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EDITORIAL

Direction

A new school year offers an opportunity for increased involvement in your professional organization.

- Plan to attend the NCTM Canadian Regional Conference. Bring a friend.
- Plan to contribute to your *journal*. The themes of the next two issues are "Problem Solving in the Junior High School" and "Technology in Mathematics Education." Anticipated publishing dates are February and May, respectively.
- Plan to have your students submit solutions to the Student Problem Corner.
- Plan to contribute, or encourage your colleagues to contribute, to the special publications your executive has authorized. A special issue, "Mathematics and Early Childhood Education," is planned, as well as a monograph, "Make It and Take It."
- Plan to renew your membership in MCATA, and to encourage a colleague to become a member.

Comment

The focus of this issue is "Effective Teaching of Mathematics," and there are many factors contributing to this topic. Knowledge of mathematics and how mathematics ideas and concepts develop is a factor. Knowledge of how children learn mathematics - and use of that knowledge - is also a part of effective teaching. Is mathematics a set of rules, cases, and procedures, or is it a thought system that utilizes a particular structure? Research on the subject of teaching effectiveness abounds. Do effective teachers of mathematics incorporate into their lessons this research on teaching strategies and classroom management, as well as the results of research on teaching and thinking and the use of technology? Effective teaching might also include student involvement, student accomplishments, and student use of mathematics.

The Minnesota Department of Education and Minnesota Council of Teachers of Mathematics investigate what mathematics teachers may do to teach thinking skills, as well as mathematics. **Sol Sigurdson** examines learning theories and proposes a "constructivist" view toward learning principles and their implementation. **Dr. Ediger** examines the scope of the mathematics curriculum and, in particular, the role of the textbook. **John Heuver** makes a critical analysis of some of the texts used in Alberta. **H. Skolrood and M.J. Maas** show parallelism in the reading process in mathematics and social studies, and identify four reading situations. **Professor Schrage and Dr. Jerry Becker** identify three limitations in the use of microcomputers for teaching mathematics. Dr. Duncan and Dr. **Litwiller** examine a multiplication table, and interesting matrices are the result. **S. Jervis**, a Grade 12 student, discusses infinity. Craig Loewen illustrates the effective use of the overhead projector in teaching geometry. **Jacqueline Fischer** shares ideas on creative problem solving, and **Oscar Schaaf** provides a geometry lesson that is especially appropriate for teaching problem solving. **Hank Boer** is the contributor to the Student Problem Corner.

John B. Percevault

Higher Order Thinking Skills and Mathematics Education

Minnesota Department of Education
and Minnesota Council of Teachers of Mathematics

EDITOR'S NOTE: This position paper was formulated at a conference in May 1985 at Ruttger's Bay Lodge, and was presented to the MCTM Meeting in Washington in April 1986. Permission to publish this paper was obtained from David Dye, Minnesota Department of Education, who was a member of the writing team for the Ruttger's Bay Conference.

Higher order thinking skills need greater emphasis in American schools. At least, this is one conclusion that can be drawn from the recent flurry of reports on the state of education in the United States. While this concern crosses discipline boundaries, it is clear that the curriculum of mathematics can provide a powerful medium for attacking this problem.

In response to the national outcry and in an attempt to bring a focus to the mass of information and opinion that have been printed on the issue of higher order thinking skills, the Minnesota Department of Education gathered together a group of mathematics educators in May of 1985. The participants, representing all levels of mathematics instruction in Minnesota, were asked to express concerns and provide direction for continued effort regarding this critical problem.

In order to initiate the discussion, five of the participants prepared presentations to pose questions on specific topics. These topics were:

1. Problem Solving
2. Decision Making
3. Logic
4. Analysis, Synthesis, and Evaluation
5. Understanding Concepts

A small group of participants then met with each presenter to develop a report outlining their reactions to the issues that had been raised. During this meeting, the participants agreed that the outcome of the conference should result in the preparation of: (1) a position paper on higher order thinking skills, (2) a working definition of higher order thinking skills in mathematics, and (3) a conference report.

Defining Thinking Skills

Many writers do not attempt to define "thinking skills." However, to clarify the group's understanding for purposes of discussion, the following was written as a *working definition:*

> Thinking skills are the dynamic mental processes, both intuitive and logical, used in collecting, organizing, interpreting, and applying information for the purpose of arriving at decisions and/or gaining new knowledge.

The conference participants generated a partial list of thinking skills. These thinking skills were then grouped into six main categories, five of which were the topics used to initiate the discussion of higher order thinking skills. The following examples have been grouped with the understanding that the skills listed may not be exclusively associated with any one particular heading.

Problem Solving -
selecting strategies, comparing, contrasting, ordering, grouping, labeling, categorizing, sorting, identifying relevant and nonrelevant information, modeling, examining special cases, being flexible, and breaking a mind set

Quantitative Thinking -
estimating, sequencing, using algorithmic skills, recalling, and recognizing

Logic -
proving, using analogies, reasoning inductively, and reasoning deductively

Analysis, Synthesis, Evaluation -
asking appropriate questions, generalizing, inventing, creating, evaluating, observing, generating unifying concepts, seeing relationships, using patterns, translating, distinguishing between fact and opinion, recognizing systems, and condensing long lists

Understanding Concepts -
visualizing, designing algorithms, hypothesizing, verbalizing abstractions, and simplifying

Decision Making -
communicating, generating alternatives, elaborating, and evaluating anticipated outcomes

The chart on the following page is intended to show the relationship of some of the skills discussed in this paper. It incorporates quantitative thinking as fundamental to the process of understanding mathematics. Of course, since we are emphasizing higher order thinking skills, those on the upper levels of Bloom's taxonomy (analysis, synthesis, and evaluation) permeate this understanding.

Position Statement

Of all the skills learned during a lifetime, one of the most basic is the ability to think. Thinking subsumes all of the other basic skills associated with learning. While thinking is not used exclusively in the realm of mathematics, the study of mathematics provides many opportunities for teaching and learning thinking skills.

Thinking is inherent to human survival. The question is not of whether to teach students to think, but of identifying certain skills that can be practiced in a variety of situations and environments to make people better thinkers. We must provide activities and experiences that give an opportunity for practice and development of those skills. Just as musicians, athletes, and artists must develop and depend on fundamental skills in order for their talents to reach full potential, so must students be aware of fundamental thinking skills in order to develop their potential as thinkers and problem solvers. Skills must not only be learned, they must also be practiced - both alone and with others!

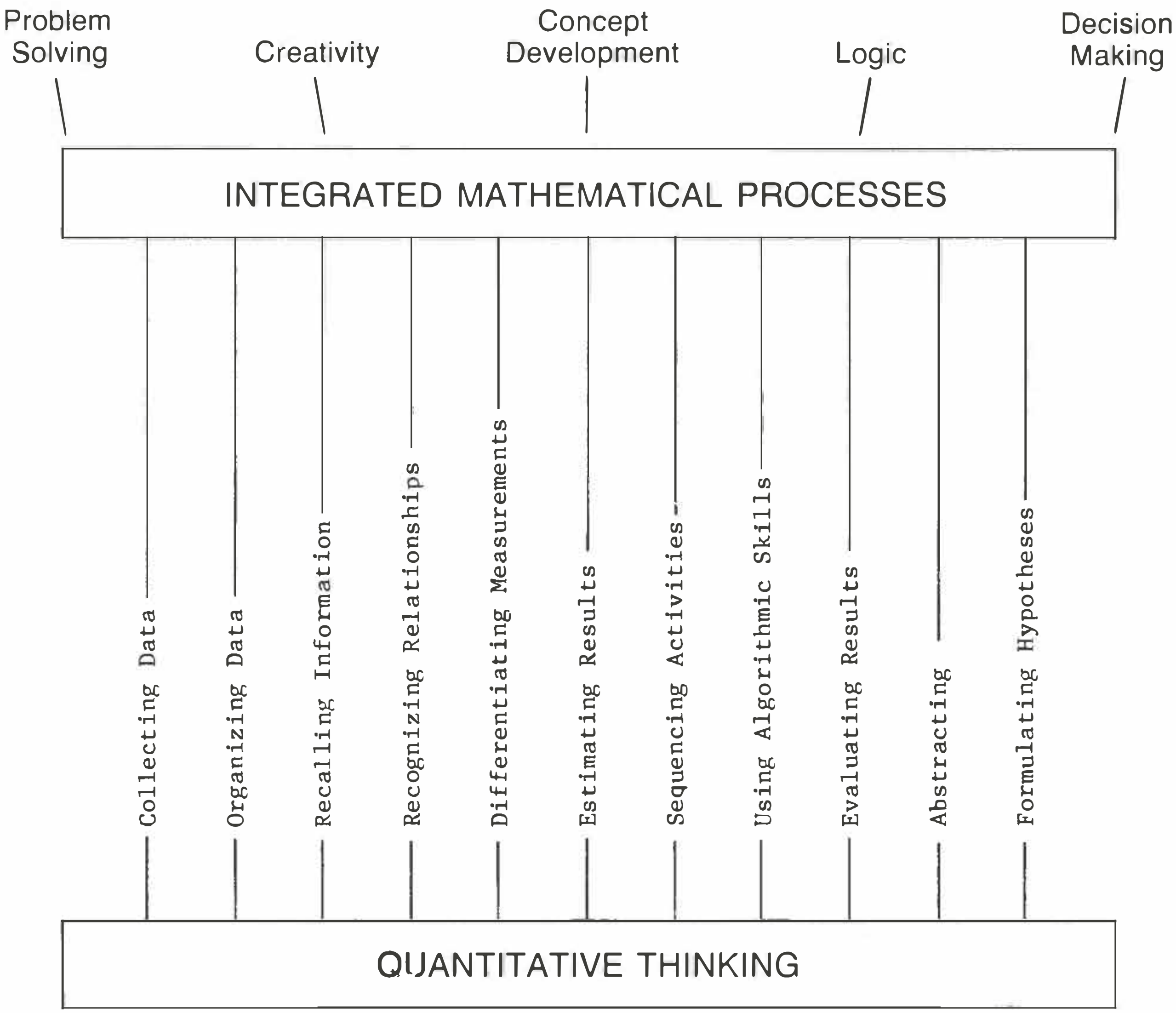
In elementary and secondary education, mathematics instruction is intended for all students, even though the expectations are different for different students. We believe that all students can be taught skills that will enable them to think better than they presently do.

The process of teaching higner order thinking skills has implications for the delivery of mathematics instruction to students. There must be a different climate in the classroom to reflect different emphases. It is important that the process whereby students are expected to memorize material, and then attempt to apply it prior to understanding the underlying concepts, be scrutinized. The participants believe - and their position is supported by a range of research -

that it is imperative for students to understand concepts at least simultaneously with, and preferably previous to, the memorization of facts and algorithms. At the same time, it is important that school personnel be aware of different learning styles among students, and make allowances for these differences.

Such changes in emphases carry clear implications for the amount of time needed for teaching mathematics. Either the time available must be increased, or deletions must be made in the present curriculum. Since it is highly unlikely that a significant amount of additional time can be extracted from an already full school day, the most obvious candidate for deletion would be the vast amount of time spent on pencil-and-paper arithmetic drill.

The proliferation of technology in schools can affect the way we teach, and an emphasis on higher order thinking skills will certainly have implications for the choice and use of that technology. Specifically, the calculator should be used to a much greater extent than it is at the present time.



Increased use of the calculator will reduce some of the time currently spent on arithmetic drill.

If we are sincere in our desire to emphasize higher order thinking skills, we must also be committed to developing suitable means for evaluating the achievement of these skills. Present testing procedures, which place a high priority on recall, recognition, and arithmetic skills, must be modified to encompass a more comprehensive approach that will assess how well the students are learning concepts and understanding processes.

Finally, there are implications for teacher training, both preservice and inservice. Teacher training programs must place more emphasis on higher order thinking skills so that teachers themselves become better problem solvers. In addition, teachers must become aware of the latest research being done in this area and its implications for their classrooms. That is, teachers must become convinced of the importance of teaching higher order thinking skills, they must learn the techniques for teaching them, and they must learn how to make room for them in the curriculum.

Guidelines and Recommendations

Finally, we are ready to make some recommendations about incorporating the teaching of higher order thinking skills into mathematics classrooms. Instructional techniques need to be carefully planned. Classroom climates conducive to exploration and experimentation must be created. These requirements should form the basis for preservice and inservice education of mathematics teachers at all grade levels.

Questioning techniques are at the heart of this kind of instruction. Teachers must learn to ask questions rather than dispense information. We recognize that good teachers have used this Socratic method extensively and that all teachers have used these techniques to some degree. We propose that teachers be encouraged to reexamine and improve questioning techniques in order for students to gain an understanding of concepts before moving to mastery of skills. Mathematics topics must be presented to students in such a way that will allow students to acquire an understanding of concepts or applications before they are asked to memorize and/or drill on techniques. This will help to ensure the transfer of knowledge to new situations, including the solution of problems in higher mathematics and related subject areas.

Objectives for mathematics classes must incorporate these new thinking skills. Expected outcomes will need revision in order to diminish emphasis on pencil-and-paper arithmetic. Testing, if it is to relate to these objectives, will also undergo radical changes. Teachers and assessment experts must use their creativity to develop testing procedures that will assess the students' ability to solve problems. In addition to formal testing, this may include observations of students while they are working.

Teachers will have to search out good sources for problems to use in their teaching of problem solving. They will have to use their creativity in selecting problems, finding appropriate situations for their inclusion, and developing procedures which will enhance the new knowledge and skills. At the same time, teachers desperately need to take time to solve mathematics problems themselves. In this way, they can be role models for students and develop an empathy for students who will be asked to solve problems. Teachers must take time, and schools must provide time, for this. Teachers must also find the time and procedures for sharing new problems with other teachers. Attendance at professional conferences and visitations to the classrooms of excellent teachers are examples of procedures for sharing information.

Curriculum writers, whether at a local or national level, must incorporate more activities involving concept development and problem solving. Since many teachers "follow the text," textbooks that provide the types of activities necessary to teach thinking skills must be developed.

Teachers and administrators must communicate the new emphases to school patrons. They will need to sell the importance of higher order thinking skills to obtain community support for spending less time on traditional topics and more time on the teaching of higher order thinking skills.

Suggested Activities for Teaching Thinking Skills

Here are some specific activities we recommend to help teach those higher order thinking skills. These are not listed in any order of priority, but are merely listed as the participants thought of them.

- Ask students to become involved in projects.
- Use manipulative materials to introduce concepts.
- Make good use of good computer software.
- Ask students to work together cooperatively to learn material.
- Augment texts with other activities and integrate discovery/exploration lessons.
- Have students write computer programs to do the algorithms of arithmetic.
- Ask students to communicate what they know, what they don't know, or what they need to know orally and in writing.
- Have students become involved in collecting/displaying data as part of an experiment.
- Develop lessons in cooperation with other subject areas.
- Focus on big ideas; for example, linear rate function leading to distance, time, and rate problems - thematic curriculum, look at special cases, emphasize structure, and develop algorithms.
- Use puzzle problems, including cryptarithmetic and toothpick problems.
- Use educational games.
- Have students make up their own games.
- Give an answer and have the students make a problem to match.
- Require book/video reports.
- Go on field trips.
- Use outside resource people.
- Use simulations on the computer.
- Let students elaborate on hobbies/personal interests.
- Allow time for students to solve problems (that is, provide a problem solving class, a problem of the week, or a problem of the day).
- Develop a school-family math program designed to get students and parents working on problem solving at home with the guidance of trained professionals.
- Have students develop test questions from different topics based on their own experiences.
- Have students explain why they used a particular algorithm or process to solve a problem.
- Encourage students to produce a videotape of an application.

No attempt has been made to classify the above activities with regard to specific course designation or grade level suitability. The suggestions are presented to aid teachers with possible instructional strategies for helping students improve thinking skills.

A Constructivist Approach to Teaching Mathematics

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Over the last few years, psychologists and educators have been interested in going beyond behavioristic and Piagetian views to new conceptualizations of learning, especially in using the computer as a "model" of how we think and learn. One of the new conceptualizations has been "information processing." Proponents of this view claim that when we think, we basically process information; it's as simple as that. This, however, leads to a further question: How do we manage this processing? To account for the management of processing, it is suggested that the learner engages other processes called metacognitive processes. But, still we might ask: What manages the metacognitive processes? Although this is not a trivial question, most proponents presently do not differentiate between levels of management, simply naming all those processes above the cognitive level metacognitive processes. In fact, the difference between cognitive and metacognitive is not always clear. For the time being, let us say that strictly mathematics propositions, procedures, and processes are called cognitive, while management decisions about such matters as when to use them, in what order, and with what degree of confidence are called metacognitive processes.

Another related view of learning has been called a theory of "personal constructs." The main tenet of this view is that all learners actively construct theories, no matter how minor, about what is appropriate action for responding to any particular situation. If a particular theory leads to inappropriate action, we revise the theory. This view, like information processing, also utilizes the notion of "metacognitive processes" managing our theory development. According to the personal constructs view, learners of differing capabilities exist because both our cognitive capacities and our metacognitive (management) capabilities differ. Another explanation, which goes beyond differences in cognitive or metacognitive components, is that some learners' perceptions are blinded (by emotion, say), so that they are unable to differentiate between appropriate and inappropriate action and, consequently, construct poor theories.

What relevance do these new conceptions have to the mathematics classroom? The one outstanding impression that the personal constructs view leaves is that our classrooms consist of 25 or so finely-tuned, sensitive, self-initiating, theory-generating, learning "beings." The metacognitive aspect, on the other hand, leads us to question how much of a committment we teachers have in attending to the *development* of metacognitive processes. The information processing aspect begs the question of how to present information for efficient storage and easy access. Psychologists and educators are still exploring answers to these questions and will be for many years. In the meantime, what aspect of these theories can be useful to teachers in dealing with the complex world of classroom instruction?

In order to make these ideas more available for teacher use, I will combine the three notions - information processing, personal constructs, and metacognitive processes - into one "constructivist" view of learning. In this article, I will describe constructivist principles of learning and further derive from them constructivist guidelines for classroom teaching of mathematics. Mathematics teachers are encouraged to think about, and use, these ideas to improve their classroom instruction. Psychologists and educators, who are continually striving for new insight into the learning process, would surely appreciate feedback from the most significant learning laboratory of all, the classroom. Curriculum examples will not be used to describe this view because these new conceptions of learning are equally relevant to all grade levels. The word constructivist has been around for many years. I am not concerned that my usage may be slightly different than that of others.

Constructivist Principles of Learning

1. **Purposeful Constructions.**
   Students construct their own theories for responding to a given situation, and, as they see their knowledge leading them to inappropriate action, they revise their theories. Learning proceeds from the current conceptions or theories of knowledge that the learner possesses. "Tuning," that is, modifying or adjusting, is an important learning process. Appropriate theories are best constructed in the light of some *acknowledged* purpose.

2. **Learning How to Learn.**
   Learners' awareness of their knowledge (mathematical content and processes, and metacognitive processes) at any time aids learning. Metacognitive processes (management of cognitive knowledge) are especially important, and these may be a major source of individual differences between slow learners and others.

3. **Confidence.**
   Because learning means taking risks and experimenting with new cognitive constructions, the atmosphere for learning must be familiar and full of trust. Inaccurate perceptions can be caused by either strong positive or negative emotions.

4. **Framework** for Information.
   Learning occurs in a context that provides a framework for the organization of information. The most appropriate context is one which is most applicable to the future situation in which the knowledge will be used. A framework for mathematical knowledge can consist of mathematical, everyday, and scientific elements.

5. **Structure of Knowledge.**
   All mathematical knowledge consists of propositional (conceptual and relational) structures and procedural (algorithmic and methods) structures. The process through which we understand and manipulate mathematical situations is grounded in specific content structures.

6. **Complexity of Concepts.**
   Propositional structures and procedural structures are complex content structures, a fact which is often disguised through rote learning and teaching. Although, traditionally, we teach through analyzing and breaking down knowledge, the constructivist sees "building up" as an equally valid learning process. Procedural

structures (algorithms) are linked in important ways to propositional structures (concepts).

7. **Transfer of Knowledge.**
As we learn, we learn context, as well as content and process. Transfer of knowledge must not be assumed; it occurs only as a new context is "seen" as the learned one.

Although a deeper understanding would require considerable elaboration on all of these principles, perhaps we can employ a constructivist teaching tactic, and let the reader come to understand the principles as they are *used* to develop the "guidelines for classroom teaching." Classifying something as complex as human learning in "seven principles" seems to be an utterly futile undertaking. However, I would like to elaborate slightly on the structure and complexity principles. Recognized in the structure principle, first of all, is the importance of relationships among all mathematical concepts and that any *understanding* of mathematics is a matter of recognizing all these relationships. Also implied in the structure principle is that all mathematical activity, such as problem solving, is highly dependent on these structures. The complexity principle, while acknowledging the many-faceted aspect of even apparently simple concepts such as multiplication, stresses that understanding and use of knowledge must take into account all, or most, of these facets.

Of course, these learning principles can be applied to the teaching of any subject, but our concern here is what this might mean for the teaching of mathematics. In deriving these guidelines for classroom teaching, it became apparent that several possible interpretations would be valid. Once again, I have opted for seven, knowing that these can only serve as general suggestions.

Constructivist Guidelines for Classroom Teaching

1. **Unit Context.**
Mathematics should be taught in the context of a three- to four-week unit constructed around a mathematical, everyday, or scientific application of the content. Students should feel comfortable and familiar with this application context.
RATIONALE: The purposeful constructions and the framework principles are satisfied by this. The actual application context would not only be a function of the content, but also of the grade level of the class, the characteristics of the students, and the school environment.

2. **Curriculum Tasks.**
The tasks which comprise the unit should be conducted with a view to the students engaging their current conceptions, mastering the task, and learning from it. The focus of the task should be central to the unit application.
RATIONALE: The learning how to learn and the confidence principles suggest that the task be a manageable part of the unit. The structure principle suggests that relevant mathematics knowledge be an integrated part of the task.

3. **Managing the Task.**
All students should be given assistance in dealing with the task - determining task difficulty, monitoring their understanding of it, apportioning time for it, and predicting how well they can perform it. The teacher should pay special attention to the students' perception of the task. Individual differences should be noted and provided for in this aspect.
RATIONALE: The purposeful constructions and learning how to

learn principles are important here, especially in helping students become aware of their knowledge and knowledge processes. This guideline is the core of the instructional process.

4. **Task Variety.**
Tasks should include a range of learning activities, such as direct examples, reviewing, textbook use, note taking, concrete materials, understanding, amplification of basic concepts, problem solving, self-inquiry, practice exercises, group activities, discussion and questioning.
RATIONALE: The purposeful constructions principle does not imply that student learning should be of a discovery nature, but only that learning should have some purpose. The complexity principle not only suggests that a considerable amount of guidance, even direct examples, is appropriate, but also that a variety of approaches is necessary to achieve an understanding of a mathematical topic.

5. **Assessment Tasks.**
Assessment should be carried out primarily within the context of the unit.
RATIONALE: The transfer principle suggests that we should first apply learning to the context of the unit. If we do testing beyond the context of the unit, we should be conscious of how the new context relates to the learned one. In actual (real-life) use of mathematics, contexts that are important to the student are most often familiar ones.

6. **Mathematical Learning.**

(a) Readiness.
Readiness for content learning must be noted, but only in the context of the learning task. What does the learner bring to the situation? Students' awareness of their own readiness is also important.
RATIONALE: Purposeful constructions are derived from previous "theories" that the student has. This is the central premise of the constructivist view. The learning how to learn principle suggests a self-awareness of these previous theories.

(b) Concepts.
Concepts, the pivotal ingredients of mathematics learning, must be constructed from the student's prior knowledge. Learning of complex subject matter is achieved through many different propositional structures. Specific instructional devices, such as concept maps and structured apparatus, should be employed.
RATIONALE: The framework, structure, and complexity principles all indicate the necessity of a thorough conceptual basis for mathematics learning.

(c) Skills.
Skill development, as it relates to the curriculum unit, is important. Care should be taken in selecting the application context for curriculum units. Skills and algorithms (procedural structures) are founded upon certain propositional structures. Skills should be learned as broader "method" approaches.
RATIONALE: Although our principles do not address the matter of skills directly, the structure principle advocates a solid basis for all procedures, while purposeful constructions implies that all skill learning be in context.

(d) Applications.
All applications occur in the context of the unit. They should be dealt with as an indication of the use, and usefulness, of mathematics, and also as a way of relating

the real world to the development of mathematics.

RATIONALE: The framework principle means that applications can be an important contribution to the framework for learning mathematics. The purposeful constructions principle suggests applications as a primary reason for studying mathematics. Lastly, the teacher must be constantly aware of transfer and the problem of the context of learning.

(e) Problem Solving.
Problem solving should be approached through a study of the particular kinds of problems in each unit. Problem solving is a particular way of knowing content.

RATIONALE: The structure principle suggests that all mathematics is dependent on specific knowledge. The metacognitive processes of the learning how to learn principle manage only cognitive knowledge. A constructivist view does not support broad generalizable problem solving strategies.

7. **Goals of Mathematics Learning.**
The major goals of mathematics teaching are that students gain understanding of complex areas of mathematical knowledge, use this knowledge in relevant situations, and understand their own processes and capabilities for functioning in a mathematical environment.

RATIONALE: The constructivist view not only provides new insight into how mathematics should be taught, but also implies a somewhat revised goal for mathematics teaching; "practice, feedback, and coaching" are not enough. Although the view expands upon what understanding means, one of the more interesting issues it raises is how teachers should regard their efforts toward improving students' capabilities for learning how to learn.

The strongest message of a constructivist approach is the desirability that teachers make clear to themselves and to students the purpose of learning mathematics. Making clear the purpose, without trivializing it, will be of great benefit in improving mathematics teaching. At this writing, I believe the weakest part of these guidelines is the matter of "context" and, therefore, the matter of what a sensible unit context might be. It seems essential that the context include, but go beyond the bounds of, mathematics itself. It certainly need not be confined to students' interests. Plausibility to the student might be a better guideline. Clearly, the broader the context, the more mathematics it will subsume. However, the greater breadth might tend to lose focus. Also, the notion of curriculum task and its position between the unit context and mathematics to be learned is somewhat problematic. An appropriate resolution of these weaknesses will need to be worked out in light of both the proposed principles of learning and the other guidelines.

Obviously, this interpretation of the constructivist perspective leaves many gaps. If a teacher were to conduct lessons solely on the basis of this statement (even assuming the availability of a textbook), I would predict chaos. The statement can only be seen as an attempt to modify already competent practice. Certainly, these are *not* prescriptions for teaching. Rather, I see them as interesting guidelines that can be tried, discussed, revised, and reinterpreted. A constructivist would see a teacher interpreting these guidelines on the basis of the teacher's existing "theories," and then, perhaps, rejecting them as invalid or "tuning" existing theories, using them, and then revising or discarding them.

At the very least, these guidelines should provide the basis for an interesting curriculum unit which would go far in explicating the guidelines. This would provide an opportunity for psychologists to say that their views have been misread or misinterpreted, which would be very useful. It might even serve to have them rethink their ideas in the light of feedback given by teachers. Whatever happens, teachers of mathematics are obligated to begin investigating ways that these new conceptualizations of learning can benefit them. Teachers certainly owe it to themselves and, in some sense, they owe it to psychologists and educators who are searching for new insight into the very important but, too often, frustrating process of learning mathematics.

During the school year 1985-86, Dr. Sol E. Sigurdson was on sabbatical leave from the University of Alberta, where he taught methods and graduate courses in mathematics education. His interests focus on classroom change brought about by inservice and curriculum change.

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Scope in the Mathematics Curriculum

Dr. Marlow Ediger
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Mathematics teachers and supervisors need to identify the scope of the curriculum. Scope answers the question of *what* pupils need to learn in lessons and units as they progress through sequential years of schooling. There are diverse means available to ascertain scope in the mathmatics curriculum.

Utilizing Basal Textbooks

Numerous mathematics teachers lean rather heavily upon the adopted single or series of textbooks to ascertain scope. The table of contents may then provide a generalized framework for what is to be taught and in which sequence. The teacher may make selected modifications and deletions in content coverage within a reputable mathematics textbook. However, the understanding, skills, and attitudinal goals emphasized in the manual section may still provide the majority of material to be learned by pupils. What, then, might selected writers of teacher education textbooks in mathematics emphasize in terms of scope? Nichols and Behr[1] discuss the following topics in a text for mathematics teachers of college/university courses:

1. numeration
2. addition and subtraction
3. multiplication and division
4. fractions
5. teaching decimals and percent
6. number patterns
7. integers, rational numbers, and real numbers
8. exploring geometric ideas
9. measurement
10. mathematical sentences
11. problem solving
12. calculators and computers in mathematics instruction
13. logical reasoning
14. teaching probability and statistics

The above-named topics may well suggest unit titles in the mathematics curriculum. A sequential program of instruction needs to be arranged so that each learner might achieve optimally in mathematics.

Learning Centres in the Mathematics Curriculum

Learning centres are a rather open-ended means for guiding student achievement. An adequate number is needed so that each learner may select which sequential tasks to pursue and which to omit. The teacher guides each student to attain optimally. No longer, then, does the teacher merely lecture and dispense subject matter, Rather, the teacher stimulates, motivates, encourages, and assists.

Student interest is an important factor in teaching and learning. If learners individually select and choose what to learn, motivation for

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[1]Eugene D. Nichols and Merlyn J. Behr, Elementary School Mathematics and How To Teach It (New York: Holt, Rinehart and Winston, 1982), Preface V.

learning should be at its highest level. Each learner sequences experiences in mathematics, rather than the teacher giving assignments, lecturing, and explaining. Biehler[2] lists the following basic ideas pertaining to humanism, as a psychology of learning:

1. Individuals act to get rid of deficiency needs (for example, hunger); they seek the pleasure of growth needs.

2. Deficiency motivation leads to reduction of disagreeable tension and restoration of equilibrium; growth motives maintain a pleasurable form of tension.

3. The satisfying of deficiency needs leads to a sense of relief and satiation; the satisfying of growth needs leads to pleasure and a desire for further fulfillment.

4. The fact that deficiency needs can be satisfied only by other people leads to dependence on the environment and to a tendency to be other-directed (for example, the person seeks the approval of others); growth needs are satisfied more autonomously and tend to make one self-directed.

5. Deficiency-motivated individuals must depend on others for help when they encounter difficulties; growth-motivated individuals are more able to help themselves.

What might be the scope of the mathematics curriculum, emphasizing humanism as a psychology of learning? The following titles of learning centres in a classroom are listed as an example of scope:

1. computation centre
2. geometry centre
3. problem solving centre
4. model making centre
5. mathematics laboratory centre
6. metric centre
7. programmed learning centre
8. basal textbooks centre
9. problem writing centre
10. instructional management centre

The breadth of offerings in terms of understanding, skills, and attitudes represents the scope of the above-named centres in the mathematics curriculum.

To achieve sequence in learning, each student needs to order tasks appropriately. Thus, ideally, each task is selected by the involved learner based on personal interests, needs, and purposes. Adequate provision in tasks must be made for slow and average learners, as well as for the gifted and talented. Each student needs guidance to attain optimally in ongoing units of study.

Mastery Learning and the Student

The total number of measurably stated objectives for learners to attain represents the scope of the mathematics curriculum within the framework of mastery learning. Measurably stated ends must be arranged in ascending order of complexity. Teachers and supervisors need to determine whether or not the specific ends are truly sequential. W. James Popham[3], an advocate of behaviorism as a psychology of learning, advocates the following model in developing teaching units:

1. precise instructional objectives
2. pretest
3. day-by-day activities
4. criterion check
5. posttest

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[2]Robert F. Biehler, Psychology Applied to Teaching, 3d ed. (New York: Holt, Rinehart and Winston, 1978), p. 517.

[3]W. James Popham, Teaching Units and Lesson Plans, (Los Angeles, California: Vincet Associates), filmstrip and cassette.

6. resources
7. backup lesson

In analyzing the above named teaching unit model, James Popham emphasizes for step one the writing of measurably stated, not general, objectives. Clarity of intent as to what teachers are to teach and learners are to learn is highly significant. Vague objectives need to be eliminated. Step two emphasizes a pretest be developed by the teacher or a team of teachers. The pretest should cover all the stated specific objectives. Paper-pencil test items (true and false, multiple choice, matching, essay, and completion items) may be utilized in the pretest. However, the pretest should not consist solely of teacher-written test items. Discussion, among other informal procedures, might also be utilized to ascertain present learner achievement in terms of pretesting. Based on pretest results, each pupil might then achieve new attainable ends.

Step three in the Popham model emphasizes using vital learning activities to realize new achievable ends. Each activity chosen must match up directly with a specific objective. It might be necessary to utilize more than one learning opportunity to guide a pupil to attain a measurable objective. In step four, a criterion check is utilized. The criterion check emphasizes measuring pupil progress continually to determine whether specific objectives are being achieved. Formative evaluation emphasizes appraising learner progress *during* the time a unit is in progress. A new teaching strategy may need to be used with those pupils not achieving vital objectives.

Step five in the Popham teaching unit model emphasizes the posttest concept. Thus, at the end of a unit, the teacher wishes to ascertain what learners have accomplished from the entire unit. Summative evaluation is then in evidence. Step six (resources) advocates teachers recording which audiovisual aids, objects, and reading sources will be used within the unit. The backup lesson (step seven) provides teachers with security; if materials for any lesson in the unit do not materialize, other activities need to be available to take their place in the backup lesson.

In any unit of study in mathematics, objectives for learners to attain must possess quality sequence. Thus, objective number one needs to be achieved prior to objective number two. Objective number two needs attainment in order that end number three can be mastered, and so on. If objectives truly contain recommended sequence, each learner should be able to achieve success in learning if initial readiness was in evidence. Before any given student moves on to the next sequential goal, a prior end must be attained. The teacher can then measure if a learner has or has not achieved an objective.

Mastery learning advocates believe that:

1. proficient mathematics teachers can select vital measurable goals for students to achieve.

2. essential activities and experiences can be chosen to guide student attainment of each specific end.

3. measurable results can be obtained from each student.

4. objectives and learning activities can be ordered appropriately to guide optimal student achievement.

5. students either do or do not reveal that a behaviorally stated (measurable) end has been achieved.

6. a modified teaching strategy can be devised which assists a learner to achieve a goal not previously acquired.

Woolfolk, et al.[4] wrote the following pertaining to mastery learning:

> Mastery learning is an approach to teaching and grading based on the assumption that, given enough time and the proper instruction, most students can master a majority of the learning objectives.
>
> To use the mastery approach, teachers must break a course down into small units of study. Each unit might involve mastering several specific objectives. Students are informed of the objectives and the criteria for meeting each objective. Often a variety of learning experiences is available to help students reach the objectives. In order to leave one unit and move on to the next, students usually have to attain a minimum mastery of the objectives. This may be defined as a certain number of questions answered correctly on the unit test. Letter grades for each unit can be based on levels of performance on the unit test. Students who do not reach the minimum level of mastery and students who reached the minimum level but want to improve their performance (thus raising their grade) can recycle through the unit and retake another form of the unit test.
>
> Under a mastery system, grades can be determined by the actual number of objectives mastered, the number of units completed, the proficiency level reached on each unit, or some combination of these methods. Students can work at their own pace, finishing the entire course quickly if they are able, or taking a long time to reach a few objectives. Of course, if only a few objectives are met by the end of the marking period, the student's grade will reflect this.

[4] Anita Woolfolk, et al. Educational Psychology for Teachers (Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1980), p. 505.

In Conclusion

There are numerous means available in developing the scope of the mathematics curriculum. The use of basal textbooks to ascertain scope assumes that textbook writers possess the knowledge and abilities necessary to determine what subject matter students need to learn. Humanism advocates that learners choose, within a framework, which activities and experiences to pursue, as well as omit. Decision making is, thus, emphasized in stimulating learners to achieve. Behaviorism emphasizes mathematics teachers writing specific sequential objectives for pupils to master. Measurable results are then significant.

Teachers and supervisors need to guide students to optimally achieve understanding, skills, and attitudinal goals in mathematics.

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Mathematics and the Alberta High School Curriculum

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From time to time, the teaching of mathematics changes. Since about 1980, the Alberta high school syllabus has undergone a certain reform, and while some of the reasons for such change seem sound, others are more obscure and questionable. The adoption of the metric system created a necessity for an update. The easy access to hand-held calculators required a different emphasis in the area of logarithms. Such traditional topics as geometry were to be treated from a different perspective because of developments in mathematics that had filtered down to the secondary school level. The inclusion of nontraditional areas, such as statistics and the minor topic of exponential growth and decay, have raised eyebrows. In this article, an attempt will be made to identify, by subject area, a few of the anomalies and difficulties that occur in our curriculum and textbooks.

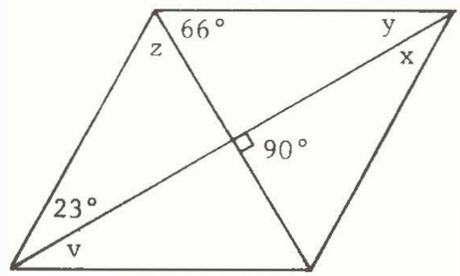
Geometry as a High School Subject

For the high school curriculum, the question of what part of geometry we present is a rather existential problem. Our present Grade 10 texts treat it very casually and with little sense of purpose, made worse by the fact that, in many places, the textbooks contain grave errors.

In the *Holt Mathematics 4* text (Hanwell, Bye, and Griffiths, p. 230), the following exercise occurs:

> Consider a parallelogram with three angles given and calculate the angles x, y, z, and v.

fig. 1



The unaware reader obtains results that correspond with the answers given in the back of the textbook. However, the exercise is completely ludicrous. Pictures and numbers have collided in a strange way. (A proper answer would be: This is a rhombus in which the diagonals are perpendicular and the diagonals also bisect the angles of the rhombus. Hence, the answer in the textbook is incorrect, and so is the "given" part.)

In order to see some of the difficulties encountered when deciding on which part of geometry should be presented in the school curriculum, we have to consider the development of geometry from a historical perspective and the more recent outlook on mathematics itself.

Once a proposition in mathematics has been settled, it becomes generally accepted. The acceptance is based on what we call proof. Over time, the significance of the proposition may change as it becomes part of a larger body of knowledge, but its quality stays the same. Since the time of Euclid, the validity of propositions in elementary geometry has been based on an axiomatic system, a collection of statements accepted as true. From these initial statements, a large collection of propositions is deduced by agreeing upon certain rules of inference. In 1931, Kurt Goedel proved that there exist axiomatic systems from which certain propositions belonging to the system can neither be proved nor disproved.

An illustration of Goedel's contention, which is even presentable in the classroom, is Goldbach's conjecture. The conjecture states that every even natural number greater than two is expressable as the sum of two primes, where primes are natural numbers divisible by one and themselves only. Up to now, no even number has been found that is not the sum of two primes. The conjecture may be true, but may not be derivable from the axioms of arithmetic. The same may apply to what is known as Fermat's last theorem. This theorem states that there are no natural numbers a, b, and c such that $a^n + b^n = c^n$ for "n" greater or equal to three and "n" a natural number. These conjectures have the charm that they can serve as illustrations in the relatively simple setting of elementary mathematics, and that, even today, these draw considerable interest from mathematicians.

The closer scrutiny of the axiomatic system was largely caused by the development of different types of geometry. In Riemannian geometry, for example, Euclid's axiom that through a point P in the plane not on line "l" a line can be drawn parallel to "l" is denied. Of course, philosophical questions arise regarding the plausibility of these geometries.

The classical belief that the properties of Euclidean geometry are valid for the world in which we live has been undermined, as it becomes evident that other geometries are equally valid. In an article entitled "Elementary Geometry, Then and Now," I.M. Yaglom (Davis, Gruenbaum, and Scherk, p. 165) speaks about geometries that draw considerable attention in this half of the twentieth century and makes a comparison to developments in the previous century. He says:

> In contrast to discrete geometry, combinatorial geometry so far has no serious practical applications; in this respect, it resembles "classical" elementary geometry, which considered properties of triangles and circles, which beautiful though they were, were scientifically blind alleys - leading nowhere, giving nothing to science at large. Still "nineteeth-century elementary geomtery" was closely bound up with what might be called the "scientific atmosphere" of those years. . . .

There are two pedagogical consequences to be drawn from Yaglom's argument. Certain aspects of geometry are culturally bound and do not necessarily lend themselves to so-called practical applications. The present curriculum seems to be preoccupied with these applications. Secondly, since Euclidean geometry is not the only valid system, we have to conclude that one of the significant objectives is to teach our students the method of a deductive system. The deductive character of a system is more easily established in Euclidean geometry than in any other part of high school mathematics. (For the 13-23-33 sequence of mathematics courses, a different perspective should prevail.)

Exponential Growth and Decay

Euclidean geometry has been, traditionally, part of the secondary school curriculum. This cannot be said of the particular minor topic presented in both approved texts for Grade 12. In order to see what is going on, we will have to go through a more or less technical explanation with omission of mathematical techniques. In the *FMT Senior* text (Dottori, Knill, and Stewart, p. 153), the exponential growth rate is explained on an intuitive basis. Since bacteria multiply by splitting, the population increases by a power of two. Without much ado, the growth function is declared to be an exponential function with base two for any increasing biological population whatsoever. It could include mice. The model in the textbook is quite reasonable as long as the bacteria are declared immortal. Such a representation violates the laws of nature.

A correct way to derive the appropriate formula for the growth rate would be by means of a simple differential equation, which is beyond the scope of high school mathematics. The proper formulation of the problem lies in the assumption that a biological population has a growth rate that is proportional to its size. In this formulation of the problem, the mortality rate is included in the hypothesis. A simple technique of elementary calculus yields the correct result. In this derivation, the base two of the textbook can be shown not to be unique. Thus, a mice population increase no longer creates a hazard for the formula.

For decay of radioactive materials, the rate of decay is again assumed to be proportional to the original mass of the material. Again, the proper formula is derived by the same differential equation. However, the textbook explanation requires the observer to watch the material for 25 years to obtain half the mass, and another 25 years to again halve the mass. After some mysterious reasoning, an exponential function emerges with the not unique base two. In *Calculus*, Volume I, Tom Apostol (p. 229) says:

> Actually, the physical laws we use here are only approximations to reality, and their motivation properly belongs to the sciences from which the various problems emanate.

The opinion has been voiced that high school courses should contain practical applications. However, some sobering thoughts come to mind if one considers the examples cited here.

1. The problem of exponential growth and decay requires mathematical techniques that are not available to the high school student.
2. If a student were to try out the methods from the textbook on a science project, it would be doomed to failure. It would also require estimation of the constants in the formula that demands the method of least squares, which is also beyond the secondary school level.
3. It seems that so-called applications borrowed from mathematical literature past the high school level lead to disastrous results.

The final conclusion has to be that this topic should be abandoned unless somebody can come up with a proof that is presentable at the high school level.

Statistics in High School

The field of statistics has grown enormously in this century and the results are being felt in almost every aspect of life. Who can imagine a political election without a poll? By its overwhelming presence, statistics has also found its way into the high school curriculum. In Grade 12, we study something about the normal distribution which, in two dimensions, is graphically represented by a bell-shaped curve. Assumptions about this distribution are, as a rule, verified by hypothesis testing. However, in high school, the experiment is absent, and so we are told that all necessary assumptions hold in order to simplify the case. Suddenly, the conclusion is drawn that we have obtained a "standardized normal distribution."

About 15 percent of the questions on the departmental exams are based on this topic. The value of this type of mental exercise is highly questionable. At present, the student has been taught to manipulate some formulae that appear out of the blue yonder.

It may be necessary to look at the historical development of statistics in order to come up with a suitable secondary program. At the moment, we only deal with the normal distribution. The danger is that we give students the impression that this is the only distribution there is, which is not true. It is also very hard to explain that mean and standard deviation have the same meaning as the first two moments of a mass in physics. Interrelationships are not established. In *Mathematics and Logic - Retrospects and Prospects*, Marc Kac and Stanilaw Ulam (p. 50) say:

> The theory (or calculus) of probability has its logical and historical beginnings in the simple problems of counting.

Indeed, it is simpler to present, in the classroom, the phenomena of tossing coins and dice than to give sound reasons for the continuous normal distribution. Since there is no long tradition in the teaching of statistics at the secondary level in any country, we are treading on very thin ice. It seems safer to go back to its original beginning and show something about the essence of its method than to show off with impressive-looking results. The normal distribution is a powerful tool in statistics, but the ability to see the full scope of its impact belongs to the professional statistician.

Conclusion

There is a great need for rethinking parts of the mathematics program. I.M. Yaglom (Davis, Greenbaum, and Scherk), in his article "Elementary Geometry, Then and Now," speaks about leading mathematicians who have written texts for secondary students. One of these is A.N. Kolmogorov, the Russian mathematician, who has written a text that is used by all secondary students in Russia. He speaks also about the French mathematician Jean Dieudonne, who wants to see geometry reduced to linear algebra and who has written a text for this purpose. Our school system cannot directly take over these ideas, but they can form a subject for study and comparison. If we want proper programs for our secondary schools, then we cannot leave the writing of textbooks to the book publishers and the forces of the marketplace.

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Curricular Implications of Microcomputers for School Mathematics

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We are all aware that the microcomputer is a potentially valuable tool in mathematics teaching. Indeed, there are many who feel the microcomputer is a very powerful educational tool. But to explore the potential of this new technology means that we must also examine its limitations. In this paper, emphasis will be placed on what cannot or should not be done with computers in mathematics education. There are three types of limitations concerning microcomputers in Grades K through 12:

1. limitations due to educational responsibility
2. limitations due to practical technical restrictions
3. limitations due to logical and conceptual restrictions

Limitations Due to Educational Responsibility

The first point is certainly the most problematic, maybe even controversial. There is no doubt that microcomputers are affecting *what* students are learning and *how* they are learning. But the more sophisticated a tool, the more we must care about its use; and it is exactly the power and versatility of microcomputers which threaten a danger of their misuse.

Propaganda about the educational use of microcomputers is pervasive in our society today. We are referring to the vast promotion of educational software for curricular subjects. Currently, the number of educational programs available is estimated at 80,000, and and that number is doubling each year! Most of this software is of tutorial or animated drill and practice type, which is usually very good from a technical point of view, but is of questionable educational value. We must use great care in selecting software in mathematics teaching or else we may be risking misuse of the microcomputer.

We strongly support a reasonable use of microcomputers in the classroom. As mathematics teachers, we would like to have a microcomputer, connected to a screen, available to use in making demonstrations which might become objects of discussion for the whole class. Further, we believe the microcomputer can be very useful for:

1. stimulating mathematical thinking and supporting mathematical problem solving,
2. visualizing mathematical concepts, and
3. simulating processes that can be handled by mathematical models.

The computer enables us to handle mathematical objects, operate with numbers, present and transform geometrical figures, and visualize relationships among data. That microcomputers can be very useful in supporting and enriching mathematical problem solving is very clear. Problem solving is central to any mathematical activity. The computer enhances our problem solving capacities, and that is what students should experience in today's mathematical education, regardless of whether this occurs via BASIC, LOGO, PASCAL, or some other programming language. (Of course, we do not mean to imply that we regard all programming languages as equally suitable.)

Solving a problem using a computer is typically a sophisticated process that includes very different kinds of activities; for example, developing mathematical models, generating data for a problem, analyzing relationships, designing alogrithms, and constructing programs. With these kinds of activities, students can get to the mathematical heart of the matter. If the problems are well chosen, students will experience the intellectual challenge of mathematics, as well as the satisfaction provided by the solution of a difficult problem.

In mathematics, the solution to a problem is not nearly as interesting as the *method* used to get it, and many problems can be solved by quite different methods. Teachers should encourage their students to look for different approaches to a problem and to compare and evaluate them. We illustrate this by two examples below.

EXAMPLE 1:

The following is a nice problem for students at the elementary or junior high school level:

> On a farm there are 178 animals - cows and geese. Altogether they have 562 legs. How many cows and how many geese are on the farm?

There are many ways to solve this problem.

(a) We can actually make a list and check all combinations:

cows	geese	legs
0	178	356
1	177	358
2	176	360
.	.	.
.	.	.
.	.	.
50	128	456
.	.	.
.	.	.
.	.	.
90	88	536
.	.	.
.	.	.
.	.	.
103	75	562

(b) We can use a computer to generate the list. The following LOGO program does the job:

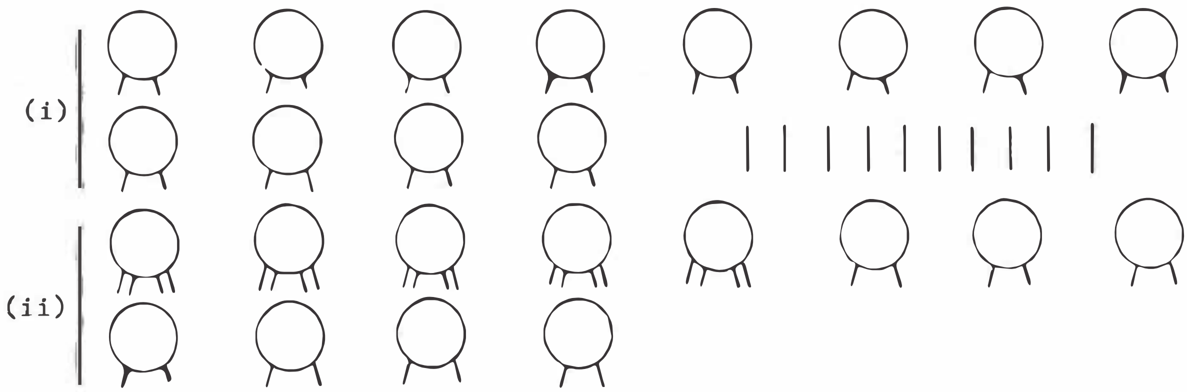
```
TO RANCH :ANIMALS :LEGS :COWS
MAKE "GEESE :ANIMALS-:COWS
(PRINT :COWS :GEESE 2*: GEESE +
  4*:COWS)
IF 2* :GEESE + 4*:COWS = :LEGS
  [STOP]
RANCH :ANIMALS :LEGS :COWS + 1
END
```

If we type in "RANCH 178 562 0," then the computer prints the above list and, therefore, solves the problem. This is a simple program, and the algorithm is brief and easy to understand and write.

(c) Some students attack the problem by using linear equations:

$$x + y = 178$$
$$4x + 2y = 562$$

(d) We know an eight-year old girl who solved the problem with smaller parameters - 12 animals and 34 legs - in the following manner:



She symbolized the animals by circles, gave two legs to each, and assigned the remaining ten legs in pairs. Once she had solved this "smaller" problem and *understood the concept*, she was then able to solve the problem for arbitrary parameters.

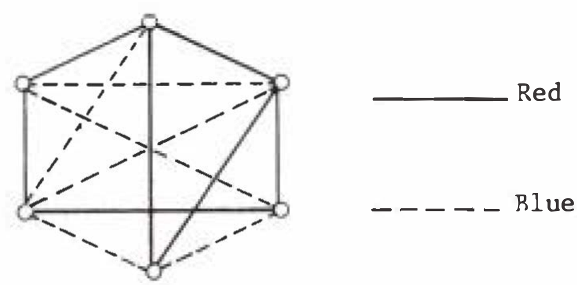
Which solution is the "best" one? We think the last, because it is the *simplest* and most straighforward one. It avoids the tedious work of the first, as well as the advanced algebraic tool of the third one. Further, it is available to an elementary school student, and we think it is the best solution from the point of view of a mathematician, too. The second (computer) solution represents a possible approach, too, but it should not stand alone.

We cannot define mathematical beauty, except to say that is has to do with simplicity and the use of straightforward arguments, simple but powerful ideas, and avoiding the use of sophisticated tools. Teachers should always strive to help students get a feeling for the beauty of mathematical ideas and methods. This is all the more important when we have a computer which we can program to supply solutions very quickly.

EXAMPLE 2:

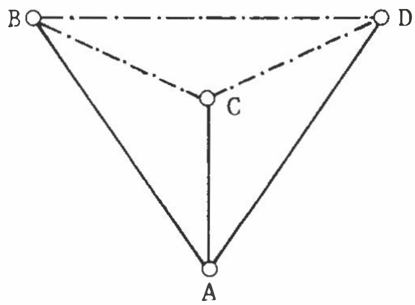
The following is a strategy game for students:

We start with six vertices of a hexagon. Two players alternately take turns, each time connecting two so far unconnected vertices. The first player uses a red pencil, and the second player uses a blue one. A player loses if he or she generates a triangle with all sides in his or her color. The result is a draw if all possible 15 lines have been drawn without producing a one-colored triangle.



After some experimentation, it is observed that someone always loses. The following conjecture arises: if each of the 15 connecting lines of a hexagon is colored either red or blue, then there will be at least one red or blue triangle.

If we have a computer available, the conjecture can be proven by systematically checking all possible blue and red colorings. If no coloring without a one-colored triangle is found in this process, then we are done. But, compare this with the following method. Consider an arbitrary vertex of the hexagon, say A. There are 5 emanating lines. At least three must have the same color. Without loss of generality, let us assume that there are three red lines. The end points of these three lines may be labeled B, C, and D.



If one of the lines BC, BD, or CD is red, then there is a red-colored triangle; if these three lines are all blue, then we have a blue triangle (that is, BCD). We think this is a beautiful proof, demonstrating the superiority of mathematical reasoning over brute computer force.

What should we learn from these examples? When teaching problem solving, we should always encourage our students to look for different ways to get solutions. They should also be aware of the tools available and select the most suitable one(s). It isn't necessary to use a bomb to kill a fly!

Limitations Due to Technical Restrictions

Virtually any finite mathematical problem can be solved by a computer, simply by checking all possible states of the problem. But many mathematical problems, especially combinatorial ones, are of such exploding complexity that even the most powerful computer may never be able to handle them. Let us demonstrate this, again by a simple example:

EXAMPLE 3:

Consider the numbers 1 and 2. There are two different arrangements to write these numbers in sequence, namely 12 and 21, and each such arrangement is called a permutation. There are six permutations of the three numbers 1,2,3: 123, 132, 213, 231, 312, 321.

It is not difficult to write a program to find out all possible permutations of n elements. But suppose we want to get a list of all possible arrangements of the numbers 1 to 15. This would seem to be a simple problem for a computer. The number of permutations of n elements is n!=1*2*3*... *(n-1)*n. The table below provides the values 1! to 15!

n	# Permutations
1	1
2	2
3	6
4	24
5	120
6	720
7	5,040
8	40,320
9	362,880
10	3,628,800
11	39,916,800
12	479,001,600
13	6,227,020,800
14	87,178,291,200
15	1,307,666,368,000

Now, imagine a very powerful computer, capable of determining and listing 1,000 different arrangements each second. This computer would have to work 40,000 years to finish its job!

There are many problems having important applications, which are practically unsolvable because of their algorithmic complexity. It is a subject of greatest scientific and economical importance to determine the complexity of algorithms and, if possible, to find algorithms for a given class of problems by which solutions can be obtained in a reasonable amount of time. We should strive to help students learn by simple examples from, say, combinatorics and number theory, that there are many problems which cannot be solved by computers for practical reasons, even if it is easy to develop a program that seems to solve the problem.

Limitations Due to Logical and Conceptual Restrictions

There are other problems which have been proven to be unsolvable by any computer. The best known of these problems is the so-called "halting problem." The unsolvability of the halting problem means that it is impossible to construct an algorithm which can decide for any arbitrary program and its data if it will ever stop or if it will be caught in a never ending loop. We think that the treatment of the halting problem and related problems is, perhaps, beyond the usual mathematics curriculum, though it is not really difficult. But there are other problems by which students can become aware of what a computer actually can and cannot provide in order to find a solution.

EXAMPLE 4:

Recently, we asked some students in a problem solving course to prove that $\sqrt{2}$ is irrational; that it cannot be represented as p/q with integers p and q. One student wrote the following: "With the help of a computer, we can determine that $\sqrt{2}$ is equal to 1.414213 . . . , never ending and never repeating. Therefore, it cannot be a rational number." The student, of course, had a fundamental misunderstanding of the conceptual potency of computers.

EXAMPLE 5:

The Collatz Problem (also known as the Ulam Problem, the Syracuse Problem, or the Hasse-Kakutani Problem). Consider the following algorithm in Pascal:

```
INPUT N
WHILE N > 1 DO
IF ODD (N) THEN N:=3*N+1 ELSE N:=
  N DIV 2
END.
```

(NOTE: DIV denotes whole number division.)

The input number N is the *seed* of a sequence either ending with 1 or never ending. Here are some examples:

(a) 10 5 16 8 4 2 1
(b) 42 21 64 32 16 8 4 2 1
(c) 120 60 30 15 46 23 70 35 106 53 160 80 40 20 10 5 16 8 4 2 1

It is still an unproven conjecture that for any positive integer N the algorithm will come to 1 eventually. Recent issues of the *American Mathematical Monthly* and *The Mathematical Intelligencer* contain papers devoted to this problem.

A good deal of experimental work has been done concerning this problem. Using powerful computers, it has been proven that the algorithm stops for any positive integer $N < 2^{40} \approx 1.2 * 10^{12}$. This gives certain evidence about the conjecture, but it doesn't prove anything concerning the general problem. Why, then, all this effort? There are two possibilities:

(a) The conjecture is true. This can never be proven by computer experimentation, because we can only check a finite number of integers, and therefore, an infinite number of possible seed numbers will forever remain unchecked.

(b) The conjecture is false; that is, there is a positive integer input N such that the algorithm never comes to 1. This can be due to either of the following reasons:

- There is a seed number N such that the sequence generated by N diverges to infinity. The existence of such a number can never be proven by running the algorithm, because you have to stop this calculation after awhile not knowing if, at sometime in the future, the algorithm would come to the end.

- There is a seed number N such that the sequence N, N_1, N_2, . . . generated by N is caught in a loop; that is, after a while, part of the sequence will be periodically repeated.

Such a loop can be detected by a computer, thus proving that the conjecture is wrong. (If, in the above algorithm, N: 3*N+1 is replaced by N:=3*N-1, we can find seed numbers producing infinite sequences. For example, 80 40 20 10 5 14 7 20 10 5 14 7 . . .) These are the only logical possibilities.

Conclusion

There is presently a great deal of discussion about computer literacy. A major factor in computer literacy, we believe, is the competence to make reasonable use of the power of computers, which means to be aware of the computer's limitations. We have all heard the term "computer revolution" in education. One characterisitic of a revolution is that it completely changes traditional values, structures, and ideas. The computer is a powerful tool that can affect *what* students will learn and *how* they will learn. But we should not forget the great mathematical ideas as developed by Euclid, Archimedes, Euler, Gauss, and others over hundreds of years. They are still the great ideas of tomorrow and tomorrow's tomorrow. Further, the importance of these ideas continues to grow. We must not allow the availability of computers to make mathematics superfluous; on the contrary, it requires improved mathematical education. The computer itself can help us to improve and enrich the curriculum. If we make sensible use of the computer, its impact should result in a permanent educational *evolution*, instead of revolution.

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Enhancing Comprehension Through Reading Instruction

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In a previous article published in *delta-K* (Volume XXV, Number 2, March 1986), the authors explored the relationship that exists between the process of problem solving in mathematics and the social inquiry process in social studies. Fundamental to understanding the content of a problem or social issue is the ability of students to read and interpret the printed word before they can develop a mental construct of the intended solution or decision to be made.

We have often heard the statement, "My students cannot read the prescribed material. They have difficulty with the words and do not understand the meaning of the content." What do we mean by reading? We know that it is an activity in communication, basic to securing some comprehension from the printed page. Reading is a decoding or deciphering process through which we translate the written symbols into an expression of meaning. In the process, meaning is attached to the written symbols. Students need help and training to learn a process for translating symbols into meaningful understanding, a fact crucial to determining the task inherent in a problem. The often repeated phrase, "every teacher is a teacher of reading," is more than a cliche; it is basic to the teaching of any subject. Perhaps it is more accurate to say that every teacher is a "teacher of reading and interpretation in a specific subject."

In both mathematics and social studies, where the focus is problem solving, the ability of the students to read, interpret, and infer meaning from a problem is crucial for insight into the process of solution. The directed reading process, traditionally used by the language arts teacher, can be just as effective in social studies or mathematics. The directed reading process can help the teacher and the students better read and understand the problem.

Teachers need to be more cognizant of how the specialized vocabulary of a subject has specific meaning or connotation in context. Teaching strategies that emphasize accurate definitions, the relating of word meaning to the personal experience of students, and the identification of the root, prefix, or suffix of a word help students to understand new vocabulary. Direct vocabulary teaching may also be necessary before beginning to determine the intended solution of the problem.

A series of lessons might be used to teach students a systematic approach to understanding a problem. Another way would be a simplified combination of steps conducted in a single lesson, which requires less practice. Extended practice would occur through the working out of problems.

Lesson One. In the first lesson, students are given a word problem to read. The students are to answer: "What is the question?" or "What are we to find?" It is not sufficient for students simply to read the question as it is stated in the problem. Rather, students should be asked to state the question in their own words. Several problems should be given to the students so that they become proficient at determining the question and restating it in their own words. Once the students can do this with little difficulty, problems may be developed and shared with the class. If students understand the problem, similar problems may be developed, or students may rewrite or retell them in their own words. In restating the problem, students should be encouraged to use appropriate synonyms related to the subject area.

Lesson Two. The second lesson should be built upon the first and focus on the ability to describe what quantities are involved, or what information is given. Adequacy and relevancy of the information should be determined. Again, students should be asked to state these quantities or information in their own words.

Lesson Three. In the third lesson, the teacher and students can begin to describe the process(es) that may be used to solve the problem. In social studies, the intended outcome, in terms of predicting a solution, may of itself determine the process; for example, historical research versus map study require different processes. The teacher is still not asking students to solve the problem, only to consider the kinds of process(es) that could be used. The students are encouraged to come up with as many different ways to solve the problem as possible. This helps students to overcome the idea that there is only one correct way to solve a problem. Problems in mathematics may be solved in more than one way, as well. Whatever process is chosen, students should be able to support their choice.

Lesson Four. Lesson four in the directed reading process is the actual solving of the problem. This step should not be introduced until students are comfortable and proficient with the other three steps. If the directed reading process has been followed up to this point, the teacher should feel confident that growth in the students' vocabulary development, reading development, and subject skills and ability to fully comprehend the meaning of the problem has occurred. Once the solution has been obtained for the mathematics problem, or a decision made on the social issue in social studies, the students should be encouraged to recheck their work to verify the accuracy of what has been done.

If students are exposed to the above process at the beginning of a semester, subsequent experience in its application would be an integral part of their thinking in terms of problem solving. They will develop a model for thinking that has transfer value in other subject areas. Thus, the product of such formal instruction should be students who will have the necessary cognitive skills to approach a problem in mathematics, or the social issue in social studies, in a systematic manner.

The charts on the following pages illustrate the process of reading a problem, as described in this article.

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Directed Reading Process

	STUDENT ACTIVITIES	
LESSON 1.	Mathematics	Social Studies
WHAT IS THE PROBLEM? - from textbook curriculum guide - teacher - student	State problem in own words. Tell a friend. Write problem in own words. Develop similar problem.	State as a "should question" - What ought to be? List key words. Define terms.
LESSON 2.		
WHAT INFORMATION IS NEEDED? 1. Information within problem (adequacy of information given). 2. Insufficient/sufficient information. 3. Recall of pertinent information. 4. Reference to data sources: charts, graphs, tables. 5. Additional information needed (research).	Underline key words. Share with a friend. Compare notes. Supply missing information. List relevant information. Cross out irrelevant information. Rewrite, deleting extraneous information. Definition/clarification of terminology. Recall formal equation. Construct similar problem. Identify facts needed in operation.	Underline key words. State concern with problem. Identify difficult words. State problem in own words. Definition/clarification of terminology. Identify specific factual data inherent in the issue. Restate the issue more accurately. Supply additional information through experience/ library research - use resources. Restate issue orally in terms of understanding its intent.

	STUDENT ACTIVITIES	
LESSON 2. (cont'd.)	**Mathematics**	**Social Studies**
	Supply additional information for interpretation. Restate problem orally in terms of understanding its intent.	
LESSON 3.		
WHAT PROCESS IS USED TO SOLVE THE PROBLEM?	Supply formula. Identify process. Use problem-solving steps. Cue words. Trial solution.	Indicate steps of social inquiry as per curriculum guide.
LESSON 4.		
SOLVE THE PROBLEM. Verification.	Solve the problem. Check process. Check reasonableness. Verify solution.	Identification of conflicting values. Make decision on the issue. Select a value position. Consider application of decision: desirable/undesirable; feasible/infeasible.

Rhombi Ratios on the Extended Multiplication Table and Hundred Square

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Mathematics teachers are interested in situations that can be used for drill and practice using either paper and pencil or a calculator. It is ideal if these situations give rise to pattern discovery in settings involving finite mathematical notions. One of these settings involves the multiplication table, as shown in Figure 1.

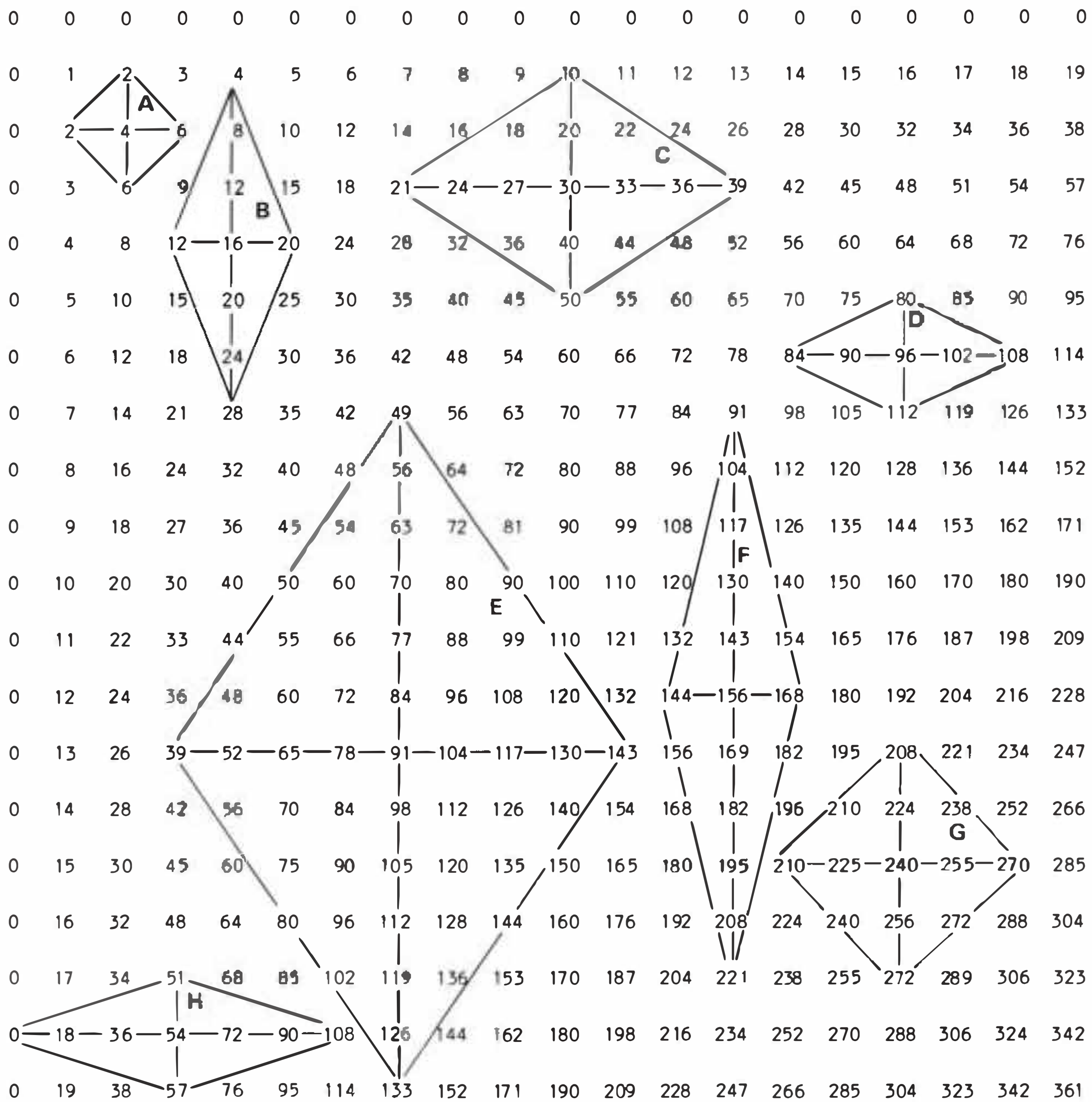
FIGURE 1. Multiplication Table

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Figure 2, on the following page, displays the interior of an extended multiplication table. Rhombi of varying sizes have been drawn on the table. For each rhombus:

1. Find the sum of the vertices (V).
2. Find the sum of the interior numbers (I).
3. Find the sum of the entries on the horizontal diagonal (HD) and the sum of the entries on the vertical diagonal (VD).

FIGURE 2.



0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57
0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114
0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133
0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152
0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171
0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190
0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209
0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228
0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247
0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266
0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285
0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304
0	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323
0	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342
0	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361

Table I reports the sums, together with the number of entries that were summed. In each case, the sum is listed first with the number of entries in each sum in parentheses.

TABLE I.

Rhombus	V(#)	I(#)	HD(#)	VD(#)
A	16(4)	4(1)	12(3)	12(3)
B	64(4)	80(5)	48(3)	112(7)
C	120(4)	330(11)	210(7)	150(5)
D	384(4)	288(3)	480(5)	288(3)
E	364(4)	4095(45)	819(9)	1183(13)
F	624(4)	1404(9)	468(3)	1716(11)
G	960(4)	1200(5)	1200(5)	1200(5)
H	216(4)	270(5)	378(7)	162(3)

To find a pattern, form ratios by dividing each sum by the parenthesized number which follows. When this is performed, a constant ratio results for each rhombus. Table II reports these constants.

TABLE II.

Rhombus	Constant Ratio
A	4
B	16
C	30
D	96
E	91
F	156
G	240
H	54

In each rhombus, the common ratio is equal to the "centre" number, the point where the two diagonals intersect. Why?

Consider rhombus C. The numbers to the right and to the left of the centre number 30 on the horizontal diagonal are evenly spaced above and below 30. To the right of 30:

$$33 = 30 + 3$$
$$36 = 30 + 6$$
$$39 = 30 + 9.$$

To the left of 30:

$$27 = 30 - 3$$
$$24 = 30 - 6$$
$$21 = 30 - 9.$$

Consequently, 30 is the mean of the seven numbers on the horizontal diagonal. This implies that the sum of the entries on the horizontal diagonal is

$$30 \cdot 7 = 210, \text{ or } 210 \div 7 = 30.$$

On the vertical diagonal below 30:

$$40 = 30 + 10$$
$$50 = 30 + 20.$$

Above 30:

$$20 = 30 - 10$$
$$10 = 30 - 20.$$

Again, 30 is the mean of the five numbers on the vertical diagonal. This implies that the sum of the vertical diagonals is $30 \cdot 5 = 150$ or $150 \div 5 = 30$.

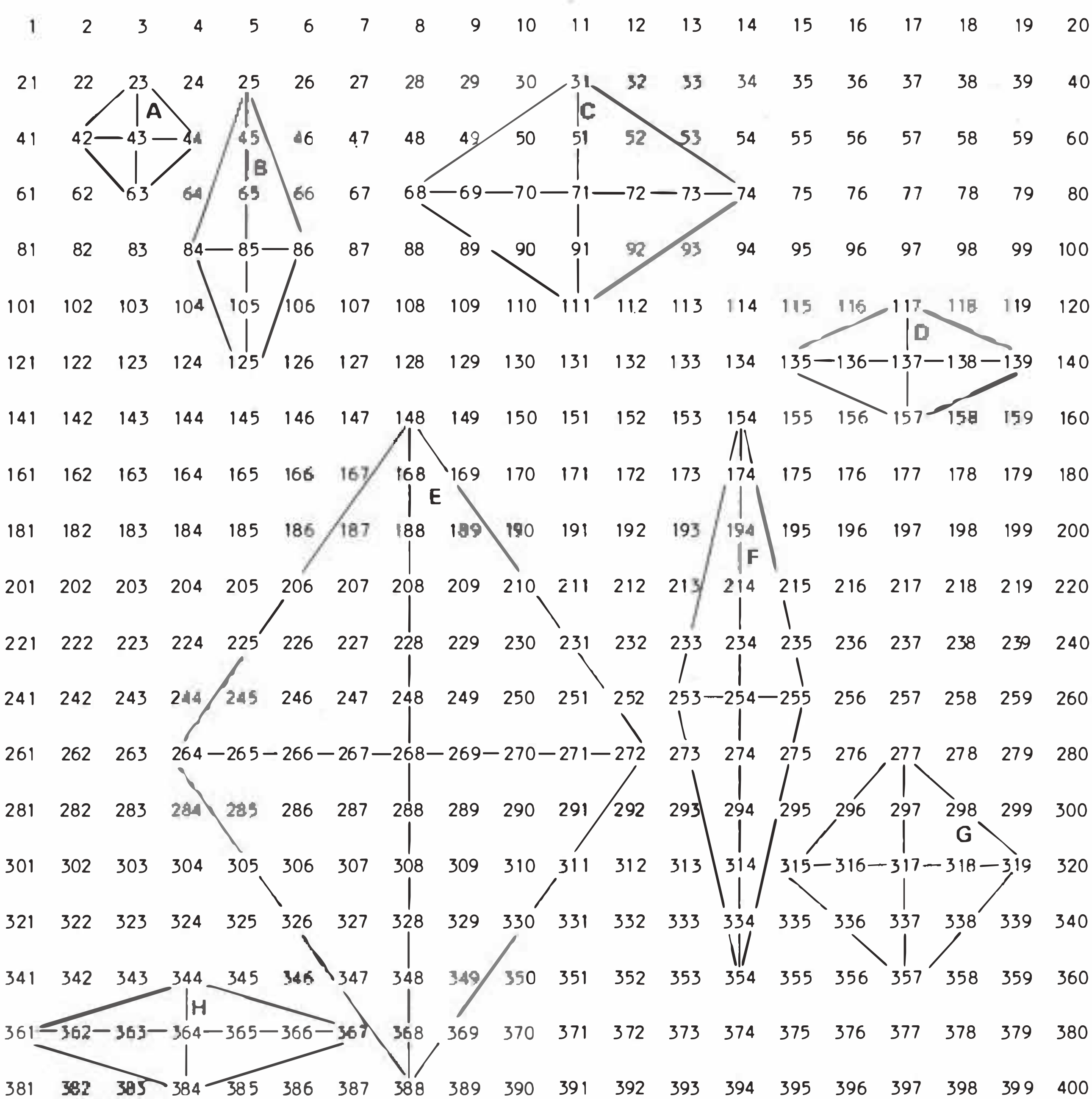
Similar arguments can be generated for the four vertex numbers and the interior numbers of C. These arguments would apply equally well to any of the other rhombi of Figure 2.

The equality of these ratios may be expressed in other ways. For example, in rhombus C, form the ratio V to I ($\frac{120}{330}$). Note this $\frac{4}{11}$ or $\frac{\#V}{\#I}$. We had previously noted that $\frac{120}{4} = \frac{330}{11}$; this observation notes that $\frac{120}{330} = \frac{4}{11}$. This is one of the standard properties of proportions.

Consider the ratio resulting from comparing HD to VD. What pattern do you observe?

Let us investigate patterns on an extended hundred square. We have made a "400 square," and rhombi have been drawn on it. See Figure 3.

FIGURE 3.


1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60
61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80
81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100
101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120
121 122 123 124 125 126 127 128 129 130 131 132 133 134 135 136 137 138 139 140
141 142 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160
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381 382 383 384 385 386 387 388 389 390 391 392 393 394 395 396 397 398 399 400
A
B
C
D
E
F
G
H

On Figure 3, rhombi A through H have been drawn in exactly the same positions as they occupied on Figure 2. Again, find the same sums and ratios as in the previous activity. Tables III and IV report these results.

TABLE III.

Rhombus	V(#)	I(#)	HD(#)	VD(#)
A	172(4)	43(1)	129(3)	129(3)
B	340(4)	425(5)	255(3)	595(7)
C	284(4)	781(11)	497(7)	355(5)
D	548(4)	411(3)	685(5)	411(3)
E	1072(4)	12060(45)	2412(9)	3484(13)
F	1016(4)	2286(9)	762(3)	2794(11)
G	1268(4)	1585(5)	1585(5)	1585(5)
H	1456(4)	1820(5)	2548(7)	1092(3)

TABLE IV.

Rhombus	Constant Ratio
A	43
B	85
C	71
D	137
E	268
F	254
G	317
H	364

Observe that the same patterns hold as in the extended multiplication table, and for the same reasons.

Challenges for the Reader:

1. Draw other rhombi on Figures 2 and 3. Compute the appropriate sums and ratios. Do the same patterns hold?

2. Compute other sums and ratios using the rhombi of Figures 2 and 3. For example, find the sum of the entries which lie on the perimeter of each rhombi. What patterns hold?

3. Draw other geometric shapes on Figures 2 and 3. Find sums and ratios. What patterns hold?

4. Draw extended addition and subtraction tables. If rhombi are drawn and sums and ratios are computed, do the same patterns hold?

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Infinity

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Infinity, or the boundless, is beyond the minds of most men. The concept has existed for centuries, over which many men have tried to propose theories. Relating each of these theories, one may get an idea of what infinity entails.

The original symbol for infinity, which is used today, is the lemniscate or ∞. This symbol was introduced in the seventeenth century, and appeared on the juggler or mangus card of the tarot cards. One of the main concepts of infinity is its endlessness, and this is why the lemniscate is used. One can travel around its periphery endlessly. The *quabalistic* symbol associated with this particular card was the Hebrew letter aleph or א. George Cantor, founder of the modern mathematics theory of the infinite, used the symbol $\aleph_0$ (aleph-null) to stand for the first infinite number.

The Greeks used the term apeiron to describe their concept of the infinite. The word literally meant unbounded and eventually came to describe general things such as disorder or the extremely complex. This apeiron may have no finite definition. For many Greek mathematicians, the concept of apeiron was unacceptable, even in the simplest form of a decimal expansion on the simplest number.

Blaise Pascal once described his feeling of being overwhelmed by the infinite:

> When I consider the small span of my life absorbed in the eternity of all time, or the small part of space which I can touch or see engulfed by the infinite immensity of spaces that I know not and that know not me; I am frightened and astonished to see me here instead of there . . . now instead of then.

Aristotle believed that "being infinite was a privation, not a perfection, but the absence of a limit." He saw that aspects of the world are apeiron - that time will not end, space is infinitely divisible, and that a line contains an infinite number of points. Aristotle invented the idea of potential and actual infinity. He proposed that the set of natural numbers is potentially infinite in that it has the ability to go on forever, yet it is not actually infinite because it does not exist as a finished thing.

Many men have expressed their beliefs of the infinite. Plutinus believed God to be infinite. St. Augustine added that God was not only infinite but could also think infinite thoughts. However, later medieval thinkers did not go so far as to believe that God was infinite. Although He has unlimited power, He does not have the ability to create an unlimited thing. (A "thing" cannot be unlimited, as it takes on the definition of being limited by nature.)

A problem was brought to the attention of mathematicians concerning the infinities of the world. On one hand, it would seem that God, being infinitely powerful, should be able to get an infinite number of angels to dance on the head of a pin, for example. On the other hand, it would seem that, in a created world, no actually infinite collection of angels could exist. Infinity appeared to be a self-contradictory argument. A line with a length twice that of another line would appear to have a larger infinity of points than the smaller. Yet a point on the smaller line would correspond with the point on a larger line, proving that infinity can be equal and different at the same time, which, in fact, seems to contradict logic.

Galileo Galilei offered that the smaller length could be turned into the longer length by adding an infinite number of small spaces. Galileo realized that there were problems with his solution, for the human mind can only think in finite terms. He stated that while looking at most natural numbers, many of them will not be perfect squares; thus, there must be a smaller set of perfect squares than natural numbers. There exists a paradox, however, that every natural number is the square root of a larger natural number. It would therefore seem that there are as many perfect squares as natural numbers. Galileo stated that:

> We can only infer that the totality of all numbers is infinite and that the number of squares is infinite. . .; neither is the number of squares less than the totality of all numbers, not the latter greater than the former; and finally, the attributes "equal," "greater," and "less" are not applicable to the infinite, but only to finite qualities.

It is essentially impossible for the finite being to contemplate the infinite. If a man were asked to calculate the largest possible number imaginable, this would, of necessity, be bounded by the finite period of his lifetime. On his deathbed, a large number would probably have been reached. As he gasped his last breath, an observer could merely add one and would start at that point.

Lucretius, in his theory *De Rerum Natura*, suggested: "Suppose for a moment that the whole of space were bounded and that someone made his way to the uttermost boundary and threw a flying dart." He then went on to consider that the dart could go past the boundary or it would stop. In either event, infinity is demonstrated. There is either a boundary stopping the dart, in which case there is something or someplace beyond, or there is no boundary, allowing the dart to continue upon its infinite path.

In more recent history, the traditional scientific view of infinity might be challenged by the so-called "big bang" theory. Such a theory is now widely accepted. However, such theory tends to suggest a beginning and an end. With the acceptance of the big bang theory, scientists now contemplate what was before the bang, and what will happen at the end of this universe. One answer that has been suggested is that the universe is an oscillating system, which endlessly expands and contracts to infinity.

It seems that the more common view of infinity is that of a series of numbers having no end. In fact, infinity has an equal place at or before the beginning of things. It is impossible to state the smallest or first number. Numbers are either infinitesimally small, or large, or somewhere in between. The paradox stated by Zeno seems to show that one can never leave the room which one is in. This, of course, is clearly ridiculous subject to the acceptance or otherwise of the Berkelian theory of existentialism. Zeno reasoned that in order to reach the door, one must first cross half the distance there. This would leave half the room to be crossed, but first one would have to cross half that distance, and so on. The modern answer to the paradox is to say that the sum of the infinite series 1/2 + 1/4 + 1/8 . . . = 1. Even so, this is not perfectly satisfactory. The paradox can be put in a different way. In a practical world, we say that a number with a decimal expansion of .99999 is the same as 1. It can be put this way:

$$\begin{array}{r} 10K = 9.999... \\ -\ \ K = \ .9999.. \\ \hline 9K = 9 \\ K = 1 \end{array}$$

Thus, we have the practical answer as compared to the theory of Zeno who regarded space as an undivided whole that cannot be broken down into parts.

If one were to take Zeno's paradox literally, any counting in whole numbers (for example, 1, 2, 3) would be impossible. If the average man on the street were asked to count to infinity, he would say that it is impossible. If he were accommodating, he might start counting for a day. Perhaps he would get up to 170,000. But, he would be unaware of Zeno's paradox. I suspect he would start counting with number one. In order to get to one, he would first have to pass 0.5 and, thus, would never "leave the room."

It is always interesting to consider the combination of random and infinity. In the unbounded time of infinity, literally anything is possible.

It has been said that if a group of monkeys were given an English dictionary, the monkeys would eventually, by random chance, utter the entire works of William Shakespeare in the exact order in which they were written. In the absence of infinity, such would not be probable.

If we add to this theory the additional fact of human intelligence, it would be reasonable to conclude that man will learn all the secrets of the universe, including the mystery of infinity, within infinity.

Sarah Jervis was a Mathematics 30 student at the Lethbridge Collegiate Institute. Sarah enrolled in an honors mathematics program, and this paper was submitted to partially fulfill the requirements for the program.

Illustrating with the Overhead

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University of Alberta

"Seeing is believing." It is easier to convince a listener if he can see demonstrated that which he has difficulty accepting. This is true whether the topic is sports, politics, oddities of nature, or mathematics concepts.

Most teachers are trained in the implementation of visual aids, and few mathematics classrooms can be found without blackboard compasses and protractors or an assortment of plastic geometric figures. There is another tool available to the teacher. That tool is the overhead projector.

Often the projector is used only to provide occasional drawings, sample problems, or class notes. It can be used more effectively to:

(a) break down learning barriers for students who have less ability in visualizing mathematics concepts;

(b) assist all students in remembering demonstrated properties, formulae, and theorems; and

(c) expedite the process of communicating information.

Of course, in order for the overhead projector to become such an efficient and effective teaching/learning tool, proper materials must be developed. This paper presents four topics, found in the junior high mathematics curriculum, which could be taught using the overhead projector.

TOPIC ONE: Deriving the Formula for the Area of a Triangle

The teacher can allow the class to derive the area formula for a triangle. First, ask the students to supply the formula for the area of the square outlined on an overhead transparency. Next, place a precut paper triangle inside the square. The students easily recognize the shape of the triangle and recognize that the area of the triangle is one-half the area of the square, or one-half the base multiplied by the height. This process can be repeated using a rectangle or parallelogram in place of the square.

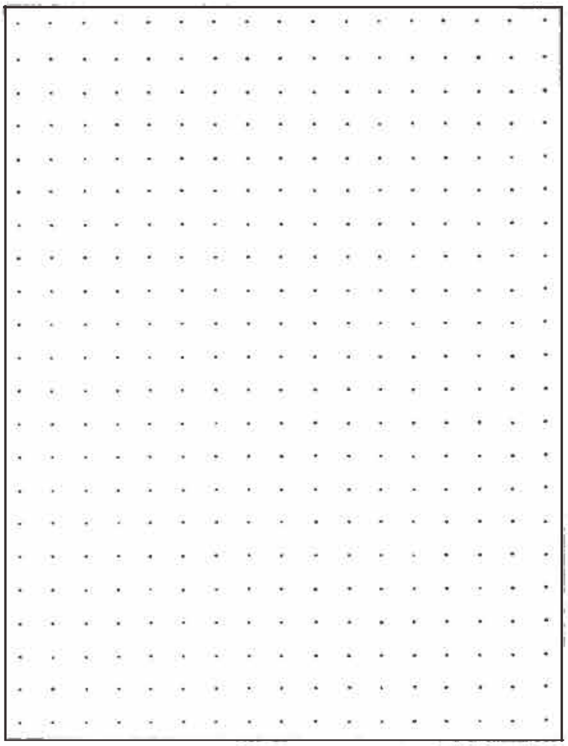
This demonstration assumes that students have learned area formulae associated with the square, rectangle, and parallelogram. The illustration's strength rests in enabling students to visually encounter the relationship between the area of a given triangle and a square, rectangle, or parallelogram. The overhead projector allows this association to be made quickly, yet requires only a few very simple materials.

TOPIC TWO: Translations, Rotations, and Reflections

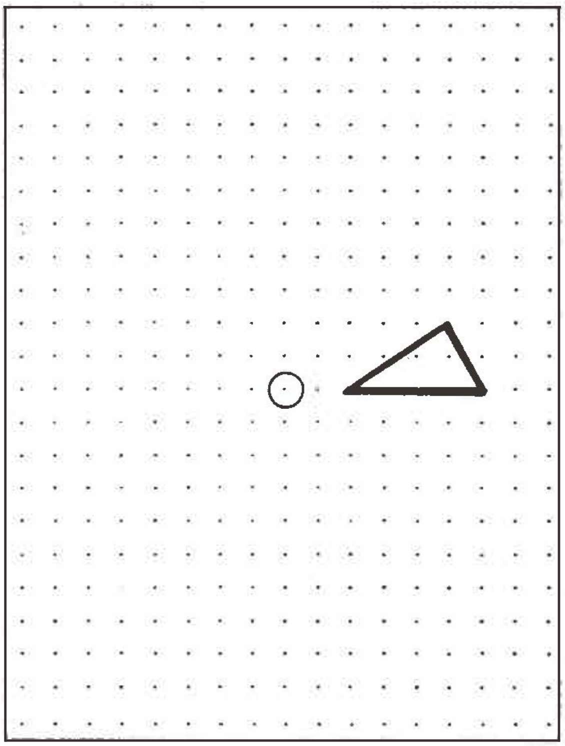
Motion geometry is easier to illustrate on the overhead projector than on the blackboard. To demonstrate translations, a transparency of dot graph paper is constructed (see Sample Transparency #1). Simply by sliding a paper triangle (or other figure) to various points on the transparency, students are able to visualize the process of a translation and the relative orientation of the resultant figure to the original.

A similar process using transparencies and paper triangles would

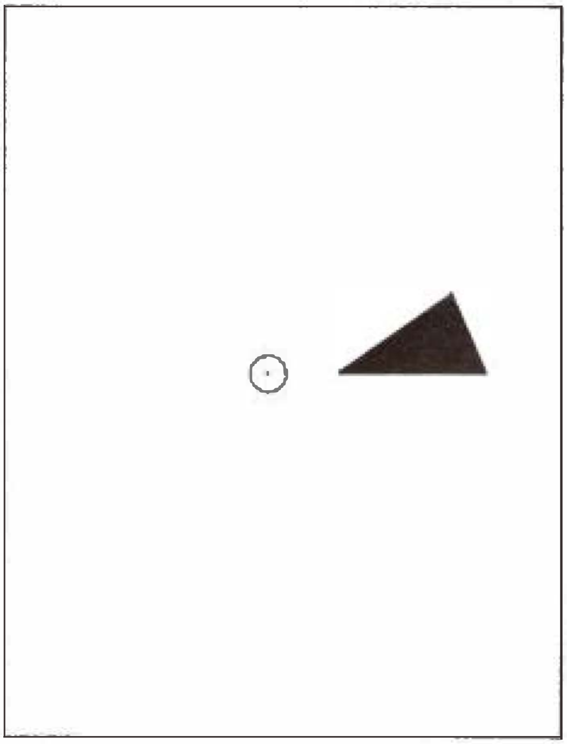
Sample Transparencies



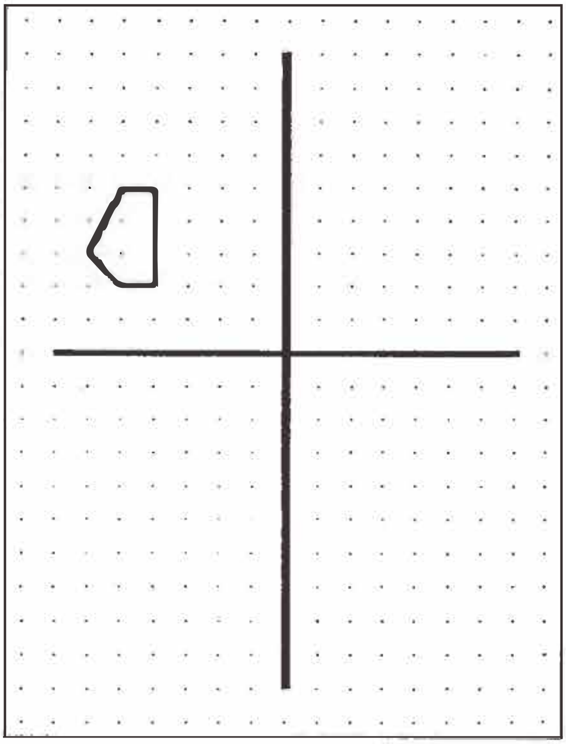
TRANSPARENCY #1



TRANSPARENCY #2



TRANSPARENCY #3



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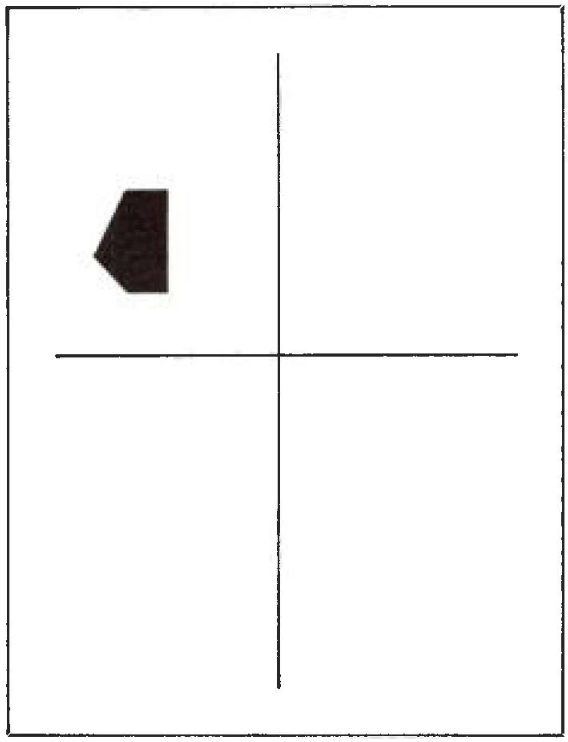

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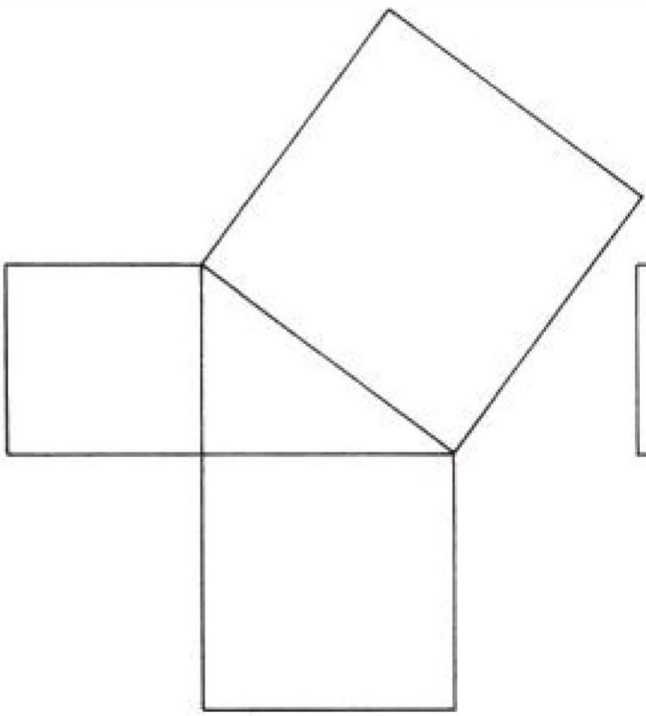


DIAGRAM 1.

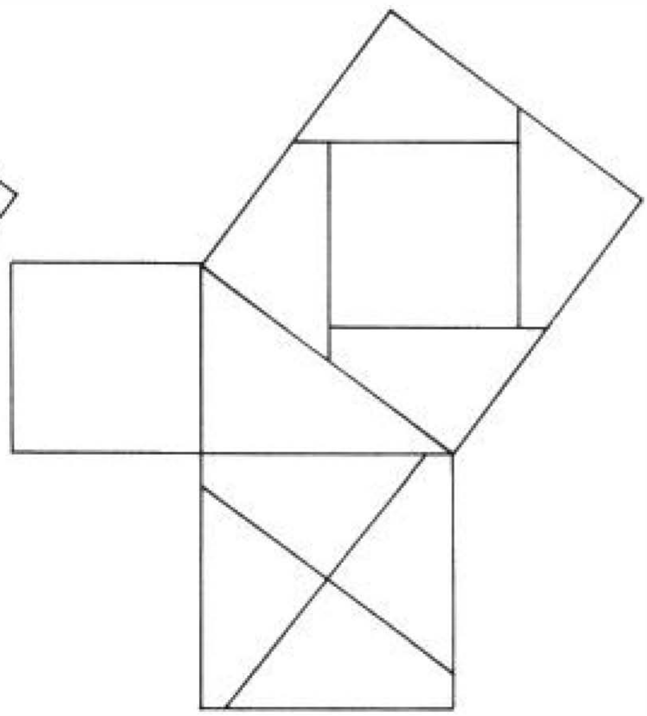


DIAGRAM 2.

enable students to visualize the other two forms of geometric motion.

Orientation resulting from a rotation is demonstrated by placing a second transparency on the dot transparency and rotating it about a selected point (see Sample Transparencies #2 and #3). Reflections are illustrated by folding a transparency back upon itself. The fold acts as the line of reflection when the mirror image is created (see Sample Transparencies #4 and #5). Resultant figures can be compared to original figures in terms of location, orientation, direction, and distance from a point of rotation or line of reflection. These comparisons allow students to summarize the intended properties.

TOPIC THREE: Equilateral Triangles

Illustrating the properties of an equilateral triangle with the overhead projector does not constitute a proof of these properties. At the junior high level, the recognition of these properties is more important than the proof.

Place an outline of an equilateral triangle on a transparency, and then rotate a paper triangle of equal size showing that the paper fits into the outline from three directions. The student concludes that an equilateral triangle not only has three equal sides but has three equal angles. The demonstration is simple, yet effective, and allows students to draw the correct conclusion: an equilateral triangle is also equiangular.

TOPIC FOUR: Pythagorean Theorem

To show the Pythagorean theorem, a transparency of the figure in Diagram 1 is created. A piece of paper the same size as the square on the hypotenuse must be cut into pieces as shown in Diagram 2. These pieces are rearranged once the theorem is presented, and the squares are identified and labeled to show that the area of the square on the hypotenuse equals the sum of the areas of the squares on the other two sides. This illustration does not prove the hypothesis, but provides students with a visual experience to aid in understanding the theorem and remembering that $a^2 + b^2 = c^2$.

The overhead projector is not the right tool to teach every concept found in the mathematics curriculum, but it can be used for more than simple diagrams or lecture notes. By experimenting with the projector's possibilities, the teacher can assist most pupils to develop better visualization skills while enhancing understanding of many mathematics topics.

Craig Loewen has taught at Rosalind School, County of Camrose. Currently, he is enrolled in a master of secondary education program at the University of Alberta.

Creative Problem Solving Activity

Jacqueline Fischer
University of Lethbridge

The following activity was developed after attending a workshop on creative problem solving, which was presented by Julie Ellis of the University of Lethbridge. It provides a creative way to teach and reinforce estimation skills and is most suitable for students at the Division II level.

Fast Food Establishments and Estimation

The following steps should be carried out in the order in which they are listed below:

1. As a class, have students brainstorm all the fast food outlets they enjoy. The teacher should list these on the chalkboard.

2. Ask students to pick *one* of the fast food places on the list. The choice may represent a favorite place, the place the student usually goes, or the outlet that is closest to home. Each student should write down his or her choice on a sheet of paper.

3. As a class, brainstorm all of the "groups" with whom the students would attend these places (for example, ball team or family). The teacher should write the list of suggestions on the chalkboard. Only groups of four or more people may be included in this list.

4. Ask students to pick *one* type of group, and have each student write down his or her choice on the paper with the fast food outlet choice.

5. Ask students to go to the fast food outlet they have selected, and write down a price list from the menu. This part of the activity allows students to do homework in a favorable atmosphere.

6. From the menu, students should choose *one* type of beverage, main course, and dessert.

7. Ask students to *estimate* the cost of their choices in item 6 above. Their estimation should be derived from the price list acquired earlier.

8. Ask students to *estimate* the total cost of a complete meal for their chosen group. The method used to arrive at the estimation should not be stipulated by the teacher. That is, students may decide to add up the totals of the entire meal selection for each member of the group, or may add up all of the beverages, main courses, and desserts chosen for the entire group.

9. Ask students to check the accuracy of their estimations by using a calculator. Students will use the calculator to add up the actual cost and compare it to the estimated total.

The following sample activity illustrates the steps outlined above.

Sample Activity

STEP 1:
(on chalkboard)

Fast Food Outlets

Al Submarine
Top Pizza & Spaghetti House
A & W Restaurant
Dairy Queen
Taco Time
Kentucky Fried Chicken
Boston Pizza
Brownie Fried Chicken
Baaco Pizza
McDonald's
Burger King
The Sub Hut
Mary Brown's Chicken
Poppa's Pizza

STEP 3:
(on chalkboard)

Types of Groups

Family
Hockey Team
Ball Team
"The Gang"
Birthday Party
Brownies
Cubs
Scouts
Girl Guides
Classroom Students

STEPS 2 and 4:

If a student were to choose *Taco Time* with his or her *family*, the remaining steps in the activity might turn out as the following do.

STEP 5:

Beverages	
large	$ .89
regular	.69
children's	.49
coffee	.35
milk	.65
hot chocolate	.45

Tacos	
natural soft taco	$2.49
soft super taco	2.19
soft taco	1.94
taco	.99

Burritos	
soft meat burrito	$2.34
crisp meat burrito	1.59
soft combo burrito	1.84
soft bean burrito	1.34

Specialties	
taco salad	$1.94
nachos	1.79
casita burrito	2.99
tostado delight	2.54
torta con carne	1.79
refritos	1.29
mexi-fries	.85

Desserts	
apple or cherry empanada	$ .75
crustos	.44

STEPS 6 and 7:
(This estimation is to the nearest dollar.)

regular beverage	$1.00
soft taco	2.00
cherry empanada	1.00
	$4.00

STEP 8:

Family of 4

2 regular drinks	2 @	$1.00 =	$ 2.00
2 large drinks	2 @	1.00 =	2.00
2 tacos	2 @	1.00 =	2.00
1 soft taco	1 @	2.00 =	2.00
1 soft super taco	1 @	2.00 =	2.00
3 crustos	3 @	0.00 =	0.00
1 cherry empanada	1 @	1.00 =	1.00
		Estimated Total	$11.00

STEP 9:

The actual work, which may be completed on the calculator, will have the following figures:

regular beverage	$ .69
soft taco	1.94
cherry empanada	.75
Actual	$3.38

Family of 4

2 regular drinks	2 @	$ .69	=	$ 1.38
2 large drinks	2 @	.89	=	1.78
2 tacos	2 @	.99	=	1.98
1 soft taco	1 @	1.94	=	1.94
1 soft super taco	1 @	2.19	=	2.19
3 crustos	3 @	.44	=	1.32
1 cherry empanada	1 @	.75	=	.75
Actual Total Cost				$11.34

Extensions or Alternatives

The estimations in this activity may be made to the nearest dime, quarter, or half dollar, or all of these may be tried to find the most efficient method. This type of activity can also be developed for other subject areas.

Jacqueline Fischer graduated from the University of Lethbridge in May 1986, with an education major in mathematics.

A Problem Solving Geometry Lesson Using Groups of Four

Oscar Schaaf
University of Oregon

In the September 1981 issue of *Learning*, Marilyn Burns describes a classroom management scheme that encourages students to learn by working cooperatively and independently. Certainly, this is a worthy educational goal for any classroom. Her article is based upon ideas given to her by Carol Meyer, a classroom teacher in Davis, California.

The scheme called "Groups of Four" requires reorganizing the classroom physically, redefining the students' responsibilities, and carefully structuring the role of the teacher. Students are randomly assigned to groups of four, and the assignments are changed regularly throughout the year. There are three rules for the students in their groups of four:

1. Each student is responsible for his or her own work and behavior.
2. Each student in the group is responsible for every other group member.
3. A student may ask for help from the teacher only when everyone in his or her group has the same question.

I suggest you read Marilyn Burns' article; it has many good suggestions.

"Groups of Four" is just one of several management schemes a teacher should use. However, this scheme is especially appropriate when problem solving is the goal of instruction. Why don't you try "Groups of Four" with the high school geometry lesson "An Investigation: Polygons and Line Segments," which is given on the following page.

Commentary and Answers for "An Investigation: Polygons and Line Segments"

Hand out the lesson sheet, one to each student, and have the students get into their groups of four. Follow the "Groups of Four" rules given above, especially rule 3. Most groups should be able to figure out on their own what to do through question 3. Use your time observing and taking notes on how the groups are functioning and on the different approaches used in solving the problems. This information will be useful later, when summarizing the lesson with the class. The table below contains the data collected for questions 1, 2, and 3, and the correct predictions called for in questions 4 and 5.

s	d	D	L
4	1	2	6
5	2	5	10
6	3	9	15
7	4	14	21
8	5	20	28
9	6	27	36
10	7	35	45
3	0	0	3
20	17	44	55

An Investigation: Polygons and Line Segments

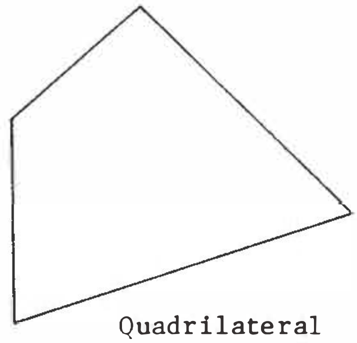
s	d	D	L
4			
5			
6			
7			
8			
9			
10			
3			
20			

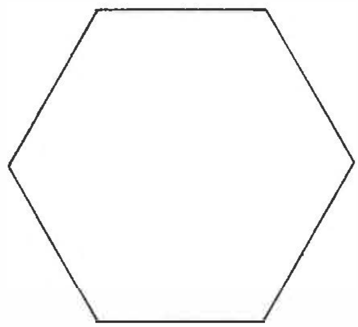
s = number of sides

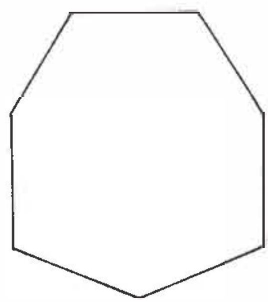
d = number of diagonals from one vertex

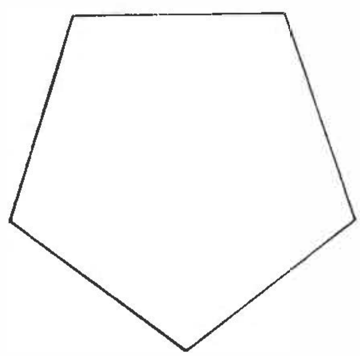
D = total number of diagonals

L = total number of line segments


Quadrilateral


Hexagon


Heptagon


Pentagon

1. Draw the diagonals from a single vertex in each of the polygons shown. Record the number in the table.
2. Draw all the other diagonals for each polygon. Record the total number in the table.
3. Record the total number of sides and diagonals for each polygon.
4. Study the patterns in the table. Predict the value for d, D, and L for an octagon, nonagon, decagon, triangle, and icosagon.
5. Check your predictions by making drawings.
6. Write the formula for the relationships suggested in the table.
   (a) d in terms of s
   (b) D in terms of s and d
   (c) D in terms of s
   (d) L in terms of s
7. Do your formulas work for all the data recorded in your table? If not, make adjustments until the equations accurately describe the situation.
8. Graph the formulas in 6a, 6c, and 6d. Let **s** be the horizontal axis in each case. By careful planning and labeling, the three graphs can be placed on the same chart.
9. Does it make sense in this lesson to draw the straight or curved line suggested by each graph? Why or why not?
10. Does it make sense to use the formulas for finding values for d, D, and L, when **s** is any whole number? Explain.

6. (a) $d = s - 3$

   (b) $D = \frac{sd}{2}$

   (c) $D = \frac{s(s - 3)}{2}$

   (d) $L = \frac{s(s-3) + s}{2}$

   $= \frac{1}{2}s(s + 1)$

7. Groups will use various strategies to get their formula. Encourage groups and individuals to keep track of the strategies they used. These should be discussed when the lesson is summarized later.

8. Attention needs to be given to the scale used for each graph. I suggest a scale of 2 cm per unit for the horizontal axis and 1/2 cm per unit for the vertical axis.

9. No. Whole numbers from 3 and after make sense, but a mixed number such as 6 1/2 for the number of sides and diagonals of a polygon does not make sense.

10. No. It does not make sense to say that a polygon has 0, 1, or 2 sides.

Dr. Oscar Schaaf is professor emeritus, Faculty of Education, University of Oregon. Oscar has been a contributor to past issues of delta-K *and has also been a speaker at MCATA conferences.*

ERRATUM

In "The Road to Four Villages" problem appearing on page 43 of the last issue of *delta-K* (Volume XXV, Number 3, July 1986), a square root sign was omitted, making the problem meaningless. Please accept our apologies for this oversight. The problem should have read:

> Four villages are situated at the vertices of a square of sides which are one mile long. The inhabitants wish to connect the villages with a system of roads, but have only enough material to make $\sqrt{3} + 1$ mile(s) of road. How do they proceed?

STUDENT PROBLEM CORNER

Students are encouraged to examine the problem presented below. Send your explanation or solution to:

The Editor
delta-K
c/o 2510 - 22 Avenue S
Lethbridge, Alberta
T1K 1J5

delta-K will publish the names of students who successfully solve the problem.

The Big Steal

Hank Boer

It was a cool morning in June as Inspector Marchand looked over the scene in the Bronxton Mansion. The imperial diamond was gone, taken the night before, sometime between 21:00, Thursday, and 22:00, Friday. He thought about the method the burglar used to enter the grounds, because the mansion was surrounded by a 4 metre electrified fence some 134 metres from the house. A pathway lead from the garden to a small pool measuring 3 metres by 4 metres and 1 metre deep. The back edge of the pool stopped 0.5 metres from the patio doors at the rear of the house. The burglar had entered the house through the back patio doors and made his or her way to the upstairs reading room, where the diamond was kept in a safe. The safe was not damaged, but somehow the burglar had dialed the correct computer-controlled combination. Inspector Marchand suspected an inside job.

The yard between the fence and house was patrolled by 5 guard dogs. The inspector found no evidence that the dogs had been drugged. However, close to the front of the house, he found the bone to a tenderloin roast.

The inspector sat on the bench by the pool as he reviewed his suspect list. Each had a possible motive and a good alibi.

- Mary was the chambermaid; her alibi was that she and her cousin had gone out to the movies. At the time, Mary was suffering from bad lung flu.

- Sebastian was the butler; he had evidence that he was visiting his mother during the robbery. It was a well-known fact that he had a deathly fear of heights.

- Ronald was an internationally known jewel thief, who had been in the area during the last three months. He said that he had been at a dance at the convention centre. His height was 1.94 metres.

- Jean was Mr. Bronxton's daughter, who had left home to attend college in the next city. She said that she was studying in the college library. She enjoyed collecting art.

- Freddie was the gardener; he said he was shopping at Rainbow Mall on the other side of town. He was almost blind in one eye.

- Sharon was Mr. Bronxton's wife; she said that she was visiting a friend in the hospital at the time. She was somewhat overweight at 165 kilograms.

- Earle was a close friend of Mr. Bronxton, and he said that he was playing squash at the country club at the time. Earle enjoyed deep sea diving as a hobby.

The inspector looked at the edge of the pool. A waterline was quite visible along the entire edge of the pool. He pulled out his measuring tape and measured the distance from the top of the calm water in the pool to the waterline that appeared above it. He wrote down 2.1 centimetres in his notebook.

The inspector walked around the pool. At the far end of the pool, away from the house, he noticed a white cloth tangled in the branches of a tall lilac bush. Upon closer inspection he found a parachute hidden behind the bush. The instruments on the parachute pack indicated that the burglar had fallen about 1020 metres. Looking up, tne inspector saw a wristwatch caught in the branch of the same bush. He measured the distance from the watch to the ground; it was 2.05 metres.

The inspector now knew how he would catch the thief. He arranged to have all the suspects meet at the Bronxton house at 13:00. The suspects gathered around the pool. The inspector asked Earle to go into the house to change into a bathing suit. When Earle came back, the inspector asked him to go into the water in the pool and completely submerge himself. While Earle was under the water, the inspector measured the depth of the water with his measuring tape. The depth was 1.04 metres.

The inspector then announced to the crowd that all the evidence was now in, and that he now knew who had committed the robbery. Who did it, and how did the inspector know?

Hank Boer is the mathematics and science consultant with the Lethbridge Public School System. Hank was president of the South Western Regional of MCATA, and was responsible for the highly successful annual meeting held in Lethbridge in October 1985.

College and University Responsibilities for Mathematics Teacher Education

College faculty must become actively involved in the education of teachers if the teaching of mathematics in the schools is to improve significantly. Active leadership and support of college and university mathematicians, mathematics educators, and administrators is essential if our nation is to increase the number of qualified teachers and strengthen their education. For this reason, the Mathematical Association of America and the National Council of Teachers of Mathematics have adopted the following recommendations for all individuals, in whatever department, who are engaged in teaching mathematics or mathematics education for current or prospective teachers:

1. Colleges and universities should assign significantly higher priority to mathematics teacher education.

2. All individuals who teach preservice or inservice courses for mathematics teachers should have substantial backgrounds in mathematics and mathematics education appropriate to their assignments.

3. Mathematics methods courses should be taught by individuals with interest and expertise in teaching and continuing contacts with school classrooms.

4. All individuals who teach current or prospective mathematics teachers should have regular and lively contact with faculty in both mathematics and education departments; for example, by regular meetings, seminars, joint faculty appointments, and other cooperative ventures.

5. All college and university faculty members who teach mathematics or mathematics education should maintain a vigorous dialogue with their colleagues in schools, seeking ways to collaborate in improving school mathematics programs and supporting the professional development of mathematics teachers.

6. Faculty advisors should encourage their mathematically talented students to consider teaching careers.

7. Colleges and universities should vigorously publicize the need for qualified mathematics teachers and strive to interest and recruit capable students into the profession; for example, by organizing highly visible campus-wide meetings for students to inform them of the opportunities, advantages, disadvantages, and requirements of a career in teaching mathematics.

8. Tenure, promotion, and salary decisions for faculty members who teach current or prospective mathematics teachers should be based on teaching, service, and scholarly activity that includes research in mathematics or mathematics education.

9. Faculty members in mathematics and in mathematics education who are effective in working with activities in the schools and in the mathematical education of teachers should be rewarded appropriately for this work.

10. All institutions involved in educating mathematics teachers should provide specialized classroom and laboratory facilities equipped with state-of-the-art demonstration materials, calculators, and computers at least comparable to those used in the best elementary and secondary schools so that prospective teachers, like graduates from other professional programs, can be properly prepared for their careers.

Geoffrey James Butler – 1944-1986

EDITOR'S NOTE: Geoffrey Butler served as mathematics representative on the Mathematics Council from the fall of 1979 until ***his*** *death. The following is reprinted from the August 7, 1986, issue of* Folio, *a University of Alberta publication.*

Geof Butler was born on March 4, 1944, in Gillingham, England. He grew up and received his early schooling in Bournemouth. In 1965, he earned a B.Sc (Special) degree, and in 1969, his Ph.D., both at University College, London, England, the latter under the tutelage of C.A. Rogers.

In 1968, he came to the University of Alberta as a post-doctoral fellow, was appointed to the academic staff in 1971, promoted to associate professor in 1974, and professor in 1980.

His research activities included convexity, ordinary differential equations, and modeling in population biology. He supervised three Ph.D. students: J. Chapin, G.S.K. Wolkowicz, and J. Roessler. His involvement with students included chairing the Canadian Mathematical Olympiad Committee and leading the Canadian team in the International Mathematical Olympiad.

He has presented many papers in several continents. In 1982, he was awarded a McCalla Professorship for excellence in research. Just before his illness, he was appointed chairman of the Mathematics Department.

Dr. Butler died on July 13 peacefully in his sleep, only 70 days after being diagnosed as having cancer. He is sorely missed by those who knew and loved him, but his kind nature, his good thoughts, and his marvelous ideas will be with us always.

Geoffrey James Butler Memorial Fund

The Mathematics Department wishes to announce the establishment of the Geoffrey James Butler Memorial Fund. Monies from the fund will be used for student scholarships at all levels, as well as to fund the Geoffrey James Butler Memorial Lectures, the first of which will be given at a conference dedicated to Dr. Butler during the summer of 1988 at the University of Alberta. Details of the conference will be announced in due course.

Tax deductible contributions payable to the University of Alberta should be sent to H.I. Freedman, Mathematics Department, University of Alberta, Edmonton, Alberta T6G 2G1.

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