance of teaching higher order thinking skills, they must learn the techniques for teaching them, and they must learn how to make room for them in the curriculum.

Guidelines and Recommendations

Finally, we are ready to make some recommendations about incorporating the teaching of higher order thinking skills into mathematics classrooms. Instructional techniques need to be carefully planned. Classroom climates conducive to exploration and experimentation must be created. These requirements should form the basis for preservice and inservice education of mathematics teachers at all grade levels.

Questioning techniques are at the heart of this kind of instruction. Teachers must learn to ask questions rather than dispense information. We recognize that good teachers have used this Socratic method extensively and that all teachers have used these techniques to some degree. We propose that teachers be encouraged to reexamine and improve questioning techniques in order for students to gain an understanding of concepts before moving to mastery of skills. Mathematics topics must be presented to students in such a way that will allow students to acquire an understanding of concepts or applications before they are asked to memorize and/or drill on techniques. This will help to ensure the transfer of knowledge to new situations, including the solution of problems in higher mathematics and related subject areas.

Objectives for mathematics classes must incorporate these new thinking skills. Expected outcomes will need revision in order to diminish emphasis on pencil-and-paper arithmetic. Testing, if it is to relate to these objectives, will also undergo radical changes. Teachers and assessment experts must use their creativity to develop testing procedures that will assess the students' ability to solve problems. In addition to formal testing, this may include observations of students while they are working.

Teachers will have to search out good sources for problems to use in their teaching of problem solving. They will have to use their creativity in selecting problems, finding appropriate situations for their inclusion, and developing procedures which will enhance the new knowledge and skills. At the same time, teachers desperately need to take time to solve mathematics problems themselves. In this way, they can be role models for students and develop an empathy for students who will be asked to solve problems. Teachers must take time, and schools must provide time, for this. Teachers must also find the time and procedures for sharing new problems with other teachers. Attendance at professional conferences and visitations to the classrooms of excellent teachers are

examples of procedures for sharing information.

Curriculum writers, whether at a local or national level, must incorporate more activities involving concept development and problem solving. Since many teachers "follow the text," textbooks that provide the types of activities necessary to teach thinking skills must be developed.

Teachers and administrators must communicate the new emphases to school patrons. They will need to sell the importance of higher order thinking skills to obtain community support for spending less time on traditional topics and more time on the teaching of higher order thinking skills.

Suggested Activities for Teaching Thinking Skills

Here are some specific activities we recommend to help teach those higher order thinking skills. These are not listed in any order of priority, but are merely listed as the participants thought of them.

- Ask students to become involved in projects.
- Use manipulative materials to introduce concepts.
- Make good use of good computer software.
- Ask students to work together cooperatively to learn material.
- Augment texts with other activities and integrate discovery/exploration lessons.
- Have students write computer programs to do the algorithms of arithmetic.
- Ask students to communicate what they know, what they don't know, or what they need to know orally and in writing.
- Have students become involved in collecting/displaying data as part of an experiment.
- Develop lessons in cooperation with other subject areas.
- Focus on big ideas; for example, linear rate function leading to distance, time, and rate problems - thematic curriculum, look at special cases, emphasize structure, and develop algorithms.
- Use puzzle problems, including cryptarithmetic and toothpick problems.
- Use educational games.
- Have students make up their own games.
- Give an answer and have the students make a problem to match.
- Require book/video reports.
- Go on field trips.
- Use outside resource people.
- Use simulations on the computer.
- Let students elaborate on hobbies/personal interests.
- Allow time for students to solve problems (that is, provide a problem solving class, a problem of the week, or a problem of the day).
- Develop a school-family math program designed to get students and parents working on problem solving at home with the guidance of trained professionals.
- Have students develop test questions from different topics based on their own experiences.
- Have students explain why they used a particular algorithm or process to solve a problem.
- Encourage students to produce a videotape of an application.

No attempt has been made to classify the above activities with regard to specific course designation or grade level suitability. The suggestions are presented to aid teachers with possible instructional strategies for helping students improve thinking skills.