Appendix I: Supplementary Problems

Problem 1

There are 1001 pebbles in a heap. The heap is divided into two, the number of pebbles in each is counted and the product of these two numbers is written down. A heap containing at least two pebbles is then chosen, divided into two, the pebbles are counted and the product is written down. This procedure is continued until every heap contains one pebble. Find the maximum value of the sum of the 1000 products written down.

Problem 2

A difficult mathematical competition consisted of a Part I and a Part II with a combined total of 28 problems. Each contestant solved exactly seven problems altogether. For each pair of problems, there were exactly two contestants who solved both of them. Prove that, if every contestant solved at least one problem in Part I, then at least one contestant solved at most three problems in Part II.

Problem 3

Use the hexominoes (see Figure 1a in Kathy Jones' article) to construct a 15 by 15 square with a centrally symmetric 3 by 5 hole.

Problem 4

Use all polyiamonds up to and including the heptiamonds (see Figure 1b in Kathy Jones' article) to construct the star of David with a hole as shown in Figure A. Note that there is a diamond inside the hole!

Problem 5

You have an inexhaustible supply of 5-cent and 8-cent stamps. What amounts of postage cannot be made exactly if only these stamps can be used?

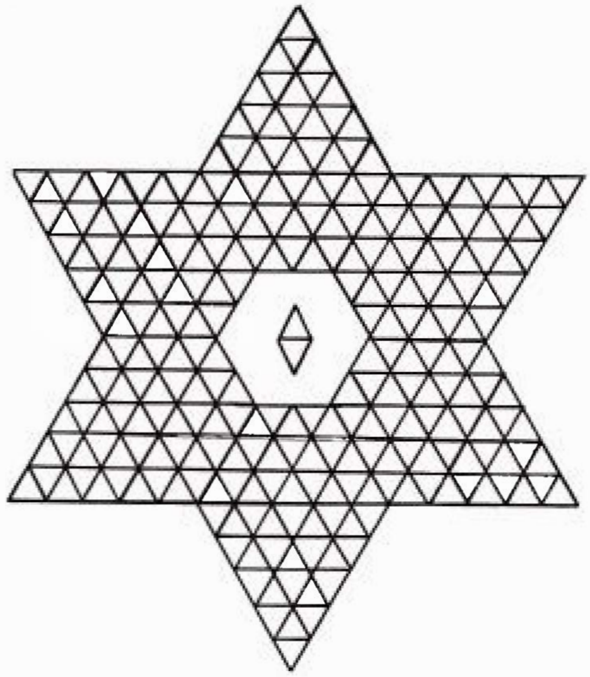

Figure A

Problem 6

A prime triple (x, y, z) consists of three prime numbers x, y and z such that $y - x = z - y$. The common value of $y - x$ and $z - y$ is called the common difference of the prime triple.

(a) Find a prime triple with common difference 2.
(b) Find another prime triple with common difference 2. How many others are there?
(c) Find a prime triple with common difference 3. Are there any?
(d) Find a prime triple with common difference 4.
(e) Find another prime triple with common difference 4. How many others are there?
(f) Find a prime triple with common difference 5. Are there any?
(g) Find a prime triple with common difference 6.
(h) Find another prime triple with common difference 6. How many others are there?

Problem 7

Prove that the shape in Figure B cannot be constructed using a complete set of pentominoes.

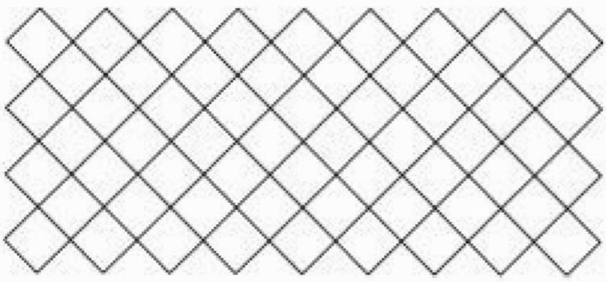

Figure B

Problem 8

Prove that the shape in Figure C cannot be constructed using a complete set of pentominoes.

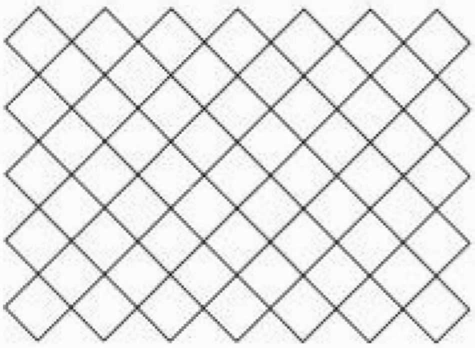

Figure C

Problem 9

Determine the maximum value of $m^2 + n^2$ where m and n are integers satisfying $m, n \in \{1,2,\ldots,1981\}$ and $(n^2 - mn - m^2)^2 = 1$.

Problem 10

Superchess is played on a 12 by 12 superboard and it uses superknights which move between opposite corner cells of any 3 by 4 subboard. Is it possible for a superknight to visit exactly once every other cell of the superboard and return to its starting cell?

Sources

Problem 1 is taken from the Soviet journal *KVANT*.
Problem 3 is taken from the instruction booklet of Sextillions (a registered trademark of Kadon Enterprises).
Problem 5 is taken from Tony Gardiner's *Discovering Mathematics*.
Problem 7 is taken from Solomon W. Golomb's *Polyominoes*.
Problem 9, proposed by Jan van de Craats, appeared in the 1981 International Mathematical Olympiad.
Problems 2, 4, 6, 8 and 10 are the editor's composition.
Problem 2 appeared in the 1984 U.S.A. Mathematical Olympiad.
Problem 10 was proposed for the International Mathematical Olympiad but not used and appeared instead in the problem set of the 1986 International Mathematical Congress.