Intersecting Lines: Problem Solving in Geometry and Algebra

Mary-Jo Maas

Mary-Jo Maas teaches at G.R. Davis School in Fort Macleod and continues to serve as secretary for the MCATA.

The 1980s have been, and continue to be, the decade in which problem solving has come to the forefront. One has only to look at a convention program to realize the impact that problem solving has had on the field of mathematics. As teachers, you have been overwhelmed with a blizzard of problem-solving sessions, workshops, articles, and books. The probability of there being a mathematics teacher who hasn't been exposed to this storm is slim, indeed.

As the 1980s draw to a close, we are faced not with the task of exposing teachers to more problem-solving exercises, but with that of helping them to use problem-solving more effectively as an integral part of their lessons. All teachers have a nice set of exercises that they use to teach problem solving. Now teachers must take these exercises and extend, modify, and redesign them so that they complement the curriculum and can be used at a variety of grade levels. Problem solving was not meant to be a totally separate topic, but rather a strand of the curriculum that is to be found in all topics.

The following is a problem that can be extended to complement the regular curriculum:

PROBLEM:

> Seven line segments intersect a circular area to produce the maximal number of areas.

or

> Using only 7 straight cuts, what is the maximal number of pieces of pizza that can be obtained?

or

> Given a square pigpen and 7 straight fences, what is the maximal number of pigs that can be placed in this pen?

This problem is generally given to upper elementary or junior high students as an enrichment problem. It could be just as easily used in the geometry unit during a discussion of intersecting lines and could also be used during a study of number patterns.

The beauty of this problem is the diversity of ways in which it can be solved. Students can create simpler problems to solve it. They can also make toothpick models, draw pictures, create a chart, organize their data, look for patterns, hypothesize, or use a combination of any of these. All of these methods are problem-solving skills and strategies.

A possible solution employing several strategies might look like this:

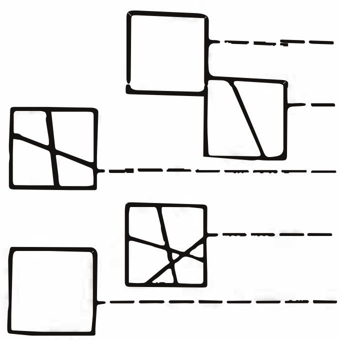

Diagram	# of Lines	Maximal Areas
	0	1
	1	2
	2	4
	3	7
	4	?

From a chart such as this, students are encouraged to explore the number patterns they see.

Intersecting Lines	Maximal Areas	Number Pattern
0	1	1
1	2	1+1
2	4	1+1+2
3	7	1+1+2+3
4	11	1+1+2+3+4
?	?	?

Students might then hypothesize how many spaces would exist for 10 lines, or 12 lines, or n lines.

This problem, however, is not just for the junior high student. At the senior high level, this problem could be presented as an application question in the study of arithmetic sequences and series. The students would use all of the previously mentioned problem-solving skills and strategies, as well as formulas such as the sum of an arithmetic series. The students would discover that given n lines, the maximum number of areas obtained would be $1 + S_n$.

I hope that, after reading this article, you have become more aware of the possibility of using problem-solving exercises as a means to explore and apply new mathematical concepts and topics. It is important that we look at our list of problem-solving exercises not as extras to use if time permits, but as teaching aids in our regular lesson plans.

These exercises should also be reviewed to see if they can be expanded and used at a variety of grade levels instead of earmarked for a particular grade. It is time that the blizzard of problem-solving exercises, workshops, and articles be turned into useful tools to truly capture the spirit of NCTM's Agenda for Action.