A Problem Solving Geometry Lesson Using Groups of Four

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In the September 1981 issue of *Learning*, Marilyn Burns describes a classroom management scheme that encourages students to learn by working cooperatively and independently. Certainly, this is a worthy educational goal for any classroom. Her article is based upon ideas given to her by Carol Meyer, a classroom teacher in Davis, California.

The scheme called "Groups of Four" requires reorganizing the classroom physically, redefining the students' responsibilities, and carefully structuring the role of the teacher. Students are randomly assigned to groups of four, and the assignments are changed regularly throughout the year. There are three rules for the students in their groups of four:

1. Each student is responsible for his or her own work and behavior.
2. Each student in the group is responsible for every other group member.
3. A student may ask for help from the teacher only when everyone in his or her group has the same question.

I suggest you read Marilyn Burns' article; it has many good suggestions.

"Groups of Four" is just one of several management schemes a teacher should use. However, this scheme is especially appropriate when problem solving is the goal of instruction. Why don't you try "Groups of Four" with the high school geometry lesson "An Investigation: Polygons and Line Segments," which is given on the following page.

Commentary and Answers for "An Investigation: Polygons and Line Segments"

Hand out the lesson sheet, one to each student, and have the students get into their groups of four. Follow the "Groups of Four" rules given above, especially rule 3. Most groups should be able to figure out on their own what to do through question 3. Use your time observing and taking notes on how the groups are functioning and on the different approaches used in solving the problems. This information will be useful later, when summarizing the lesson with the class. The table below contains the data collected for questions 1, 2, and 3, and the correct predictions called for in questions 4 and 5.

s	d	D	L
4	1	2	6
5	2	5	10
6	3	9	15
7	4	14	21
8	5	20	28
9	6	27	36
10	7	35	45
3	0	0	3
20	17	44	55

An Investigation: Polygons and Line Segments

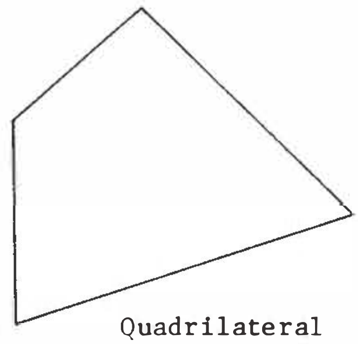
s	d	D	L
4			
5			
6			
7			
8			
9			
10			
3			
20			

s = number of sides

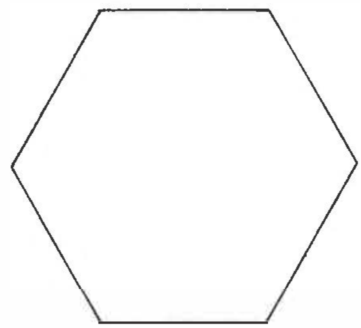
d = number of diagonals from one vertex

D = total number of diagonals

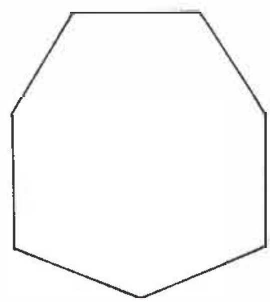
L = total number of line segments



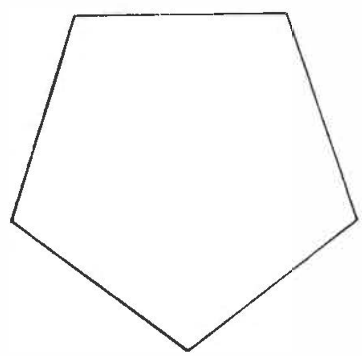
Quadrilateral



Hexagon



Heptagon



Pentagon

1. Draw the diagonals from a single vertex in each of the polygons shown. Record the number in the table.

2. Draw all the other diagonals for each polygon. Record the total number in the table.

3. Record the total number of sides and diagonals for each polygon.

4. Study the patterns in the table. Predict the value for d, D, and L for an octagon, nonagon, decagon, triangle, and icosagon.

5. Check your predictions by making drawings.

6. Write the formula for the relationships suggested in the table.
   (a) d in terms of s
   (b) D in terms of s and d
   (c) D in terms of s
   (d) L in terms of s

7. Do your formulas work for all the data recorded in your table? If not, make adjustments until the equations accurately describe the situation.

8. Graph the formulas in 6a, 6c, and 6d. Let **s** be the horizontal axis in each case. By careful planning and labeling, the three graphs can be placed on the same chart.

9. Does it make sense in this lesson to draw the straight or curved line suggested by each graph? Why or why not?

10. Does it make sense to use the formulas for finding values for d, D, and L, when **s** is any whole number? Explain.

6. (a) $d = s - 3$

(b) $D = \frac{sd}{2}$

(c) $D = \frac{s(s - 3)}{2}$

(d) $L = \frac{s(s-3) + s}{2}$

$= \frac{1}{2}s(s + 1)$

7. Groups will use various strategies to get their formula. Encourage groups and individuals to keep track of the strategies they used. These should be discussed when the lesson is summarized later.

8. Attention needs to be given to the scale used for each graph. I suggest a scale of 2 cm per unit for the horizontal axis and 1/2 cm per unit for the vertical axis.

9. No. Whole numbers from 3 and after make sense, but a mixed number such as 6 1/2 for the number of sides and diagonals of a polygon does not make sense.

10. No. It does not make sense to say that a polygon has 0, 1, or 2 sides.

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ERRATUM

In "The Road to Four Villages" problem appearing on page 43 of the last issue of *delta-K* (Volume XXV, Number 3, July 1986), a square root sign was omitted, making the problem meaningless. Please accept our apologies for this oversight. The problem should have read:

> Four villages are situated at the vertices of a square of sides which are one mile long. The inhabitants wish to connect the villages with a system of roads, but have only enough material to make $\sqrt{3} + 1$ mile(s) of road. How do they proceed?