

Writing in Mathematics

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Mathematics instruction involves the interrelationship of two languages: the native language and the language of mathematics. Just as we communicate our native language via an extensive and comprehensive set of symbols and vocabulary, so too do we communicate the language of mathematics. However, the language of mathematics differs from the native language in that it knows no cultural or geographic boundaries, thus it is unique and universal. Furthermore, vocabulary shared by both languages has a different meaning in a mathematical context than it has in a native language.

There are four developmental phases in language arts: listening, speaking, reading and writing. They are interrelated. Speaking and writing are closely related in that they impart language. They combine with the receptive phases, listening and reading, to facilitate growth in and mastery of language as a whole. The same four phases are essential to learning mathematics, the language and the body of knowledge.

It is recognized that success in mathematics is correlated with the ability to read mathematics. Just as listening and speaking are precursors to speaking, reading is a precursor to writing. Writing is, therefore, the most complex of the four phases of language arts.

This article will focus on writing and how writing should be used to reinforce mathematical content and afford students opportunities to develop higher-order thinking skills through analysis and synthesis of content. Not only will mathematical content be reinforced, but writing, which traditionally

is taught and practiced exclusively in English classes, will be freed from the confines of this single subject.

Writing is a powerful learning tool. Yet, while writing occurs in almost all classes, it is frequently in the form of notetaking or other types of informational writing. Usually classwork, homework and other assignments demand of students only rote feedback of information. Text is infrequently constructed, thus mechanical writing seems to be emphasized. Thinking is outside the context of such written expression. The powerful potential of writing cannot thereby be realized. By contrast, writing in imaginative and creative ways involves thinking at higher levels and provides opportunities for students to analyze and synthesize content and to express ideas and relationships in unique forms.

What are some of these forms? How can mathematics teachers use writing in the teaching of mathematics to reinforce mathematical content? Some examples from our teaching experience will be presented in this article.

Clearly, it is important that writing in the mathematics curriculum emphasize content. Stephen Tchudi, talking about content as the focus of the writing process, states:

Teachers . . . know their discipline well. . . . If they will simply keep a focus on helping students to express ideas in the discipline clearly, matters of content, form, style and even mechanics will be taught in context as appropriate. (Tchudi, p. 24)

Whereas writing in other content areas largely involves the use of native language to express ideas, writing in math allows students to reinforce their facility with the language of mathematics, as well. Writing in various subjects affords teachers a special opportunity to communicate with one another,

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especially with the English teacher who teaches writing as a principal part of the curriculum. Ideally, mathematics teachers who intend to assign creative writing should consult with English teachers prior to doing so to ensure that they are reinforcing the writing standards taught in English class while their students write in a mathematical context. From the same perspective, writing in mathematics should help other subject areas. All lessons require reinforcement to be learned, and writing is no different.

Our first effort in implementing a "writing in mathematics" program in our classes is to motivate our students to want to write. To do this, we must be motivated ourselves. Our attitudes are contagious. If we are involved and excited about writing, our students will share this excitement. Next, the classroom environment must be conducive to mathematics language skill development, that is, our students' use of spoken and written words must be encouraged and reinforced. Verbal exchanges in a brainstorming session are an important prewriting activity.

To create an atmosphere of discovery requires inductive teaching. It is believed that students learn more effectively through self-discovery of ideas, using language to express these ideas, verbally and in written form. The problem-solving process is at work here. This is the antithesis of the common "show-and-tell" method of teaching, which can't lead to true discovery by students. Thus, writing in mathematics must be viewed in conjunction with problem solving. This was the number one recommendation of the National Council of Teachers of Mathematics for mathematics education in this decade.

As with problem solving, writing activities should begin with the group; they should be a class endeavor. This endeavor should include the four stages in the writing process: prewriting, writing, rewriting and editing. Prewriting involves the verbal exchange of ideas. Along with the class, discuss the following: What type of writing shall we do today? Shall we write a paragraph to clarify and show the interrelationship of terms we've learned in each unit? Should we do some imaginative writing, expressing some mathematical content through a poetic form? The prewriting stage is vital to the writing process for it is through the exchange of ideas that plans are made for what to write about (the content focus) and what writing medium will be used.

Once we write, we need to examine what and how we are writing. Rewriting and editing are important activities for achieving a high quality final product.

At these stages a variety of questions should be explored: Have we kept content the focus of our writing? Are the relationships clearly expressed? Have we spelled, capitalized and punctuated correctly? How can we build on the strong points of our writing? Do we need to do some research for further writing, to expand that which we have already done? Is there anything we wish to delete? Several drafts may be necessary to achieve the finished product.

Following are several examples of writing that focus on mathematical content. Most of the writing was done by Grade 7 students at Baldi Middle School.

1. "Splish-Splash, I Was Taking a Bath: A Mathematical Version" by Michael Marcus. In *limerick form*.

Archimedes had no clothes or shoes,
As he ran through the streets of Syracuse.
He found out, and not too late,
Displaced water was his weight.
And he ran to tell the people this news.

2. *Mathaiku by Class B7-4*

The problems in math
Are real challenges to us
Be we still solve them.

3. *Hidden Sequence by Michael Ganetsky. An example of free verse.*

Once
There
Lived a
Boy, whose name
Was Leonardo.
We know him as Fibonacci.
He studied the reproduction of rabbits to find
A sequence of numbers named for him, which
can be found by counting the syllables in
Each line of this free verse which can't continue
indefinitely as does the special sequence of numbers
discovered by Fibonacci.

4. *Euclid by Jeffrey Jenofsky. A narrative poem with the rhyming pattern a,a,a,a,b,b,c and four beats per line.*

Euclid lived in 300 B.C.
He wrote 13 books on geometry.
The king asked, "Euclid," I quote he,
"Is there an easier way to geometry?"
Euclid replied with some sense,
"You must read the *Elements*.
There is no royal road to geometry."

5. A mathematical version of "One" from A Chorus Line by Class B7-4

One, mathematical sensation,
Can be called by many a name.
One, when used in multiplication,
The product will remain the same.
Whole, natural, integer, rational, real, square,
too,
Triangular, positive, cube, just to name a few.
One, in base ten numeration
Can be used in any place.
One, as a power keeps the same base, base,
Ooh, sigh, give one your attention.
Do we really have to mention,
"One's the one!"

6. Popular song titles—*mathematical, of course!*
Contributions made by all students in the class.

One More Point by Phil Colinear
Automorphic by the Seventy-Sixers
Walk Like a Mathematician by the Angles
Hold Three Now by the Thompson Triplets
I'll Be Square the Jackson Twenty-Five
The Bunny Hop by Leonardo Fibonacci
Tapestry by M.C. Escher
Lune River by Hipparchus

7. The Definition of Confusion

Expository writing by Grade 8 math students.

A *factor* of a number *divides* the number. Therefore, the *factor* is a *divisor* of that number, which in turn, becomes a *multiple* of the *factor*. When the *factor* is *divided* into the *multiple*, another *factor* is produced. The pair of *factors* gives a unique *product*. Since each *factor divides* the *product*, the *product* becomes a *dividend* in a *division* problem and the *factors* become the *quotient* and *divisor*.

8. Selections from An Anthology of Problem-Solving Poems

The following two poems were group efforts; the first was led by Michael Marcus and the second by Michael Ganetsky.

I

When we solve a problem
Here's what we do.
First, we have to read the problem through.
Next, we see what we're looking for.
Then we read the problem once more.

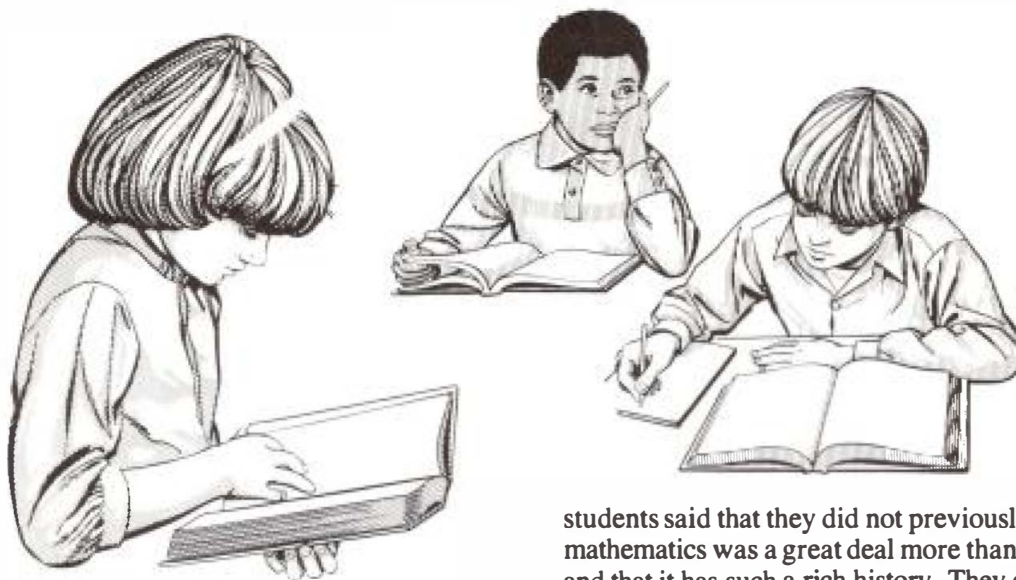
Now we write the important information,
And using these facts we look for a relation.
Next, we pick from the many strategies
And solve the problem using these.
Last, we check our answer to see
If it fits in accurately.

II

When you solve a problem
You use your head.
But you can't glance
For a problem must be read.
Next, you have to know
What you have to find.
Read the problem over again
Then shuffle it in your mind.
You must note the key facts.
Then list this important information.
Next, you decide on a strategy,
Like writing an equation.
Another one is make a chart
Or you can often guess and test.
Depending on the situation
Any one could work best.
You need to check the answer,
The last step left to do.
And if you find the answer fits,
You'll know that you are through.
But if your answer doesn't check,
And you did not succeed,
Go back and read this poem again
To find out how to proceed.

Several of the examples shown above focus on the history of mathematics. History should be part of the total math program to help students develop an appreciation of mathematics and the interrelationship of its branches. Independent research and writing projects such as students' autobiographies, biographies of famous mathematicians or articles on special historical or technical topics provide students with information to foster further creative writing.

Consistent with curricular planning, of course, most written work by students is done at home (after the classroom motivational sessions mentioned above). Occasional individual conferences can be held, if necessary, to provide comments on revision, editing and final drafting. To encourage and maintain the students' interest, comments should focus on the strengths of each student's work and on the content. The mechanics of writing should be noted but not emphasized.



When the first piece of writing has been completed, display it prominently. Share it with other staff members and pupils. Exhilaration, excitement and the desire to continue to write will likely be evident among your students. Let it be known that writing has been freed from the confines of the English classroom. This opens up a multitude of possibilities, limited only by the imagination!

We have found that students' attitudes have improved and changed this past year. Our students have been exposed to a language arts program in mathematics classes for an entire year; they recently expressed their reactions to this program in a very positive way. In an informal survey, the majority of

students said that they did not previously realize that mathematics was a great deal more than computation and that it has such a rich history. They enjoyed writing about this history and other topics. They listened to and read each other's work. They wrote, they learned and they enjoyed!

References

- Applebee, Arthur. *Contexts for Learning to Write: Studies of Secondary Instructors*. Norwood, N.J.: Ablex, Inc., 1984.
- Earle, Richard A. *Teaching Reading and Mathematics*. Newark, Del.: International Reading Association, 1976.
- Calkins, Lucy McCormick. *The Art of Teaching Writing*. Portsmouth, N.H.: Heinemann Educational Books, Inc., 1986.
- Tchudi, Stephen. "The Hidden Agendas in Writing Across the Curriculum." *English Journal* 75, no. 7 (November 1986). Urbana, Illinois: National Council of Teachers of English.