Algebra Can Be a Language

George A. Calder

George A. Calder is a teacher at Franklin High School in Livonia, Michigan.

Effectively teaching word problems in algebra is probably as difficult for teachers as solving the problems is for students. Some textbooks approach word problems as individual sets. That is, age problems are treated in one section, mixture problems in another section and motion problems in a third section. Other textbooks purport to intersperse word problems throughout the text.

Most algebra teachers adapt one of the above treatments to their own teaching styles. As new textbooks are adopted, the teachers may or may not adjust. But regardless of textbook treatments and teacher styles, the topic of word problems in algebra has traditionally been a low point in most algebra classes (for both teachers and students).

One technique for teaching word problems is to treat "Algebra" (with a capital "A") as a language. Indeed, one can make a very good argument that algebra *is* the language of mathematics. What is suggested in this article, however, is that Algebra is a language, like English, Spanish or German, and that word problems can be handled as if one was "translating" from English into Algebra.

The first step in learning a new language is usually to buy a dictionary. No such item exists for Algebra. Thus, we are forced to create our own. Before introducing word problems in an algebra class, a teacher might use a worksheet similar to the one shown here.

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DICTIONARIES

English	French	German	Spanish
white	blanc, blanche	weiss	blanco, blanca
the dog	le chien	der Hund	el perro
the house	la maison	das Haus	la casa
is	est	ist	es, esta
closed	fermee	geschlossen	cerrada

Translate the following into English:

Example: Le chien blanc. Answer: The white dog.

1. Der Hund ist weiss.
2. El perro blanco.
3. La maison est fermee.
4. El perro es blanco.
5. Das Haus ist geschlossen.
6. Der weisse Hund.
7. Le chien blanc.
8. Le chien est blanc.
9. Das Hundhaus ist weiss.
10. La casa esta cerrada.

Translate the following into the language indicated:

11. Spanish.....The house is white.
12. French.....The house is white.
13. German.....The dog is white.
14. German.....The doghouse is closed.
15. Spanish.....The white dog.
16. French.....Der weiss Hund.

An example on the worksheet can be used to point out that, in some languages, the adjectives come after the nouns. Thus, while a word-for-word translation of the examples would be "the dog white," the rules of grammar for English would require "the

white dog'' as the correct translation. In the worksheet itself, however, students should be allowed word-for-word translations. The example is good to recall later when a student must translate ''three less than John's age'' as ''j − 3'' rather than word-for-word as ''3 − j'' (a common error among students). In other words, Algebra also has it's own set of ''grammar rules'' when making translations.

After students have completed the worksheet, it can be pointed out that even though they do not know all (or perhaps any) of the languages, they *can be successful* in doing something brand new when given appropriate aids (like a dictionary).

The next day's lesson, then, is spent introducing the concept of a ''dictionary'' for algebra word problems. Simple age problems are ideal for introducing this concept. Here is an example:

Problem

Mary is three times as old as Bob. The sum of their ages is 28. How old is Bob?

DICTIONARY

	Age in years
Mary	3b
Bob	b *

Students are taught to complete the dictionary in the following order:

1. List the ''characters'' of the story (word problem) down the left-hand column. (Mary and Bob)
2. List the ''characteristics'' of those characters across the top row. Be sure to include the measures, i.e., years not months. (Age in years)
3. Put an asterisk in the box you are asked to find. (Bob's age in years)
4. Choose a variable and put it in any box. Usually it will be the box with the asterisk, but not always. Sometimes it will be in the box with the least amount of information given in the problem. Don't always choose ''x'' or ''y,'' but instead, choose a variable that will remind you of what it represents. (b for ''Bob'')
5. Fill in all other boxes using information for the problem.

After the ''dictionary'' is complete, use it to translate the story from English into Algebra. Then solve the resulting algebra equation. Finally (and this is important), translate the Algebra answer back into English by writing an English sentence.

Translation	3b + b	= 28
Solving	4b	= 28
	b	= 7

Answer: Bob is seven years old.

Some students may even discover that the dictionary could be as follows:

ALTERNATE DICTIONARY

	Age in years
Mary	m
Bob	(⅓)m*

Translation	m + (⅓)m	= 28
Solving	(4/3)m	= 28
	m	= 21
	(⅓)m	= 7

Answer: Bob is seven years old.

Notice that in the alternate solution the answer to the equation (21) is *not* the answer to the story problem. This is why the asterisk is placed in the dictionary—to remind the student of what to look for.

After a day's practice with simple age problems, the students can be exposed to more difficult age problems such as follows.

Problem

Kim is four years older than Dean. Eight years ago, Kim was twice as old as Dean was. How old is Kim now?

DICTIONARY

	Age in years	
	now	8 years ago
Kim	d + 4*	d + 4 − 8
Dean	d	d − 8

Translation $d + 4 - 8 = 2(d - 8)$
Solving $d - 4 = 2d - 16$
$- 4 = d - 16$
$12 = d$
$d + 4 = 16$
Answer: Kim is now 16 years old.

Notice how the concept of "8 years ago" automatically requires the subtraction of eight ($- 8$) in equations for every character in the story, whether or not that information is later needed. A concept of "10 years from now" would, of course, require "$+ 10$" in its equations.

Later in the school year, when students have studied two variable systems of equations, the same dictionary can be used, with "$d + 4$" changed to "k" (for Kim) and "$d + 4 - 8$" changed to "$k - 8$."

Age problems can become quite complicated with the addition of more characters and the consideration of their ages at present, in the past and in the future, all at the same time.

The use of such a dictionary in all word problems not only forces the student to read and re-read the problem many times (something many students never bother to do), but also helps the teacher to pinpoint where a student's reading of the problem is incorrect. Even the simple act of placing an asterisk can show which students do not know what they are supposed to find. How can a teacher expect a student to find something in a problem, if the student's comprehension of the words is so flawed that he or she can't even tell what to look for?