A Constructivist Approach to Teaching Mathematics

Sol E. Sigurdson
University of Alberta

Over the last few years, psychologists and educators have been interested in going beyond behavioristic and Piagetian views to new conceptualizations of learning, especially in using the computer as a "model" of how we think and learn. One of the new conceptualizations has been "information processing." Proponents of this view claim that when we think, we basically process information; it's as simple as that. This, however, leads to a further question: How do we manage this processing? To account for the management of processing, it is suggested that the learner engages other processes called metacognitive processes. But, still we might ask: What manages the metacognitive processes? Although this is not a trivial question, most proponents presently do not differentiate between levels of management, simply naming all those processes above the cognitive level metacognitive processes. In fact, the difference between cognitive and metacognitive is not always clear. For the time being, let us say that strictly mathematics propositions, procedures, and processes are called cognitive, while management decisions about such matters as when to use them, in what order, and with what degree of confidence are called metacognitive processes.

Another related view of learning has been called a theory of "personal constructs." The main tenet of this view is that all learners actively construct theories, no matter how minor, about what is appropriate action for responding to any particular situation. If a particular theory leads to inappropriate action, we revise the theory. This view, like information processing, also utilizes the notion of "metacognitive processes" managing our theory development. According to the personal constructs view, learners of differing capabilities exist because both our cognitive capacities and our metacognitive (management) capabilities differ. Another explanation, which goes beyond differences in cognitive or metacognitive components, is that some learners' perceptions are blinded (by emotion, say), so that they are unable to differentiate between appropriate and inappropriate action and, consequently, construct poor theories.

What relevance do these new conceptions have to the mathematics classroom? The one outstanding impression that the personal constructs view leaves is that our classrooms consist of 25 or so finely-tuned, sensitive, self-initiating, theory-generating, learning "beings." The metacognitive aspect, on the other hand, leads us to question how much of a committment we teachers have in attending to the *development* of metacognitive processes. The information processing aspect begs the question of how to present information for efficient storage and easy access. Psychologists and educators are still exploring answers to these questions and will be for many years. In the meantime, what aspect of these theories can be useful to teachers in dealing with the complex world of classroom instruction?

In order to make these ideas more available for teacher use, I will combine the three notions - information processing, personal constructs, and metacognitive processes - into one "constructivist" view of learning. In this article, I will describe constructivist principles of learning and further derive from them constructivist guidelines for classroom teaching of mathematics. Mathematics teachers are encouraged to think about, and use, these ideas to improve their classroom instruction. Psychologists and educators, who are continually striving for new insight into the learning process, would surely appreciate feedback from the most significant learning laboratory of all, the classroom. Curriculum examples will not be used to describe this view because these new conceptions of learning are equally relevant to all grade levels. The word constructivist has been around for many years. I am not concerned that my usage may be slightly different than that of others.

Constructivist Principles of Learning

1. **Purposeful Constructions.**
   Students construct their own theories for responding to a given situation, and, as they see their knowledge leading them to inappropriate action, they revise their theories. Learning proceeds from the current conceptions or theories of knowledge that the learner possesses. "Tuning," that is, modifying or adjusting, is an important learning process. Appropriate theories are best constructed in the light of some *acknowledged* purpose.

2. **Learning How to Learn.**
   Learners' awareness of their knowledge (mathematical content and processes, and metacognitive processes) at any time aids learning. Metacognitive processes (management of cognitive knowledge) are especially important, and these may be a major source of individual differences between slow learners and others.

3. **Confidence.**
   Because learning means taking risks and experimenting with new cognitive constructions, the atmosphere for learning must be familiar and full of trust. Inaccurate perceptions can be caused by either strong positive or negative emotions.

4. **Framework** for Information.
   Learning occurs in a context that provides a framework for the organization of information. The most appropriate context is one which is most applicable to the future situation in which the knowledge will be used. A framework for mathematical knowledge can consist of mathematical, everyday, and scientific elements.

5. **Structure of Knowledge.**
   All mathematical knowledge consists of propositional (conceptual and relational) structures and procedural (algorithmic and methods) structures. The process through which we understand and manipulate mathematical situations is grounded in specific content structures.

6. **Complexity of Concepts.**
   Propositional structures and procedural structures are complex content structures, a fact which is often disguised through rote learning and teaching. Although, traditionally, we teach through analyzing and breaking down knowledge, the constructivist sees "building up" as an equally valid learning process. Procedural

structures (algorithms) are linked in important ways to propositional structures (concepts).

7. **Transfer of Knowledge.**
   As we learn, we learn context, as well as content and process. Transfer of knowledge must not be assumed; it occurs only as a new context is "seen" as the learned one.

Although a deeper understanding would require considerable elaboration on all of these principles, perhaps we can employ a constructivist teaching tactic, and let the reader come to understand the principles as they are *used* to develop the "guidelines for classroom teaching." Classifying something as complex as human learning in "seven principles" seems to be an utterly futile undertaking. However, I would like to elaborate slightly on the structure and complexity principles. Recognized in the structure principle, first of all, is the importance of relationships among all mathematical concepts and that any *understanding* of mathematics is a matter of recognizing all these relationships. Also implied in the structure principle is that all mathematical activity, such as problem solving, is highly dependent on these structures. The complexity principle, while acknowledging the many-faceted aspect of even apparently simple concepts such as multiplication, stresses that understanding and use of knowledge must take into account all, or most, of these facets.

Of course, these learning principles can be applied to the teaching of any subject, but our concern here is what this might mean for the teaching of mathematics. In deriving these guidelines for classroom teaching, it became apparent that several possible interpretations would be valid. Once again, I have opted for seven, knowing that these can only serve as general suggestions.

Constructivist Guidelines for Classroom Teaching

1. **Unit Context.**
   Mathematics should be taught in the context of a three- to four-week unit constructed around a mathematical, everyday, or scientific application of the content. Students should feel comfortable and familiar with this application context.
   RATIONALE: The purposeful constructions and the framework principles are satisfied by this. The actual application context would not only be a function of the content, but also of the grade level of the class, the characteristics of the students, and the school environment.

2. **Curriculum Tasks.**
   The tasks which comprise the unit should be conducted with a view to the students engaging their current conceptions, mastering the task, and learning from it. The focus of the task should be central to the unit application.
   RATIONALE: The learning how to learn and the confidence principles suggest that the task be a manageable part of the unit. The structure principle suggests that relevant mathematics knowledge be an integrated part of the task.

3. **Managing the Task.**
   All students should be given assistance in dealing with the task - determining task difficulty, monitoring their understanding of it, apportioning time for it, and predicting how well they can perform it. The teacher should pay special attention to the students' perception of the task. Individual differences should be noted and provided for in this aspect.
   RATIONALE: The purposeful constructions and learning how to

learn principles are important here, especially in helping students become aware of their knowledge and knowledge processes. This guideline is the core of the instructional process.

4. **Task Variety.**
   Tasks should include a range of learning activities, such as direct examples, reviewing, textbook use, note taking, concrete materials, understanding, amplification of basic concepts, problem solving, self-inquiry, practice exercises, group activities, discussion and questioning.
   RATIONALE: The purposeful constructions principle does not imply that student learning should be of a discovery nature, but only that learning should have some purpose. The complexity principle not only suggests that a considerable amount of guidance, even direct examples, is appropriate, but also that a variety of approaches is necessary to achieve an understanding of a mathematical topic.

5. **Assessment Tasks.**
   Assessment should be carried out primarily within the context of the unit.
   RATIONALE: The transfer principle suggests that we should first apply learning to the context of the unit. If we do testing beyond the context of the unit, we should be conscious of how the new context relates to the learned one. In actual (real-life) use of mathematics, contexts that are important to the student are most often familiar ones.

6. **Mathematical Learning.**

   (a) Readiness.
   Readiness for content learning must be noted, but only in the context of the learning task. What does the learner bring to the situation? Students' awareness of their own readiness is also important.
   RATIONALE: Purposeful constructions are derived from previous "theories" that the student has. This is the central premise of the constructivist view. The learning how to learn principle suggests a self-awareness of these previous theories.

   (b) Concepts.
   Concepts, the pivotal ingredients of mathematics learning, must be constructed from the student's prior knowledge. Learning of complex subject matter is achieved through many different propositional structures. Specific instructional devices, such as concept maps and structured apparatus, should be employed.
   RATIONALE: The framework, structure, and complexity principles all indicate the necessity of a thorough conceptual basis for mathematics learning.

   (c) Skills.
   Skill development, as it relates to the curriculum unit, is important. Care should be taken in selecting the application context for curriculum units. Skills and algorithms (procedural structures) are founded upon certain propositional structures. Skills should be learned as broader "method" approaches.
   RATIONALE: Although our principles do not address the matter of skills directly, the structure principle advocates a solid basis for all procedures, while purposeful constructions implies that all skill learning be in context.

   (d) Applications.
   All applications occur in the context of the unit. They should be dealt with as an indication of the use, and usefulness, of mathematics, and also as a way of relating

the real world to the development of mathematics.

RATIONALE: The framework principle means that applications can be an important contribution to the framework for learning mathematics. The purposeful constructions principle suggests applications as a primary reason for studying mathematics. Lastly, the teacher must be constantly aware of transfer and the problem of the context of learning.

(e) Problem Solving.
Problem solving should be approached through a study of the particular kinds of problems in each unit. Problem solving is a particular way of knowing content.

RATIONALE: The structure principle suggests that all mathematics is dependent on specific knowledge. The metacognitive processes of the learning how to learn principle manage only cognitive knowledge. A constructivist view does not support broad generalizable problem solving strategies.

7. **Goals of Mathematics Learning.**
The major goals of mathematics teaching are that students gain understanding of complex areas of mathematical knowledge, use this knowledge in relevant situations, and understand their own processes and capabilities for functioning in a mathematical environment.

RATIONALE: The constructivist view not only provides new insight into how mathematics should be taught, but also implies a somewhat revised goal for mathematics teaching; "practice, feedback, and coaching" are not enough. Although the view expands upon what understanding means, one of the more interesting issues it raises is how teachers should regard their efforts toward improving students' capabilities for learning how to learn.

The strongest message of a constructivist approach is the desirability that teachers make clear to themselves and to students the purpose of learning mathematics. Making clear the purpose, without trivializing it, will be of great benefit in improving mathematics teaching. At this writing, I believe the weakest part of these guidelines is the matter of "context" and, therefore, the matter of what a sensible unit context might be. It seems essential that the context include, but go beyond the bounds of, mathematics itself. It certainly need not be confined to students' interests. Plausibility to the student might be a better guideline. Clearly, the broader the context, the more mathematics it will subsume. However, the greater breadth might tend to lose focus. Also, the notion of curriculum task and its position between the unit context and mathematics to be learned is somewhat problematic. An appropriate resolution of these weaknesses will need to be worked out in light of both the proposed principles of learning and the other guidelines.

Obviously, this interpretation of the constructivist perspective leaves many gaps. If a teacher were to conduct lessons solely on the basis of this statement (even assuming the availability of a textbook), I would predict chaos. The statement can only be seen as an attempt to modify already competent practice. Certainly, these are *not* prescriptions for teaching. Rather, I see them as interesting guidelines that can be tried, discussed, revised, and reinterpreted. A constructivist would see a teacher interpreting these guidelines on the basis of the teacher's existing "theories," and then, perhaps, rejecting them as invalid or "tuning" existing theories, using them, and then revising or discarding them.

At the very least, these guidelines should provide the basis for an interesting curriculum unit which would go far in explicating the guidelines. This would provide an opportunity for psychologists to say that their views have been misread or misinterpreted, which would be very useful. It might even serve to have them rethink their ideas in the light of feedback given by teachers. Whatever happens, teachers of mathematics are obligated to begin investigating ways that these new conceptualizations of learning can benefit them. Teachers certainly owe it to themselves and, in some sense, they owe it to psychologists and educators who are searching for new insight into the very important but, too often, frustrating process of learning mathematics.

During the school year 1985-86, Dr. Sol E. Sigurdson was on sabbatical leave from the University of Alberta, where he taught methods and graduate courses in mathematics education. His interests focus on classroom change brought about by inservice and curriculum change.

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