Evaluation through Problem Solving

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This article will present the concept of evaluation **through** problem solving rather than evaluation **of** problem solving. These two concerns are related, but the latter, which is of no lesser importance, would require a different interpretation than will be focused on here.

This article is based on two of my personal beliefs:

1. Learning or demonstrating knowledge of a concept or skill in isolation or minimal context is of limited, if any, value to the learner.
2. Learning of subskills or concepts, which are developed in a linear or hierarchical order, does not necessarily result in holistic understanding of these concepts or skills for the purpose of application in any but the most routine of situations.

An illustration of the first point is the method of teaching the associative property of addition. It is often presented in symbolic form,

$$(a + b) + c = a + (b + c)$$

preceded and/or followed with examples such as,

$$\begin{aligned}(2 + 5) + 3 &= 2 + (5 + 3)\\ (7) + 3 &= 2 + (8)\\ 10 &= 10\end{aligned}$$

A number of practice exercises may follow:

$$\begin{aligned}(10 + \_\_) + 2 &= 10 + (6 + 2)\\ 3 + (9 + 11) &= (3 + 9) + \_\_\\ 6 + (\_\_ + 8) &= 6 + (4 + 8)\\ (7 + 4) + 2 &= \_\_ + 6\\ 15 + 3 &= 7 + (\_\_ + 3)\end{aligned}$$

After learning this, children may verbalize the property as "You can add the same numbers in any order and still get the same sum," yet have little sensitivity to personal application. For example, suppose the children were selling cookies at recess as a class money-raising project. The cookies cost 10¢ each and, at the end of recess, the day's sellers' recorded their sales for 1, 8, 9, 7, 3, and 2 cookies, respectively:

10
80
90
70
30
20

When adding to obtain the total amount, they would probably use the standard algorithmic method (that is, add the ones column for a total of zero), which would be recorded, and then add the tens column (1 + 8 = 9 + 9 = 8 + 7 = 25 + 3 = 28 + 2 = 30), which would be recorded for a total of 300.

Taking a moment to view this addition holistically, one may see sums of 100 and quickly add mentally (10 + 90, 80 + 20, and 70 + 30, for a total of 300).

The associative property can also be used in conjunction with estimation. For example, when going through the express lane at the grocery store, I usually estimate the total price of my purchases. Suppose I had five items, for $1.49, $.33, $1.19, $.67, and $2.79. I could group them on the counter as follows: $1.19 and $2.79 for about $4, plus $.67 and $.33 for about $1, plus $1.49 for about $1.50, making a quickly estimated total of $6.50.

For an illustration of the second point, consider the following problem. At Westvale Elementary School, a total of 189 students joined the art club and a total of 153 students joined the science club. Some of these students are in both clubs. If there are 456 students in all, how many did not join the art club? To solve this problem, one must subtract 189 from 456. Note that using the popular "decomposition" algorithm,

$$\begin{array}{r} 456 \\ -189 \\ \hline \end{array}$$

"regrouping" or "borrowing" over both tens and hundreds is required. As the algorithm increases in difficulty in the textbook--that is, has larger numbers and requires more regroupings (including the dreaded zeros!)--problems that require each level of subtraction are presented. This means that Jimmy, who has difficulty when regrouping over hundreds is required, would probably not solve (or be expected to solve) this problem. The fact of the matter is, though, that the algorithm used is irrelevant to the problem. It could be solved using the "equal additions" or the "left to right" algorithm. What is important is that Jimmy should be given the opportunity (and he well may be able) to attempt to solve the problem--to cut through the extraneous data and know why subtraction is appropriate. Being a whiz at computation will not serve this purpose for him.

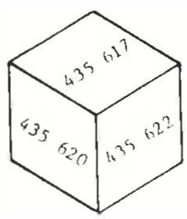
Now consider an example of what evaluation of problem solving is **not**, but often is accepted as such in textbooks. You probably can find a page in the textbook you are using that fits the following description. About 35 multiplication exercises which range from 1 digit by 2 digits to 3 digits by 4 digits. Following this are perhaps six word problems that require the application of multiplication for solution and each one is of the form, "There are n objects in each of m sets. How many objects in all?" Is there any doubt about how to solve these "problems," given the schema aroused by the 35 exercises and the fact that each problem has only two numbers? The children need not read these "problems" to solve them. In this context, they are not problems; they require only trivial application of multiplication. A problem is a situation in which a person does not know immediately what to do; it requires thought and decision making.

Consider the following problems. Solve each one and record the concepts or skills you used.

1. If you can place a 2, 4, or 7 in each box, how many different numerals can you write that are less than 35 000?

   3 $\square$ 2 $\square$ 5

2. The numbers on the faces of this cube are in consecutive order. What numbers are on the unseen faces?



3. Three of the following four numbers were added, and the sum was about 90 000. Which addends were used?

41 195
56 308
19 687
26 429

4. Arrange these digits 6, 2, 5, 7, and 4 to give the greatest product.

□ □ □
x □ □

5. Use the associative property to help you add these numbers on your calculator.

6 957 843 152
9 834 279 947
3 679 982 158
7 465 815 279

6. Without using a marked ruler, draw a triangle with sides of 4 cm, 5 cm, and 10 cm. Verify your answer, using a ruler.

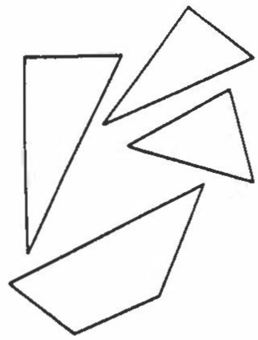
7. How long (to the second) will it be until the minute and hour hands are in a straight line?

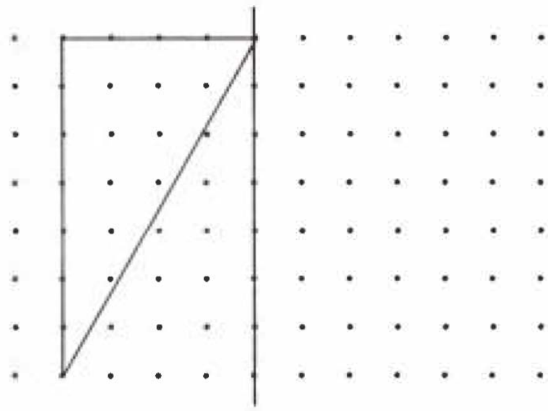
8. The perimeter of a rectangle is 14 cm.
   (a) Draw such a rectangle.
   (b) Find the area of your rectangle.

9. Marge bought three of the following items. If she got $5.60 change from a $10 bill, what did she buy?

ice cream cone	$ .65
sundae	$1.50
parfait	$1.95
banana split	$1.75
1 litre of ice cream	$2.25

10. Place these shapes so that they form the flip image of the triangle.





When you have finished solving the problems, compare them with the problems in the following exercises. Each objective is taken from the Alberta Elementary Mathematics Curriculum Guide (1982) and is followed by a typical textbook evaluation question. Each of the exercises is matched with the problem having the same number.

1. OBJECTIVE: Identifies and names place value of digits (to 999 999).

   TEST ITEM: What is the place value of the 5 in each numeral?
   (a) 45 203
   (b) 351 462
   (c) 698 520

2. OBJECTIVE: Reads, writes, and orders whole numbers to 999 999.

TEST ITEM: List from smallest to largest:
326 951
26 591
326 519
362 591

3. OBJECTIVES: Rounds whole numbers (up to nearest 10 000). Estimates sums.

TEST ITEM: Round each number to the nearest ten thousand and add to estimate the sum.
53 162
29 579
15 406
72 658

4. OBJECTIVES: Multiplies whole numbers using one-, two-, and three-digit multipliers. Estimates products.

TEST ITEMS: Multiply:

$$\begin{array}{r} 256 \\ \times\ 79 \\ \hline \end{array} \qquad \begin{array}{r} 3412 \\ \times\ 6 \\ \hline \end{array} \qquad \begin{array}{r} 795 \\ \times\ 184 \\ \hline \end{array}$$

5. OBJECTIVE: Understands the associative property of addition.

TEST ITEM: (195 + 623 + 439) + (866 + 219) = (623 + 219 + 439) + (195 + ___)

6. OBJECTIVE: Estimates and uses standard units of length.

TEST ITEM: Write your estimate of the length of this line in cm and then measure to the nearest cm.

7. OBJECTIVE: Reads and writes time to seconds.

TEST ITEM:
(a) Write the time shown.
(b) What time will it be 2h 36 min 10s later?

8. OBJECTIVES: Finds perimeter of polygons without using formulas. Finds area of polygons without using formulas. Uses standard units of linear measure.

TEST ITEM: Measure the sides of this rectangle in cm. Find the perimeter and the area.

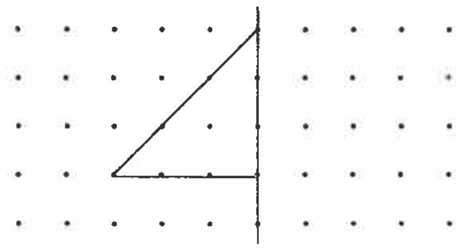
9. OBJECTIVE: Uses money (coins and bills) for purchasing and making change.

TEST ITEM: Sundaes:
lge. 65¢ med. 50¢ sm. 35¢

Joe gave the clerk $10 for 4 large, 5 medium, and 3 small sundaes. How much change should he get?

10. OBJECTIVE: Identifies and draws flips of 2-dimensional figures.

TEST ITEM: Draw the flip image of the triangle.



After you have compared the problems with the test items, consider the following comments.

- The test items tell students what to do. Applying the concept or skill routinely is all that is required.

The problems, which require students to apply the same concepts or skills, also require students to think about how the concepts or skills can be used.

- Test items usually have one answer. Problems may have several possible correct answers, as in problems 5 and 8.

- Test items usually have an answer! Problem 6 is impossible. Children must learn to be critical of data given and trust their own judgment. Explaining why a problem has no answer is a solution.

- Test items usually require only pencil and paper. Problems may require concrete manipulation, as in problem 10.

- Test items usually require manual computation. A calculator is required to complete problem 5, and it could be useful for problems 4 and 9 as well.

- Test items usually focus on one concept or skill, whereas problems may combine concepts nonroutinely. Problem 2 requires knowledge about properties of a cube as well as of numeration.

The reader may not agree with all of these ideas. In fact, it could be beneficial to do the problems and test items with a colleague and discuss interpretations. Through collaboration, a bank of suitable problems for evaluation could be developed.

The intention of this article was to stimulate reflection on the purpose of evaluation in mathematics education. There are no absolute responses to this concern, but none that has been given should be taken for granted.