Pitfalls to Avoid in Teaching Mathematics

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One of the things that makes the teaching of mathematics a difficult job is that some methods that appear to work well initially can cause problems for the student several years later. Teachers may have no suspicion of the problem, however, since they do not normally have the opportunity to see the results of their teaching techniques. This article discusses some common teaching strategies and the problems that can result when they are employed. The intent is not to criticize but to sensitize and, in so doing, to emphasize the importance of considering the future implications of the instructional methods we use.

One day, in a Grade 9 class, I asked a girl for the result when -6 was divided by 2. She responded immediately that it was -12. Without reacting to this answer, I asked for an explanation of her logic. The reply was that since numbers get smaller in proportion to the divisor when they are divided, she just thought of the number that was twice as small as -6, and this was -12!

It was clear that this student had some knowledge of integers and that she was using a logical thinking process to calculate the answer. However, she was applying an idea that works only when dividing a positive number by another positive whole number. Perhaps in an earlier grade, the teacher found it helpful to tell the class that division makes the answer smaller, not realizing the problems that could occur both when using integers and when dividing by fractions or decimals smaller than one. These kinds of rules should be avoided unless they are **always** true and unlikely to be misinterpreted by the student.

Another good case in point is the use of the phrase "two negatives make a positive." It is easy to tell if students have been taught this rule because they will tell you that -2 - 5 = +7 and vociferously defend the answer as being correct! Unfortunately, it seems to be very hard to eradicate such ideas when they have been acquired at an early age. A better statement would be: When two integers of the same sign are multiplied together, the result is positive. It is a little longer to say, but it is correct and quite hard for students to misapply.

Students in elementary school often learn the method of transposition from their teachers (or parents) and come into high school saying things like "move it over and change the sign." The very capable students may be able to apply this technique correctly, but many students will solve an equation and end the solution with the following sequence of steps:

$$9x = 9$$

$$x = \frac{9}{-9} = -1$$

When asked why they divided by -9 rather than 9, they always respond that whenever you move a number over, you have to change the sign. Technically, the rule is correct as stated because, on the left side, the 9 is multiplied, and hence, on the right side, the operation should become division. This subtlety is hard for most students to figure out, however, and so they often mix up the signs as in the example.

The problem can be avoided by not teaching transposition at all, but simply having the students add or subtract the same quantity on both sides, or multiply and divide both sides by the same number. Most students will develop short cuts in this procedure, but it will be when they are ready, and when they are comfortable with doing some of the steps mentally.

Lest the reader think that high school teachers are immune to such problems, consider the following explanation of how to factor the trinomial $x^2 + 5x + 6$. The teacher puts two brackets on the board with an x in each and says: "You need two numbers that multiply to give 6 and add to give 5. What are they?" Students will readily give 2 and 3 as the values and, after a few more examples, proceed with the rest of the problems with little difficulty. Does this mean that there is nothing wrong with this method? Nothing so long as we restrict ourselves to factoring trinomials of the form $ax^2 + bx + c$ where $a = 1$. However, the teacher in the next grade will have a hard time convincing the students that to factor $6x^2 - 13x - 5$, they are not looking for two numbers that multiply to give -5 and add to give -13!

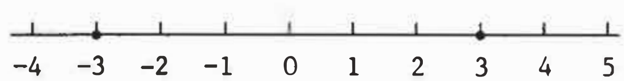
This difficulty can be overcome by using a more careful choice of words. After writing the two brackets on the board like this

complete with the curved lines to indicate where the middle term is to come from, the teacher asks: "What two numbers multiply to give 6 that will also give a middle term when multiplied out of 5x?" A small difference perhaps, but significant all the same. Students will now ask why they can't just multiply to get the last number and add to get the one in the middle. This gives the teacher a chance to emphasize why this method is a poor one.

Teachers typically teach students that inequations and equations are very similar because you can do anything to one side as long as you do the same thing to the other. The only difference is that multiplication or division by a negative value requires the inequality sign to be reversed. This method is fine as long as the students see only linear inequations, but what about solving an inequation like $x^2 < 9$? A significant number of my student teachers, all of whom have a minimum of two courses at the university level, solved this by finding the square root of both sides and giving the solution as $x < \pm 3$. This is not correct because $-4 < -3$, but -4 does not satisfy the original inequation.

Actually, there are two problems of pedagogy here. In the first place, students should be taught to solve equations such as $x^2 = 9$ by rearranging into a quadratic form such as $x^2 - 9 = 0$. This helps to ensure that they get two roots, rather than the single value that often results when the plus or minus sign is forgotten after taking a square root. In addition, if students are taught to factor, they will be less likely to use the method of finding the square root of both sides, which does not work with quadratic inequations.

An inequation such as $x^2 < 9$ can be factored to $(x - 3)(x + 3) < 0$ and then the boundary (or zero) values graphed on a number line:



We can then verify each of the three regions that these boundary values divide the number line into by picking values such as x = −4, x = 0, and x = 4 and substituting them into $x^2 < 9$ to get a true or false statement. These considerations give the correct solution that $-3 < x < 3$.

This method has the advantage of being applicable to more complicated quadratic inequations such as $x^2 + 3x - 4 > 0$. Furthermore, the idea of graphing the boundary and checking test points is consistent with the method used to graph linear (and quadratic) inequations in two variables in the x-y plane. Thus, the student learns one method that helps integrate various parts of mathematics.

Needless to say, there are many more examples that could be discussed. Indeed, many teaching techniques have inherent and unavoidable difficulties associated with them. The message, however, is clear: When planning lessons, try to be aware of future implications of the method you plan to use. Perhaps most important of all, talk to your colleagues! Find out where the ideas that you are teaching today will lead in the next grade. You will grow professionally and the ultimate winners will be your students.

MAGIC SQUARES
Addition of Fractions

PREPARATION

Photocopy 3 X 3 grids, or have students draw their own.

HOW TO PLAY
(2 or 4 players)

The numbers 1/6, 1/3, 1/2, 2/3, 5/6, 1, 1 1/6, 1 1/3, and 1 1/2 will be placed in the grid. Each number may be used only once.

The first player (or pair) places one of the fractions in one of the squares. The next player (or pair) places a different number in a different square.

Every time a player (or pair) completes a row or column or diagonal so that the sum of the three numbers equals 2 1/2, they receive one point. It is possible to write all nine numbers so that the sum of each row, column, and diagonal is 2 1/2. This is called a magic square.

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Have students make up their own magic squares, and then write the fractions to be used on the top of a new page. Let the class play the new game.

Contributed by Karen Gibling, Elboya Elementary-Junior High School, Calgary.