Problem-Solving Relationships in Algebra

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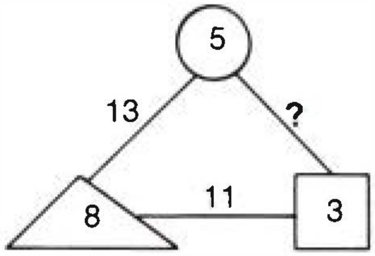
John Percevault is associated with the Faculty of Education at the University of Lethbridge. Although he is retired, his interest in problem solving continues.

Many problems prepared for upper-elementary grades may also be used as algebra readiness problems for junior high school students.

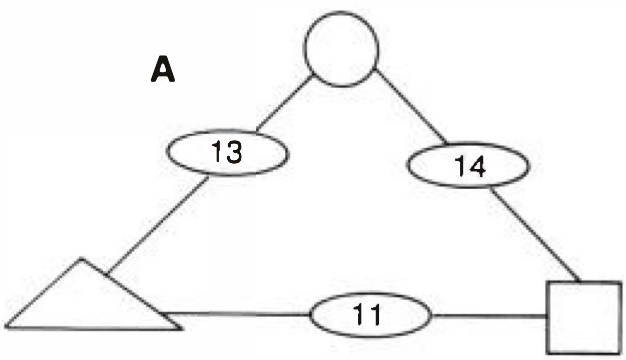
One problem that may be used in this manner is illustrated.

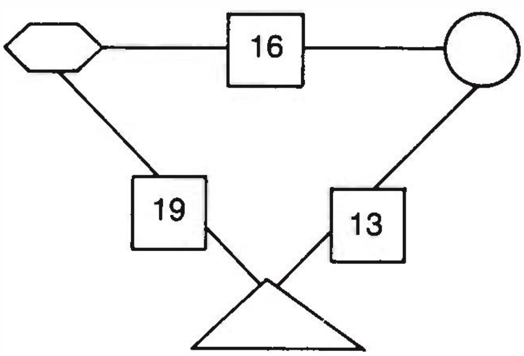


The problem solver did not complete the puzzle. How many relationships can you find? What number should replace the question mark? Why? What relationship did you use?

Set of Problems

The relationship that was found among the numbers in the sample problem applies in the following problems. Given the sums, the solver is asked to find the addends.





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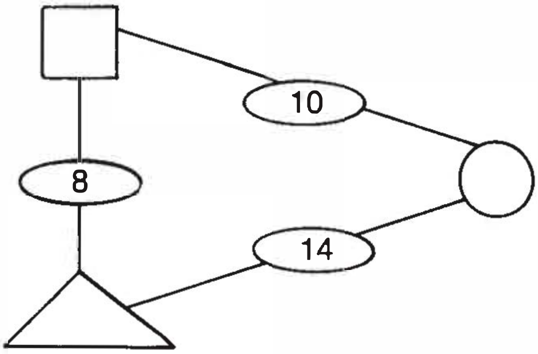

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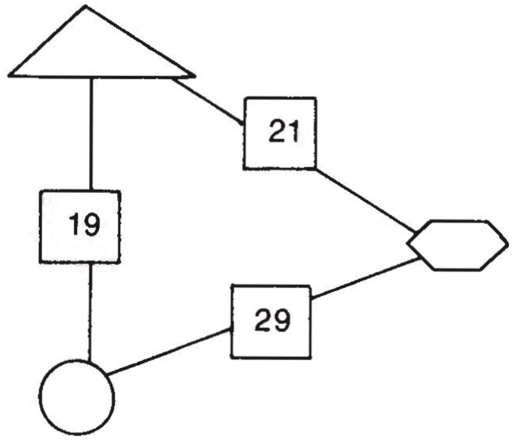

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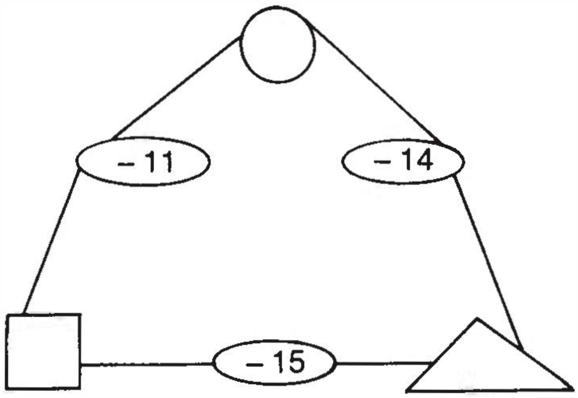

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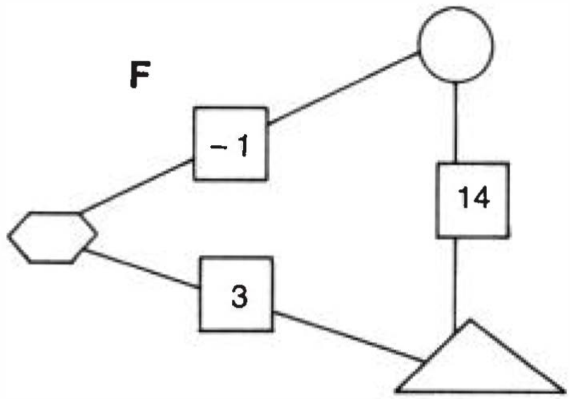
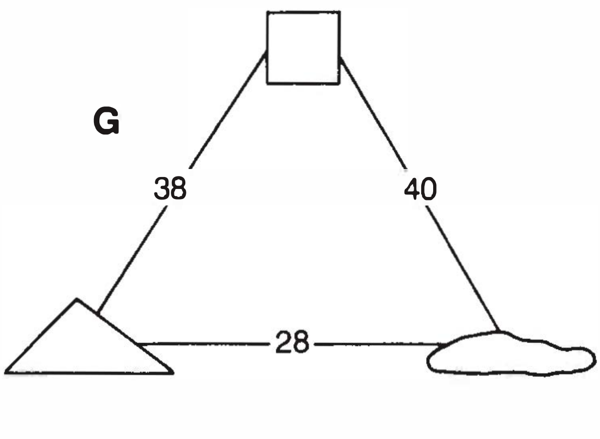


An analysis of students' approaches to solving the problems reveals that the initial strategy is guess-check-refine. Others may use an organized list of addends for each given sum. Others seem to solve the problem intuitively. Questioning will reveal that this group has analyzed the problem and determined an algebraic relationship.

Exploring Algebraic Relationships

A RELATIONSHIP

G



After students have completed the sample problems and others developed by you and them, ask them to compare the sum of the sums to the sum of the addends. Encourage the students to generalize. The relationship that should be evident is that the sum of the sum is twice the sum of the addends.

Consider the problem and express the addition facts with the symbols provided.

$$\square + \triangle = 38$$
$$\bigcirc + \triangle = 28$$
$$\square + \bigcirc = 40$$

$$2\square + 2\square + 2\bigcirc = 106 \quad \text{OR}$$
$$2(\square + \triangle + \bigcirc) = 106$$

Determine that $\square + \triangle + \bigcirc = 53$

But you also know that $\square + \triangle = 38$

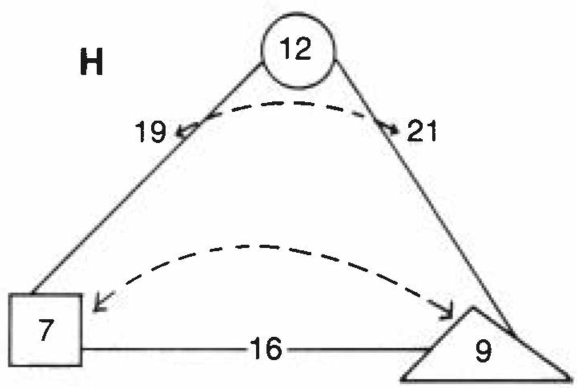
Do you know what $\bigcirc$ is?

Substitute value for $\bigcirc$ into the two equations in which $\bigcirc$ is an addend to find the value of $\triangle$ and $\square$.

Verify in the equation $\square + \triangle = 38$.

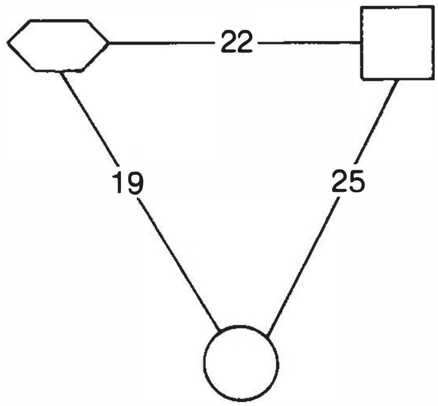
Another Relationship

Encourage students to reexamine the solved problems to discover other relationships that exist. Consider the following solved problem.



The dotted arrows suggest that the reader focus on the relationship between two sums and two addends. A difference of "2" is constant. Explore other sets of two addends and two sums. Is there a constant difference?

I



○ + □ = 25

○ + ⬡ = 19

SUBTRACT AND OBTAIN

□ − ⬡ = 6

□ = ⬡ + 6

Now the value of ⬡ and □ can be determined by substituting into

⬡ + □ = 22.

Students will formulate the generalization that the difference between the sums is the difference between the two related addends. To return to the sample problem, the students can generalize that 22 is the sum of a number and that number increased by 6, or $n + (n + 6) = 22$.

Exploration

Refer to problem H.

⬡ + □ = 22

⬡ + ○ = 19

Add 2⬡ + □ + ○ = 41

But we know that □ + ○ = 25

Can we find the value of ⬡ ?

Is the problem solved? Can the student verify by substitution? Finally, almost, take the two equations:

⬡ + □ = 22

and

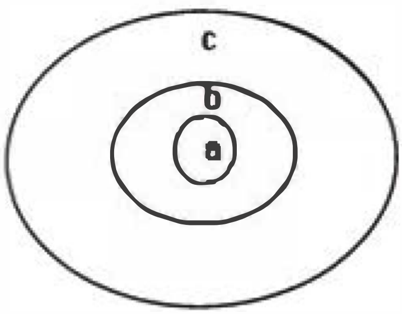
⬡ + ○ = 19

Subtract to obtain □ − ○ = 3. Use the equation, □ + ○ = 25 and substitute.

Use any other process or any conclusions.

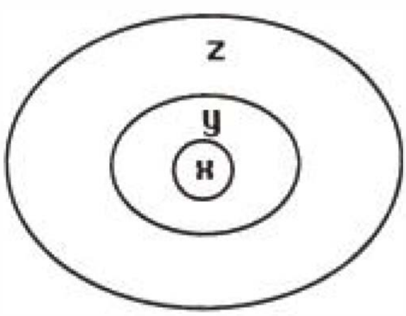
A Final Problem

Two little space travellers escape from the planet Rood. Skol really escapes. Rood is allowed to use the planet's name. Anyway, Skol and Rood land on Earth and are immediately lured to an arcade. Skol is attracted to an electronic deal game (EDG) whose score board is illustrated.



After a few random shots with his space gun, Skol realizes that his score is recorded after two shots. He determines that when he hits a and b, the computer records a score of 15. A hit on a and c is recorded by a score of 13, while hits on b and c show a score of 8.

Meanwhile, Rood is attracted to a different EDG whose score board is illustrated. After a few experimental shots, Rood determines that the same rule applies. Rood determines that:



$$x + z = 15$$
$$x + y = 7$$
$$z + y = y + z = 2$$

Post Mortem

Well! What do you do now? It's up to you to determine the questions you could pose. Change the rule? Change the variable? Others?