Probability without Formulas and Equations

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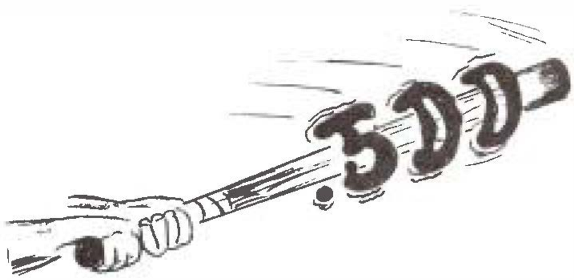
The girl by the Probability Machine, watching the balls drop and form an approximate normal distribution, may find it easy to agree with Laplace, who stated that "the theory of probability is nothing more than good sense confirmed by calculation." It is also easy to agree with Laplace when one considers the everyday probability statements on the following page.

Probability Machine from the IBM MATHEMATICA exhibition at the Chicago Museum of Science and Industry. Photo by Office of Charles and Ray Eames.



The batting champion is hitting 500.

"Our group used Suds toothpaste and we had 49 percent fewer cavities."

"Four out of five doctors recommend sugarless gum for those patients who chew gum."

Insurance records show that very few people die between the ages of 100 and 120.

"Seventy-five percent chance of rain."

Artwork by I.K. Burbank.

However, many students who have had formal exposure to probability may find it difficult to agree with Laplace if their introduction to probability was in terms of sophisticated equations, technical terminology, and meaningless tables of value. Many would say they did not have a chance to build their knowledge and understanding of probability on a "good sense" basis because they were busy memorizing definitions and equations.

The purpose of this article is to outline and discuss some basic probability topics that can be taught to students without the use of equations and technical terminology. The topics to be discussed are:

1. Probability--The Science of Chance
2. The Range of Probability
3. Applying the Theory
4. A Picture of Probability Outcomes.

The Science of Chance

GOAL: Students will experience making assessments based on given data.

PROCEDURES:

Place 12 blue cubes and 2 yellow cubes in a pail and follow these steps:

Step 1: Ask the students, "What is in the pail?" They will not be able to answer because of the lack of data.

Step 2: Shake the pail and ask, "What do you think is in the pail?" and "Are the objects hard or soft, round or square?" The students will be able to make some assessment from the information they obtain from sound.

Step 3: Let some students feel the objects (without looking) and ask them, "What do the objects feel like--are they smooth, rough, sharp?" "About how many are in the pail?" and "What color are the objects?" Now that the students have more data by feeling, they are able to give a more accurate assessment of the items in the pail. However, to state the color of the objects, they will have to look, which brings us to the next step.

Step 4: Have a student take out a cube. However, before he or she looks at it, ask: "What color do you think it is?" "Can you guess the color?" "Are you very sure you are right?" and "How much will you bet that you are right?" At this point, the student is not very confident he or she knows the right color. Allow the cube to be shown, then ask, "Do you think all the cubes in the pail are that color?" Have the student replace the cube. Let three or four other students select one cube, and ask them the same questions. Then allow a student to take three cubes, but before opening his or her hand ask, "What color are the cubes in your hand?" "Are they all blue?" "How much are you willing to bet they are all blue?" and "How much would you bet that one is blue?" As more students take turns selecting and replacing cubes, the confidence that a blue cube will be selected increases.

Step 5: Inform the students that the pail contains 14 cubes. The cubes are either blue or yellow. Have students estimate the number of blue cubes and yellow cubes.

In this simple activity, students can experience making "probability" assessments from given data, and it will help them appreciate the association of probability and chance.

The Range of Probability

Once the students understand that probability is the science of chance, they are ready to experience how the probability of an event is determined.

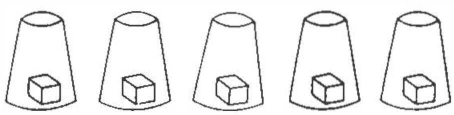
GOAL: Students will experience an activity that illustrates how probability ranges from 0 to 1.

PROCEDURE:

Place 5 cubes under 5 cups, and have students determine the chance of selecting a cup with a cube in each of the following cases. In Cases 2 through 6, replace a cube with a ball, as shown in the diagram.

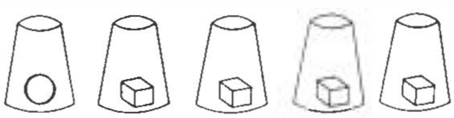
Note: P(cube) is short for "the Probability of selecing a cube."

Case 1: With 5 cubes under 5 cups, what is the probability of selecting a cup with a cube under it?



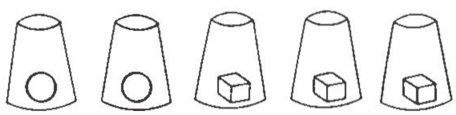
Number of choices 5
Number of cubes 5
Chance of selecting a cup with a cube is 5 out of 5 or 5/5
P(cube) = **5/5 or 1**

Case 2: Replace a cube with a ball. What is the probability of selecting a cube [P(cube)]?



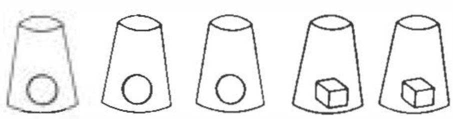
Number of choices ______
Number of cubes ______
P(cube) = ______

Case 3: What is the probability of selecting a cube in this case?



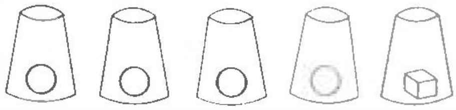
Number of choices ______
Number of cubes ______
P(cube) = ______

Case 4: P(cube)?



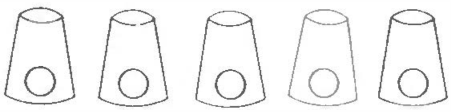
Number of choices ______
Number of cubes ______
P(cube) = ______

Case 5: P(cube)?



Number of choices ______
Number of cubes ______
P(cube) = ______

Case 6: P(cube)?



Number of choices ______
Number of cubes ______
P(cube) = ______

In this activity, the student can conclude that the outcome of an event is known for sure if probability is 0 or 1, and that all other values range between these two extremes when the outcome is not certain. All this can be learned by the student, and the only skill required is to observe and count.

The fact that the probability that an event occurs plus the probability that the event does **not** occur equals one can be illustrated in this activity by having the students fill out the following tables from their observation of cubes, balls, and cups.

	P(Cube)	P(Not a Cube)	P(Cube) + P(Not a Cube)
Case 1:	5/5	0/5	5/5 + 0/5 = 1
Case 2:	4/5	1/5	4/5 + 1/5 = 1
Case 3:	3/5	2/5	3/5 + 2/5 = 1
Case 4:	2/5	3/5	2/5 + 3/5 = 1
Case 5:	1/5	4/5	1/5 + 4/5 = 1
Case 6:	0/5	5/5	0/5 + 5/5 = 1

In filling out this table, note that the probability of not selecting a cube is the same as selecting a ball.

Applying the Theory

Now that the student knows that the probability of an event is the ratio of the number of times the event occurs over the total number of possible events, the next step is to have them apply the theory.

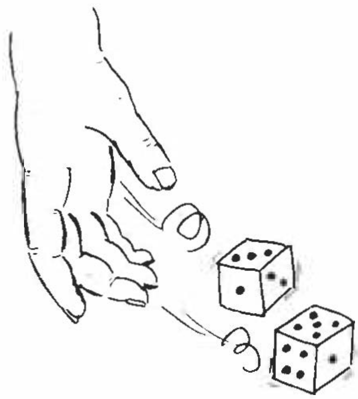
GOAL: The students will experience a probability problem in a game setting.

PROCEDURE:

Provide each pair of students with a pair of dice. Let the students decide who is to be Player A and Player B, based on the following rule:

- Player A scores a point if the sum on the pair of dice is 9, 8, 7, 6, 5.
- Player B scores a point if the sum on the dice is 2, 3, 4, 10, 11, 12.
  Have one student in each pair roll the dice 30 to 40 times and the partner record the scores for A and B.

For example:



Player A	Player B
~~||||~~	///

3 + 5 = 8

Player A scores a point.

Although Player A has 5 sums (9, 8, 7, 6, 5) and Player B has six sums (2, 3, 4, 10, 11, 12), the students soon realize that there is something strange about the outcome of the game. When they share results, they will say, "Player A won almost twice as often as Player B. Why?" In answer to this question, have the students complete the following table by circling the sums that scored a point for A and putting a box around the sums that scored a point for B. Then have them respond to the questions.

Red Die

White Die	1	2	3	4	5	6
1	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	5	6	7
2	$\boxed{3}$	$\boxed{4}$	5	6	7	8
3	$\boxed{4}$	5	6	7	8	9
4	5	6	7	8	9	$\boxed{10}$
5	6	7	8	9	$\boxed{10}$	$\boxed{11}$
6	7	8	9	$\boxed{10}$	$\boxed{11}$	$\boxed{12}$

How many sums does Player A have? ________

How many sums does Player B have? ________

What is the total number of sums possible in the roll of two dice? ________

What is the probability of A's winning? ________

What is the probability of B's winning? ________

The probability of A's winning is 24/36, and the probability of B's winning is 12/36. Theoretically, Player A should win twice as many times as Player B. Compare this theoretical probability with the students' tallies for this game by having each pair of students record their scores on the chalkboard.

For example, assume four pairs had these results:

	Player "A"	Player "B"
Joe, Mary	39	21
Bill, Ed	38	22
Sue, Jill	40	18
Dick, Anne	41	19
TOTAL	158	80

The students will note that the actual outcome of 158 to 80 was close to the theoretical outcome in which Player A wins twice as often as Player B.

From the following table of values, many additional questions can be answered by observing and counting.

		Red Die					
		1	2	3	4	5	6
White	1	2	3	4	5	6	7
Die	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Total number of sums = ________

Most frequent sum = ________

Least frequent sum = ________

P(sum of 7) = ________

P(sum of 12) = ________

P(of an even sum) = ________

P(of an odd sum) = ________

Finally, the students are ready to complete the probability of each sum from 2 to 12 and identify patterns.

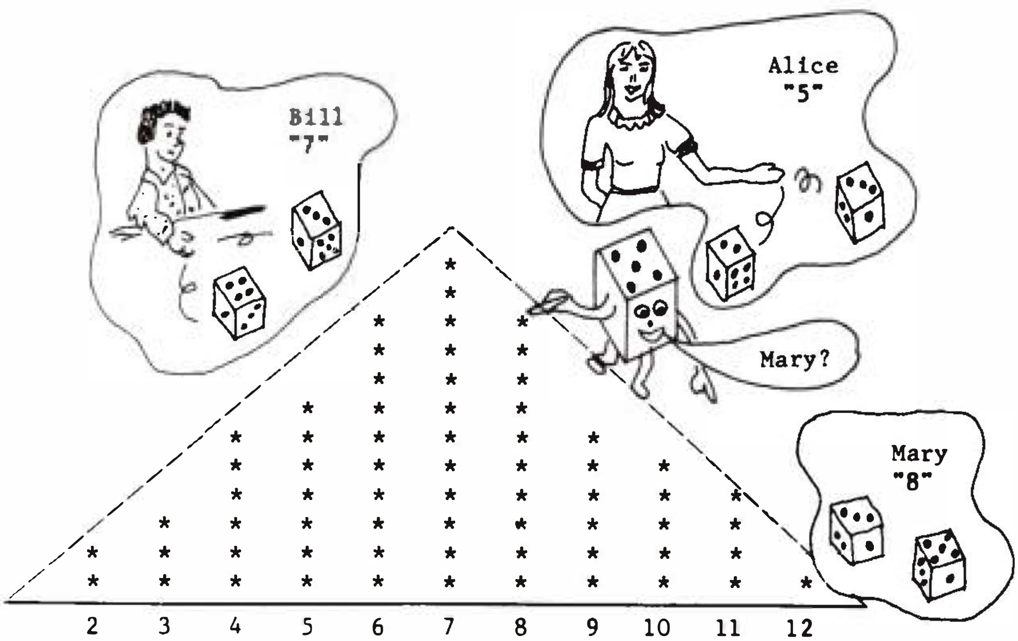
A Picture of Probable Outcomes

From the previous activities, students will realize that certain sums occur more frequently than others. The next activity will build on this perception.

GOAL: The students will experience an activity which illustrates the concept of a triangular distribution.

PROCEDURE:

Have each student roll a pair of dice 4 or 5 times and record their sums on the chalkboard in the following manner.



After the results are recorded, ask the following questions:

1. What do you notice about the sums?
2. In a throw of a pair of dice, what sums will likely show up?
3. What sums are the least likely to show up?
4. Count the number of sums less than 7 and greater than 7. How do they compare?
5. What shape does the dotted line form?
6. How many numbers are between 2 and 7?
7. How many numbers are between 7 and 12?
8. Make a statement about 7.

This activity can be extended to illustrate the concept of a normal distribution. Have each student roll 4 dice a number of times and record the sums on the board. The sums will range from a 4 (four ones) to 24 (four sixes).

In this article, probability topics such as chance, sampling, ratio, range, $P(A) + P(A') = 1$, distribution, and sample space have been illustrated without the use of equations and technical terminology. The major skills required to work through the activities were observation and counting. It is hoped that the ideas in this article will be of help to teachers so that their students may also conclude that "The theory of probabilities is nothing more than good sense, confirmed by calculation" (Laplace).