Infinity

Sarah Jervis
Lethbridge Collegiate Institute

Infinity, or the boundless, is beyond the minds of most men. The concept has existed for centuries, over which many men have tried to propose theories. Relating each of these theories, one may get an idea of what infinity entails.

The original symbol for infinity, which is used today, is the lemniscate or ∞. This symbol was introduced in the seventeenth century, and appeared on the juggler or mangus card of the tarot cards. One of the main concepts of infinity is its endlessness, and this is why the lemniscate is used. One can travel around its periphery endlessly. The *quabalistic* symbol associated with this particular card was the Hebrew letter aleph or א. George Cantor, founder of the modern mathematics theory of the infinite, used the symbol $\aleph_0$ (aleph-null) to stand for the first infinite number.

The Greeks used the term apeiron to describe their concept of the infinite. The word literally meant unbounded and eventually came to describe general things such as disorder or the extremely complex. This apeiron may have no finite definition. For many Greek mathematicians, the concept of apeiron was unacceptable, even in the simplest form of a decimal expansion on the simplest number.

Blaise Pascal once described his feeling of being overwhelmed by the infinite:

> When I consider the small span of my life absorbed in the eternity of all time, or the small part of space which I can touch or see engulfed by the infinite immensity of spaces that I know not and that know not me; I am frightened and astonished to see me here instead of there . . . now instead of then.

Aristotle believed that "being infinite was a privation, not a perfection, but the absence of a limit." He saw that aspects of the world are apeiron - that time will not end, space is infinitely divisible, and that a line contains an infinite number of points. Aristotle invented the idea of potential and actual infinity. He proposed that the set of natural numbers is potentially infinite in that it has the ability to go on forever, yet it is not actually infinite because it does not exist as a finished thing.

Many men have expressed their beliefs of the infinite. Plutinus believed God to be infinite. St. Augustine added that God was not only infinite but could also think infinite thoughts. However, later medieval thinkers did not go so far as to believe that God was infinite. Although He has unlimited power, He does not have the ability to create an unlimited thing. (A "thing" cannot be unlimited, as it takes on the definition of being limited by nature.)

A problem was brought to the attention of mathematicians concerning the infinities of the world. On one hand, it would seem that God, being infinitely powerful, should be able to get an infinite number of angels to dance on the head of a pin, for example. On the other hand, it would seem that, in a created world, no actually infinite collection of angels could exist. Infinity appeared to be a self-contradictory argument. A line with a length twice that of another line would appear to have a larger infinity of points than the smaller. Yet a point on the smaller line would correspond with the point on a larger line, proving that infinity can be equal and different at the same time, which, in fact, seems to contradict logic.

Galileo Galilei offered that the smaller length could be turned into the longer length by adding an infinite number of small spaces. Galileo realized that there were problems with his solution, for the human mind can only think in finite terms. He stated that while looking at most natural numbers, many of them will not be perfect squares; thus, there must be a smaller set of perfect squares than natural numbers. There exists a paradox, however, that every natural number is the square root of a larger natural number. It would therefore seem that there are as many perfect squares as natural numbers. Galileo stated that:

> We can only infer that the totality of all numbers is infinite and that the number of squares is infinite. . .; neither is the number of squares less than the totality of all numbers, not the latter greater than the former; and finally, the attributes "equal," "greater," and "less" are not applicable to the infinite, but only to finite qualities.

It is essentially impossible for the finite being to contemplate the infinite. If a man were asked to calculate the largest possible number imaginable, this would, of necessity, be bounded by the finite period of his lifetime. On his deathbed, a large number would probably have been reached. As he gasped his last breath, an observer could merely add one and would start at that point.

Lucretius, in his theory *De Rerum Natura*, suggested: "Suppose for a moment that the whole of space were bounded and that someone made his way to the uttermost boundary and threw a flying dart." He then went on to consider that the dart could go past the boundary or it would stop. In either event, infinity is demonstrated. There is either a boundary stopping the dart, in which case there is something or someplace beyond, or there is no boundary, allowing the dart to continue upon its infinite path.

In more recent history, the traditional scientific view of infinity might be challenged by the so-called "big bang" theory. Such a theory is now widely accepted. However, such theory tends to suggest a beginning and an end. With the acceptance of the big bang theory, scientists now contemplate what was before the bang, and what will happen at the end of this universe. One answer that has been suggested is that the universe is an oscillating system, which endlessly expands and contracts to infinity.

It seems that the more common view of infinity is that of a series of numbers having no end. In fact, infinity has an equal place at or before the beginning of things. It is impossible to state the smallest or first number. Numbers are either infinitesimally small, or large, or somewhere in between. The paradox stated by Zeno seems to show that one can never leave the room which one is in. This, of course, is clearly ridiculous subject to the acceptance or otherwise of the Berkelian theory of existentialism. Zeno reasoned that in order to reach the door, one must first cross half the distance there. This would leave half the room to be crossed, but first one would have to cross half that distance, and so on. The modern answer to the paradox is to say that the sum of the infinite series $1/2 + 1/4 + 1/8 \ldots = 1$. Even so, this is not perfectly satisfactory. The paradox can be put in a different way. In a practical world, we say that a number with a decimal expansion of .99999 is the same as 1. It can be put this way:

$$\begin{array}{r} 10K = 9.999\ldots \\ - \quad K = .9999\ldots \\ \hline 9K = 9 \\ K = 1 \end{array}$$

Thus, we have the practical answer as compared to the theory of Zeno who regarded space as an undivided whole that cannot be broken down into parts.

If one were to take Zeno's paradox literally, any counting in whole numbers (for example, 1, 2, 3) would be impossible. If the average man on the street were asked to count to infinity, he would say that it is impossible. If he were accommodating, he might start counting for a day. Perhaps he would get up to 170,000. But, he would be unaware of Zeno's paradox. I suspect he would start counting with number one. In order to get to one, he would first have to pass 0.5 and, thus, would never "leave the room."

It is always interesting to consider the combination of random and infinity. In the unbounded time of infinity, literally anything is possible.

It has been said that if a group of monkeys were given an English dictionary, the monkeys would eventually, by random chance, utter the entire works of William Shakespeare in the exact order in which they were written. In the absence of infinity, such would not be probable.

If we add to this theory the additional fact of human intelligence, it would be reasonable to conclude that man will learn all the secrets of the universe, including the mystery of infinity, within infinity.

Sarah Jervis was a Mathematics 30 student at the Lethbridge Collegiate Institute. Sarah enrolled in an honors mathematics program, and this paper was submitted to partially fulfill the requirements for the program.