

# Hexagonal Combinations on Familiar Operation Tables

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Number patterns often occur in geometric settings where they are not expected. In this article, we will present polygonal combination activities on the extended addition, subtraction, and multiplication tables. These activities result in interesting number patterns.

## Activity 1

Figure 1 is an extended addition table with hexagonal combinations drawn upon it. A hexagonal combination includes a centre hexagon with six surrounding hexagons. For example, D is a centre hexagon and it is surrounded by hexagons A, B, E, G, F, and C.

For each hexagonal combination:

1. Find the sum of the interior numbers of the centre hexagon; call this sum C.
2. Find the sum of the interior numbers of the six hexagons that surround the centre hexagon; call this sum H.
3. Divide by 6 to find the average of the interior numbers of the surrounding hexagons; call this quotient A.
4. Compare C and A.

Table 1 reports the results of our computations.

TABLE 1.

Centre Hexagon	C	Surrounding Hexagons	H	A
D	17	A B E G F C	102	17
K	43	H I L N M J	258	43
R	37	O P S U T Q	222	37
Y	53	V W Z B' A' X	318	53

Observe that in each case,  $C = A$ ; that is, the sum of the interior numbers of the centre hexagon is equal to the average of the interior number sums of the six surrounding hexagons. Draw other hexagonal combinations and check to see that the conjecture holds.

## Activity 2

Figure 2 is an extended subtraction table with hexagonal combinations drawn upon it. Perform steps 1 through 4 of Activity 1. Does the same pattern hold?

Table 2 summarizes the results of our calculations.

TABLE 2.

Centre Hexagon	C	Surrounding Hexagons	H	A
D	-3	A B E G F C	-18	-3
K	-21	H I L N M J	-126	-21
R	9	O P S U T Q	54	9
Y	-1	V W Z B' A' X	-6	-1

Observe that the same pattern holds as for that of Activity 1. Draw other hexagonal combinations and check to see that the pattern continues to hold.

## Activity 3

Does the pattern of Activities 1 and 2 also hold on the extended multiplication table of Figure 3?

The results of our computations are shown in Table 3.

TABLE 3.

Centre Hexagon	C	Surrounding Hexagons	H	A
D	36	A B E G F C	216	36
K	176	H I L N M J	1056	176
R	190	O P S U T Q	1140	190
Y	108	V W Z B' A' X	648	108

Again the conjecture holds. Draw other hexagonal combinations and check that the pattern continues to hold.

What happens if hexagonal combinations of "larger size" are shown upon the extended operation tables.

### Activity 4

On the extended addition table of Figure 4 with the hexagonal combinations, perform these steps:

1. Find the sum of the interior numbers of the centre hexagon; call this sum C.
2. Find the sum of all the interior numbers of the six surrounding hexagons; call this sum H.
3. Divide by 6, the number of surrounding hexagons; this will give the average, A.
4. Compare C and A.

The results of steps 1 through 4 are displayed in Table 4.

**TABLE 4.**

Centre Hexagon	C	Surrounding Hexagons	H	A
E	187	A B F J I D	1122	187
F	231	B C G K J E	1386	231
I	209	D E J M L H	1254	209
J	253	E F K N M I	1518	253

**CONJECTURE:** The sum of the interior numbers of the centre hexagon is equal to the average of the interior number sums of the six surrounding hexagons.

### Activity 5

On the extended subtraction table of Figure 5, check to see if the same pattern holds.

Table 5 is given so that you will be able to check your results.

TABLE 5.

Centre Hexagon	C	Surrounding Hexagons	H	A
E	-11	A B F J I D	-66	-11
F	-55	B C G K J E	-330	-55
I	55	D E J M L H	330	55
J	11	E F K N M I	66	11

The pattern holds; that is, the sum of the interior numbers of the centre hexagon is equal to the average of the interior number sums of the six surrounding hexagons.

### Activity 6

On the extended multiplication table of Figure 6, check to see if the pattern holds.

Compare your results with those of Table 6. The pattern again holds.

TABLE 6.

Centre Hexagon	C	Surrounding Hexagons	H	A
E	792	A B F J I D	4752	792
F	1144	B C G K J E	6864	1144
I	924	D E J M L H	5544	924
J	1452	E F K N M I	8712	1452

**CHALLENGE:** Draw other polygonal combinations upon the extended operation tables and find number patterns.

FIGURE 1.

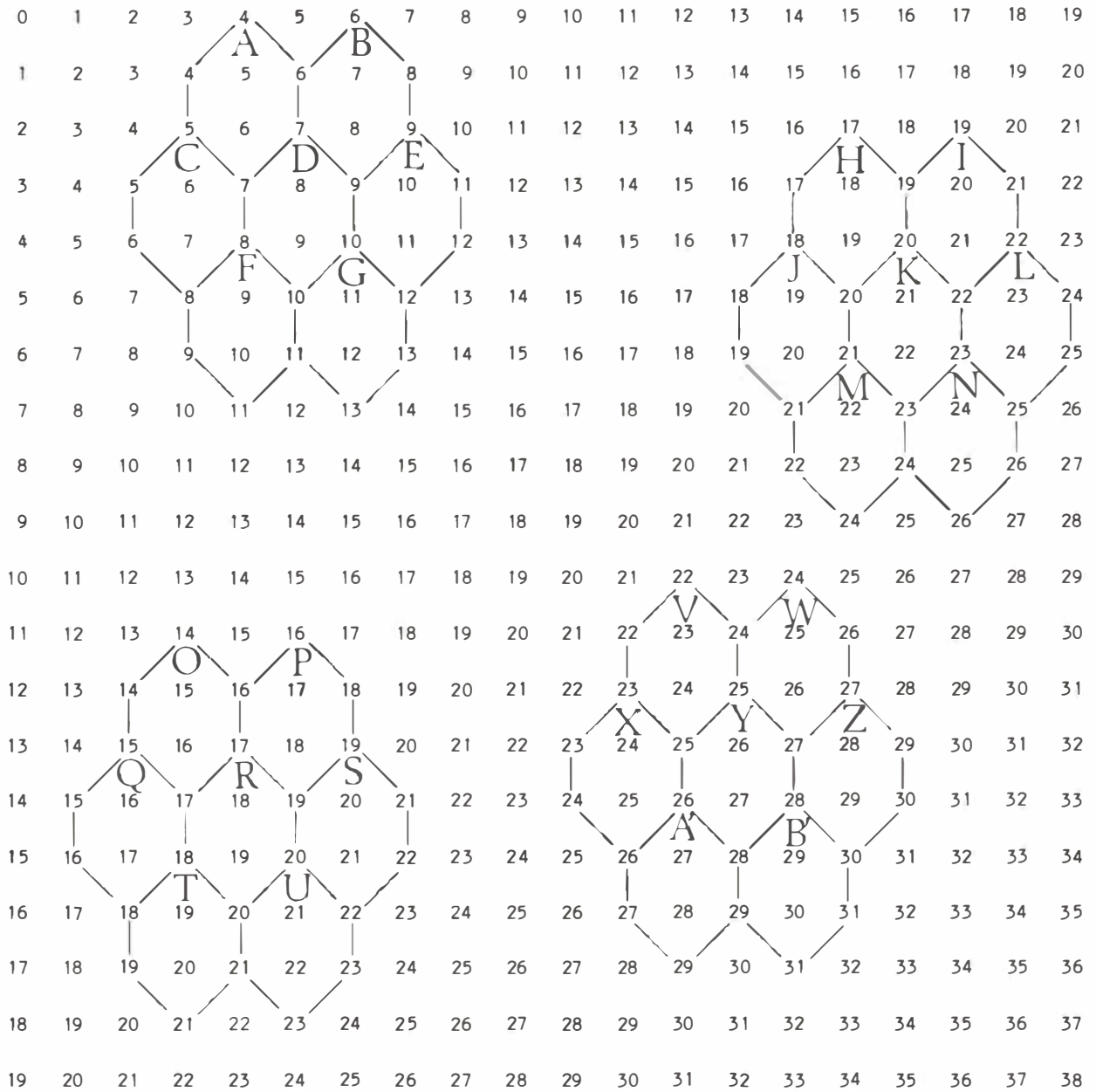


FIGURE 2.

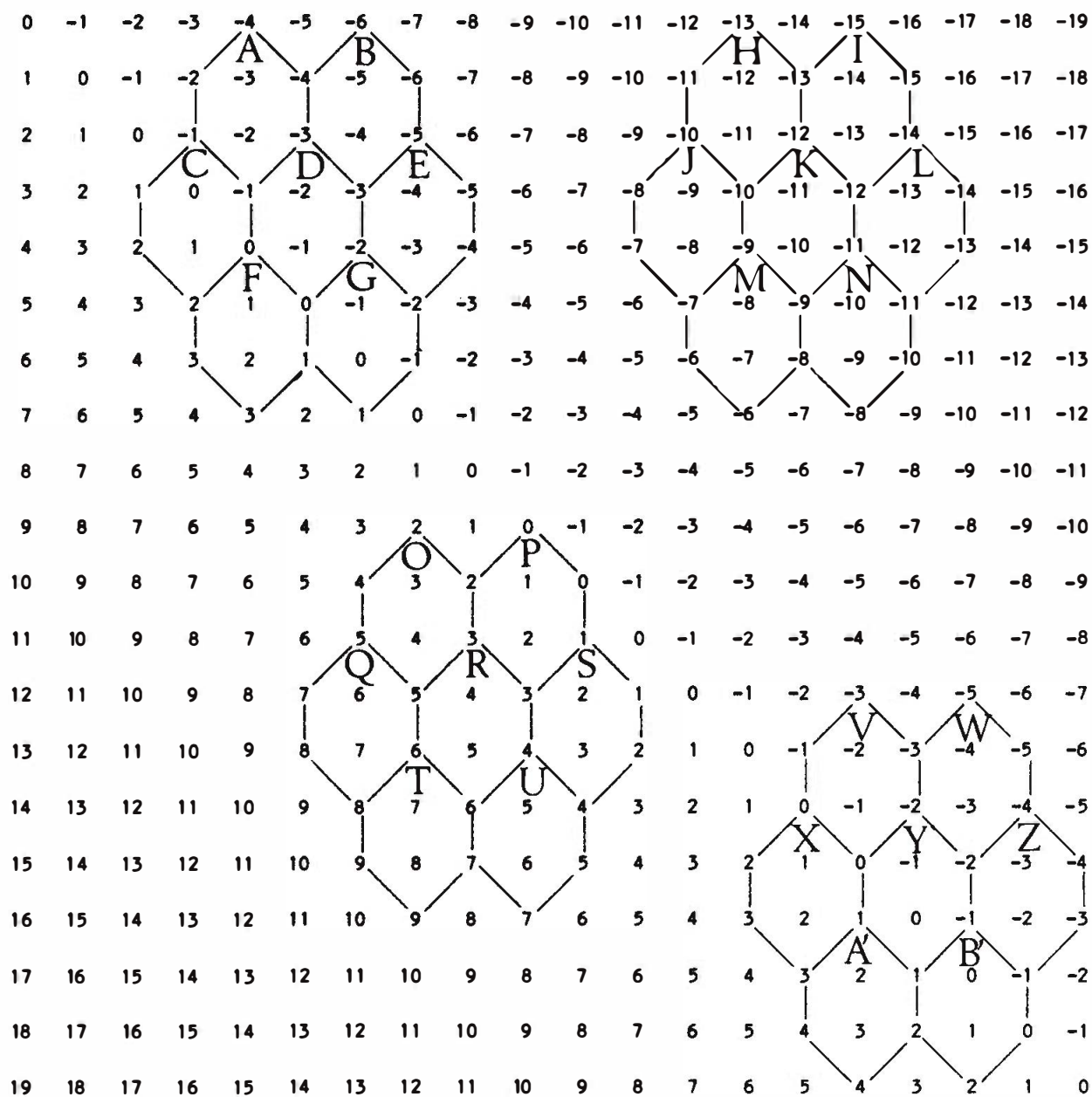


FIGURE 3.

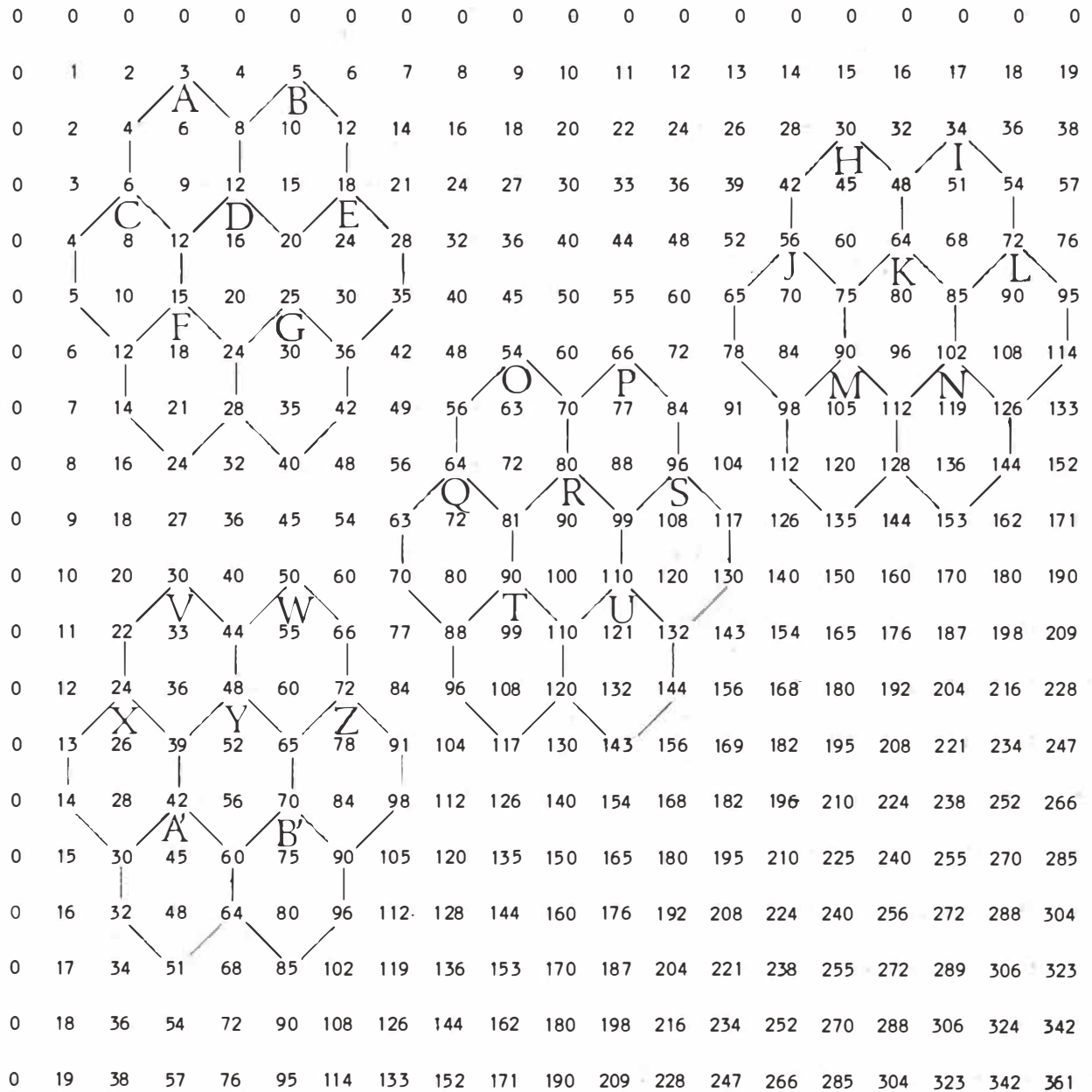


FIGURE 4.

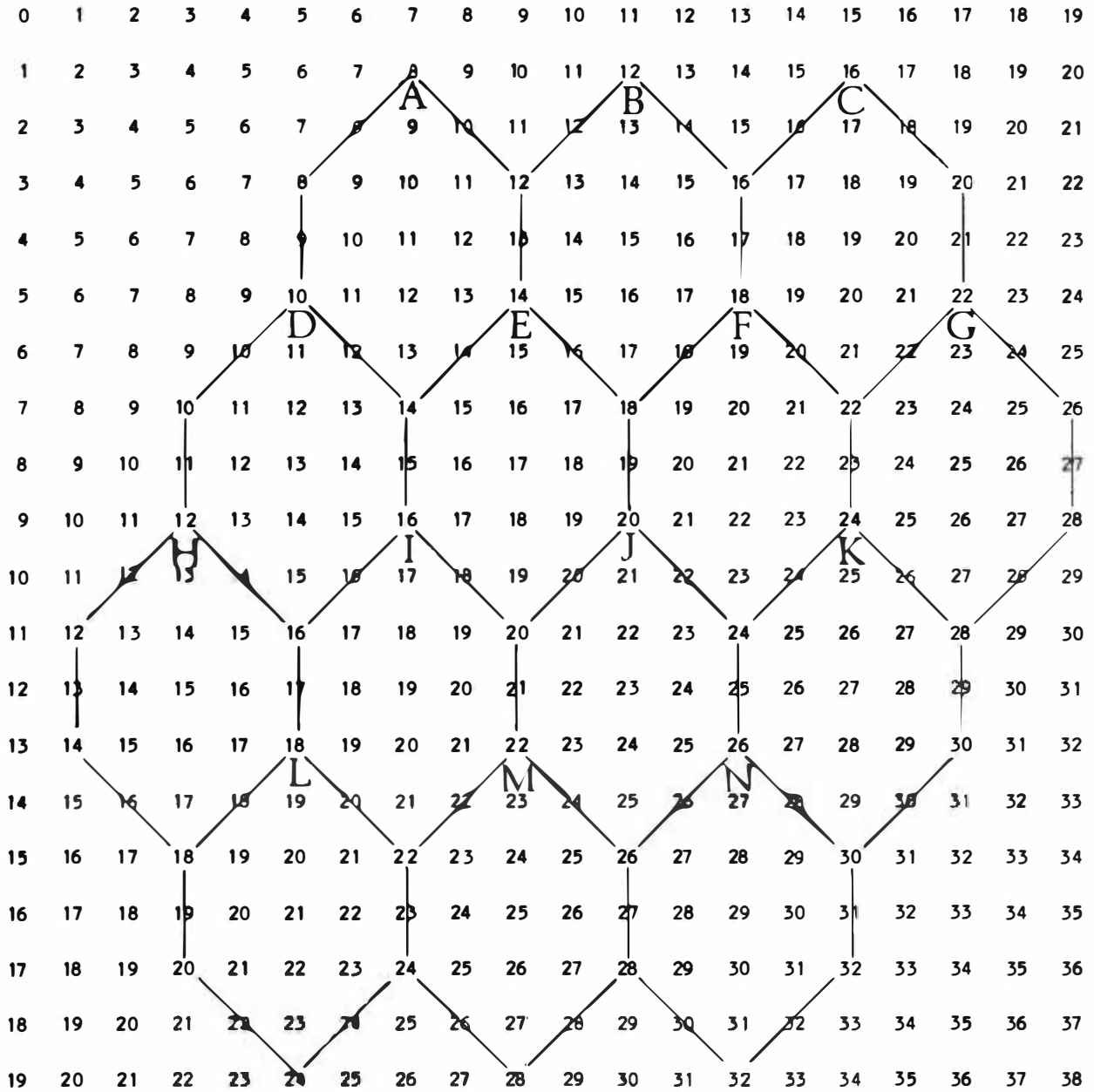




FIGURE 5.

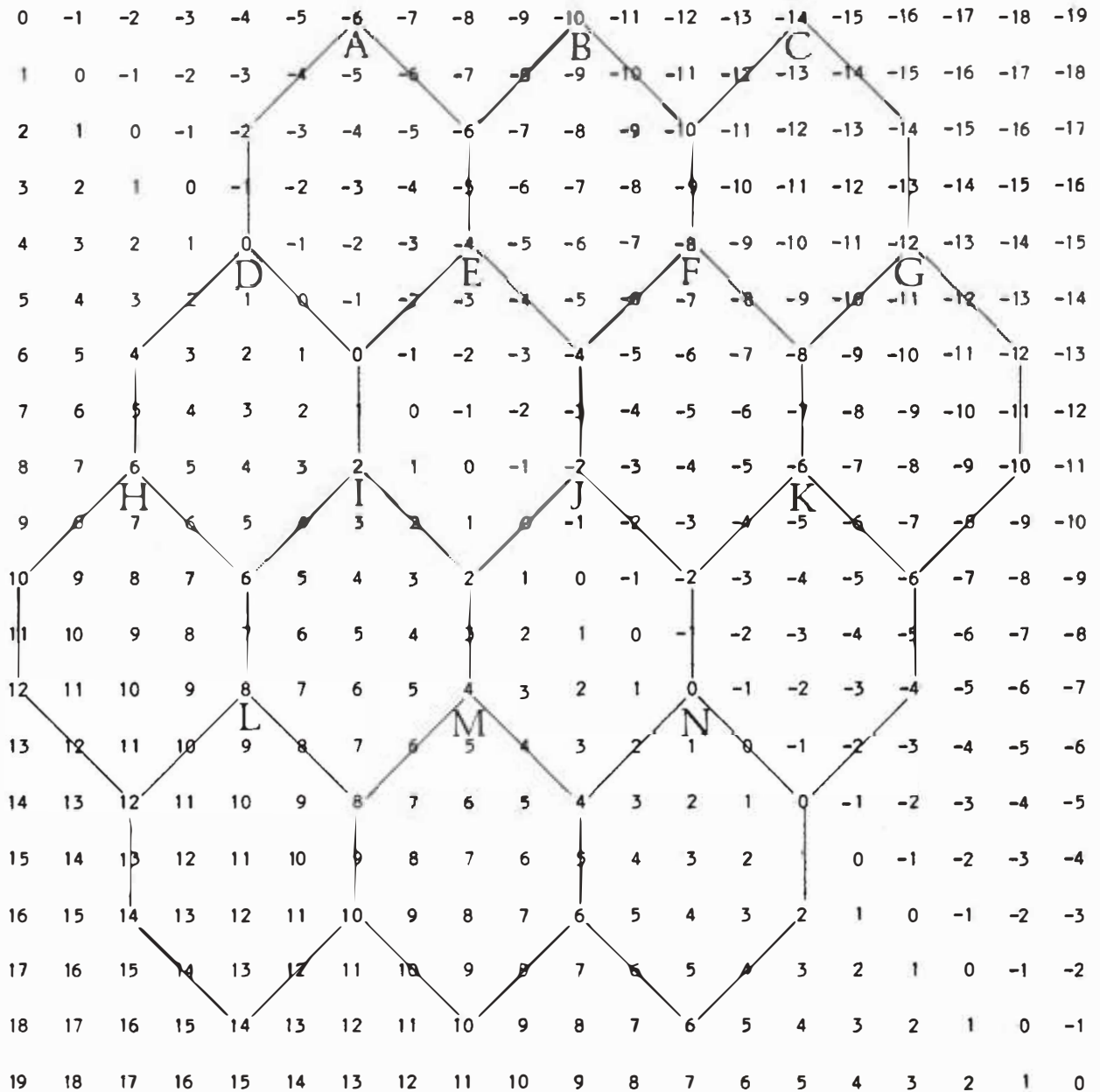


FIGURE 6.

