Appendix III: A Selected Bibliography on Popular Mathematics

There are so many good books on popular mathematics that it is difficult to list them all. This selected bibliography is limited to books which are in English and are still in print as far as is known, and which have been used in the editor's "Saturday Mathematical Activities, Recreations and Tutorials" (SMART) program of enrichment for students of Grades 4 to 9 in the greater Edmonton area. A typical problem from each book is given.

A. Martin Gardner's Scientific American Series

For over 20 years, Martin Gardner had a monthly column in *Scientific American* called "Mathematical Games." Despite the title, it covered a wide range of topics, from the very elementary to the frontiers of current research but always with a delightful element of play. It was written in a lively style indicative of the strong literary background of Martin Gardner. When the subject under discussion was difficult, he took considerable effort to smooth out the path and guide the reader along gently. The column had and still has enormous influence in the mathematics community of North America and beyond.

Unfortunately, Martin Gardner has retired. His column was replaced briefly by "Metamagical Themas" (an anagram of "Mathematical Games"). A. K. Dewdney's "Computer Recreations" now occupies that distinguished spot in *Scientific American*.

Fortunately, anthologies of Martin Gardner's columns have appeared regularly. To date, 12 volumes are in print, with enough columns left over for at least five more books. If a school library can afford only one set of books on popular mathematics, this is the one!

The Scientific American Book of Mathematical Puzzles and Diversions, 1959, Simon & Schuster.

Topics covered are hexaflexagons, magic with a matrix, ticktacktoe, probability paradoxes, the icosian game and the Tower of Hanoi, curious topological models, the game of hex, Sam Loyd: America's greatest puzzlist, mathematical card tricks, memorizing numbers, polyominoes, fallacies, nim and tac tix, left or right, as well as two collections of short problems. Here's an example.

Problem 1

An old riddle runs as follows. An explorer walks one mile due south, turns and walks one mile due east, turns again and walks one mile due north. He finds himself back where he started. He shoots a bear. What color is the bear? The time-honored answer is: "white," because the explorer must have started at the North Pole. But not long ago someone discovered that the North Pole is not the only starting point that satisfies the given conditions! Can you think of any other spot on the globe from which one can walk a mile south, a mile east, a mile north and find himself back at his original location?

The 2nd Scientific American Book of Mathematical Puzzles and Diversions, 1961, Simon & Schuster.

Topics covered are the five Platonic solids, tetraflexagons, Henry Ernest Dudeney: England's greatest puzzlist, digital roots, the soma cube, recreational topology, phi—the golden ratio, the monkey and the coconuts, mazes, recreational logic, magic squares, James Hugh Rilet Shows Inc., eleusis: the induction game, origami, squaring the square, mechanical puzzles, probability and ambiguity, as well as two collections of short problems.

Problem 2

Two missiles speed directly toward each other, one at 9000 miles per hour and the other at 21000 miles per hour. They start 1317 miles apart. Without using pencil and paper, calculate how far apart they are one minute before they collide.

Martin Gardner's New Mathematical Diversions from Scientific American, 1966, Simon & Schuster.

Topics covered are the binary system, group theory and braids, the games and puzzles of Lewis Carroll, paper cutting, board games, packing spheres, the transcendental number pi, Victor Eigen: mathemagician, the four-color map theorem, Mr. Apollinax visits New York, polyominoes and fault-free rectangles, Euler's spoilers: the discovery of an order-10 Graeco-Latin square, the ellipse, the 24 color-squares and the 30-color cubes, H. S. M. Coxeter, bridg-it and other games, the calculus of finite differences, as well as three collections of short problems.

Problem 3

One morning, exactly at sunrise, a Buddhist monk began to climb a tall mountain. The narrow path, no more than a foot or two wide, spiraled around the mountain to a glittering temple at the summit. The monk ascended the path at varying rates of speed, stopping many times along the way to rest and eat the dried fruit he carried with him. He reached the temple shortly before sunset. After several days of fasting and meditation he began his journey back along the same path, starting at sunrise and again walking at variable speeds with many pauses along the way. His average speed descending was, of course, greater than his average climbing speed. Prove that there is a spot along the path that the monk occupies on both trips at precisely the same time of day.

The Magic Numbers of Dr. Matrix, 1985, Prometheus Books.

This is different from any other book in this series. It centres around a mystic character, Dr. Irving Joshua Matrix, a professional numerologist. Dr. Matrix columns appear to be Martin Gardner's own favorite, as he returned to them periodically until the good doctor's untimely demise in the last episode.

Problem 4

```
  FORTY
    TEN
+   TEN
-------
  SIXTY
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"Each letter in that addition problem stands for a different digit," Dr. Matrix explained. "There's only one solution, but it takes a bit of brain work to find it."

The Unexpected Hanging and Other Mathematical Diversions, 1969, Simon & Schuster.

Topics covered are the paradox of the unexpected hanging, knots and Borromean rings, the transcendental number *e*, geometric dissections, Scarne on gambling, the church of the fourth dimension, a matchbox game-learning machine, spirals, rotations and reflections, peg solitaire, flatlands, Chicago Magic Convention, tests of divisibility, the eight queens and other chessboard diversions, a loop of string, curves of constant width, reptiles: replicating figures on the plane, as well as three collections of short problems.

Problem 5

Six Hollywood stars form a social group that has very special characteristics. Every two stars in the group either mutually love each other or mutually hate each other. There is no set of three individuals who mutually love one another. Prove that there is at least one set of three individuals who mutually hate one another.

Martin Gardner's 6th Book of Mathematical Diversions from Scientific American, 1983, University of Chicago Press.

Topics covered are the helix, Klein bottles and other surfaces, combinatorial theory, bouncing balls in polygons are polyhedra, four unusual board games, sliding-block puzzles, parity checks, patterns and primes, graph theory, the ternary system, the cycloid: Helen of geometry, mathematical magic tricks, word play, the Pythagoras Theorem, limits of infinite series, polyiamonds, tetrahedra, the lattice of integers, infinite regress, O'Gara the mathematical mailman, extraterrestrial communication, as well as three collections of short problems.

Problem 6

An oil well being drilled in flat prairie country struck pay sand at an underground spot exactly 21000 feet from one corner of a rectangular plot of farmland, 18000 feet from the opposite corner and 6000 feet from a third corner. How far is the underground spot from the fourth corner?

Mathematical Carnival, 1977, Vintage Books.

Topics covered are sprouts and Brussels sprouts, penny puzzles, aleph-null and aleph-one, hypercubes, magic stars and polyhedra, calculating prodigies, tricks of lightning calculators, the art of M. C. Escher, card shuffles, Mrs. Perkin's quilt and other square-packing problems, the numerology of Dr. Fliess, random numbers, the rising hourglass and other physics puzzles, Pascal's Triangle, jam, hot and other games, cooks and quibble-cooks, Piet Hein's super-ellipse, how to trisect an angle, as well as one collection of short problems.

Problem 7

An infinity of non-touching points lies inside a closed curve. Assume that a million of those points are selected at random. Will it always be possible to place a straight line on the plane so that it cuts across the curve, misses every point in the set of a million and divides the set exactly in half so that 500000 points lie on each side of the line? The answer is yes; prove it.

Mathematical Magic Show, 1978, Vintage Books

Topics covered are nothing, game theory, guess-it and foxholes, factorial oddities, double acrostics, playing cards, finger arithmetic, Mobius bands, polyhexes and polyaboloes, perfect, amicable and sociable, polyominoes and rectification, knights of the square table, colored triangles and cubes, trees, dice, everything, as well as three collections of short problems.

Problem 8

A telephone call interrupts a man after he has dealt about half of the cards in a bridge game. When he returns to the table, no one can remember where he had dealt the last card. Without learning the number of cards in any of the four partly dealt hands, or the number of cards yet to be dealt, how can he continue to deal accurately, everyone getting exactly the same cards he would have had if the deal had not been interrupted?

Mathematical Circus, 1981, Vintage Books

Topics covered are optical illusions, matches, spheres and hyperspheres, patterns of induction, elegant triangles, random walks and gambling, random walks on the plane and in space, Boolean algebra, can machines think?, cyclic numbers, dominoes, Fibonacci and Lucas numbers, simplicity, solar system oddities, Mascheroni constructions, the abacus, palindromes—words and numbers, dollar bills, as well as two collections of short problems.

Problem 9

You have six weights. One pair is red, one pair white, one pair blue. In each pair one weight is a trifle heavier than the other but otherwise appears to be exactly like its mate. The three heavier weights (one of each color) all weigh the same. This is also true of the three lighter weights. In two separate weighings on a balance scale, how can you identify which is the heavier weight of each pair?

Wheels, Life and Other Mathematical Amusements, 1983, W. H. Freeman.

Topics covered are wheels, Diophantine analysis and Fermat's Last Theorem, alephs and supertasks, nontransitive dice and other probability paradoxes, geometrical fallacies, the combinatorics of paper folding, ticktacktoe games, plaiting polyhedra, the game of Halma, advertising premiums, Salmon on Austin's dog, nim and hackenbush, Golomb's graceful graphs, chess tasks, slither, $3x + 1$ and other curious questions, mathematical tricks with cards, the game of life, as well as three collections of short problems.

Problem 10

Make a statement about n that is true for, and only true for, all values of n less than one million.

Knotted Doughnuts and Other Mathematical Entertainments, 1986, W. H. Freeman.

Topics covered are coincidence, the binary Gray code, polycubes, Bacon's cipher, doughnuts: linked and knotted, Napier's bones, Napier's abacus, sim, chomp and racetrack, elevators, crossing numbers, point sets on the sphere, Newcomb's paradox, look-see proofs, worm paths, Waring's problems, cram, bynum and quadraphage, the I Ching, the Laffer curve, as well as two collections of short problems.

Problem 11

My wife and I recently attended a party at which there were four other couples. Various handshakes took place. No one shook hands with himself or herself or with his or her spouse, and no one shook hands with the same person more than once. After all the handshakes were over, I asked each person, including my wife, how many hands he or she had shaken. To my surprise each gave a different answer. How many hands did my wife shake?

Time Travel and Other Mathematical Bewilderments, 1988, W. H. Freeman.

Topics covered are time travel, hexes and stars, tangrams, nontransitive paradoxes, combinatorial card problems, melody-making machines, anamorphic art, six sensational discoveries, the Csaszar polyhedron, dodgem and other simple games, tiling with convex polygons, tiling with polyominoes, polyiamonds and polyhexes, curious maps, magic squares and cubes, block packing, induction and probability, Catalan numbers, fun with a pocket calculator, tree-planting problems, as well as two collections of short problems.

Problem 12

A cake has been baked in the form of a rectangular parallelepiped with a square base. Assume that the square cake is frosted on the top and four sides and that the frosting's thickness is negligible. We want to cut the cake into seven pieces so that each piece has the same volume and the same area of frosting. The slicing is conventional. Seen from above, the cuts are like spokes radiating from the square's centre, and each cutting is perpendicular to the cake's base. How can we locate the required seven points on the perimeter of the cake's top?

B. Raymond Smullyan's Logic Series

While this series deals only with logic, it more than compensates by its tremendous depth. While readers may find parts of the books in this series difficult, they will not find them difficult to read. Each book takes the form of a series of logic puzzles, presented in very attractive settings. The reader can reasonably expect to have success with the earlier ones. As confidence increases, the reader will discover the skill of the author in paving a smooth path towards some very important results in logic and mathematics, particularly those associated with the name Kurt Gödel.

What is the Name of This Book?, 1978, Prentice-Hall.

A distinguishing feature of Raymond Smullyan's logic puzzles is that not all of the information provided is to be taken at its face value. This is exemplified by his favorite characters, the knights who always tell the truth and the knaves who always lie. Before one can utilize a statement made by one of them, it is important to know if it is made by a knight or a knave. Very often the clues are contained in the statement itself.

The knights and knaves made their debut in this wonderful book (what is its name again?), along with other denizens from Alice (of Wonderland fame) to Count Dracula. In solving numerous intriguing logic puzzles, the reader gets an enjoyable lesson in propositional logic, an introduction to some logical paradoxes and curiosities and a proof of a form of Gödel's famous Incompleteness Theorem.

Problem 13

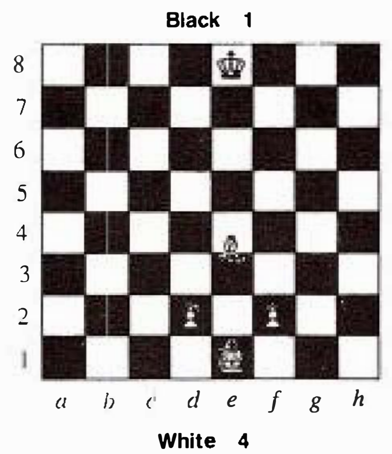
Each of A, B and C is either a knight or a knave. A stranger asks A, "How many knights are among you?" A answers indistinctly. So the stranger asks B, "What did A say?" B replies, "A said that there is exactly one knight among us." Then C says, "Don't believe B; he is lying!" What are B and C?

The Chess Mysteries of Sherlock Holmes, 1979, Alfred A. Knopf.

This and the next volume are quite different from the other books in the series in that logic is exercised over the chessboard. The reader needs to know the rules of chess, but being a good player is not essential. In fact, this is more often a handicap, as the moves that are made in the games in these books, though perfectly legal, are hardly what one would describe as good moves. The object is not to win but to deduce the past history of a game, based on the current position and possibly some additional information. In this volume, Watson serves as the narrator, with Sherlock Holmes as the chessboard detective.

Problem 14

A white bishop was placed equally between the squares *e*3 and *e*4. Thinking this was an oversight, I was about to move it when Holmes stopped me. "No, no, Watson. That's precisely the problem! On which square, *e*3 or *e*4, stands the bishop, given that in this game, no piece or pawn has ever moved from a white square to a black square, nor from a black square to a white square?"



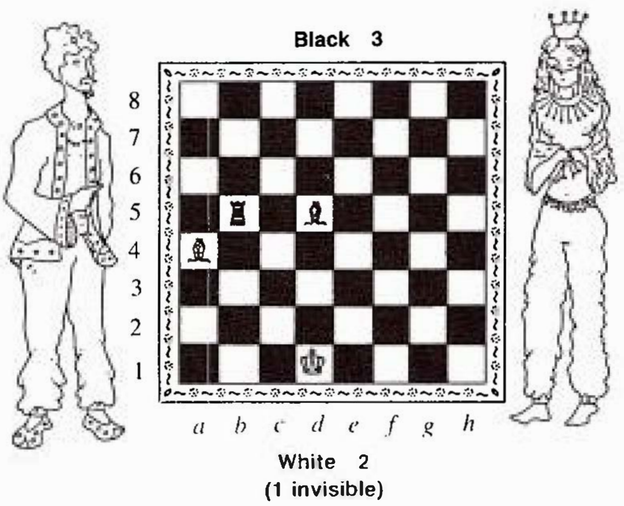
Problem 14

The Chess Mysteries of the Arabian Knights, 1981, Alfred A. Knopf.

In this volume, the characters are the White King Haroun Al Rashid and his entourage and the opposing camp headed by the black King Kazir. The setting is that of the "Tales of the Arabian Nights." The style of the narrative and the charming illustrations by Greer Fitting add to the authenticity.

Problem 15

Haroun Al Rashid—Ruler of the Faithful—had gathered from sorcerers all over the world many secrets of magic. One of his favorite tricks was the art of invisibility. So here Haroun is, standing in broad daylight, on one of the 64 squares of the enchanted chess kingdom. But nobody can see him for the simple reason that he is invisible. On what square does he stand?



Problem 15

The Lady or the Tiger?, 1982, Alfred A. Knopf.

Although the knights and knaves make only a brief appearance in the first part of this book, the other characters are unmistakably knight-like or knave-like, though with fascinating variations. The reader is also introduced to meta-puzzles, or puzzles about puzzles. The second half of the book is a novelty called a "mathematical novel." The step-by-step unveiling of the "Mystery of the Monte Carlo Lock" is an absorbing study in combinatorial logic.

Problem 16

Each of two rooms contained either a lady or a tiger. A sign on the door of the first room read, "In this room there is a lady, and in the other room there is a tiger." A sign on the door of the second room read, "In one of these rooms there is a lady, and in one of these rooms there is a tiger." One of the signs was true but the other one was false. It was possible that both rooms contained ladies or both rooms contained tigers. Which room should be chosen in order to get a lady?

Alice in Puzzle-Land, 1982, William Morrow.

This book celebrates the 150th anniversary of the birth of Lewis Carroll, of whom Raymond Smullyan has been described as a modern version. In this volume, Alice and other Carrollian characters are reunited to entertain the reader with logic and meta-logic puzzles. There are also some elementary mathematical problems. The illustrations by Greer Fitting are simply gorgeous.

Problem 17

Half the creatures are totally mad, meaning that everything true they believe to be false and everything false they believe to be true. The other half are totally sane, meaning that everything true they know to be true and everything false they know to be false. "There's the Cook and the Cheshire Cat," said the Duchess. "The Cook believes that at least one of the two is mad." What can be deduced about the Cook and the Cheshire Cat?

To Mock a Mockingbird, 1985, Alfred A. Knopf.

The first third of this book consists of the basic knight-knave type of puzzles and more meta-puzzles. The remaining part takes the reader on another tour of the realm of combinatorial logic. This second "mathematical novel" shares several common characters with the "Mystery of the

Monte Carlo Lock'' while adding many more, most of which are birds that can talk.

Problem 18

A certain enchanted forest is inhabited by talking birds. Given any birds A and B, if you call out the name of B to A, then A will respond by calling out the name of some bird to you; this bird we designate AB. Given any birds A and B, there is a bird C such that for every bird x, C's response to x is equal to A's response to B's response to x; in other words, $Cx = A(Bx)$. Prove that for any birds A, B and C, there is a bird D such that for every bird x, $Dx = A\ (B(Cx))$.

Forever Undecided, 1987, Alfred A. Knopf.

This is the most challenging yet of Raymond Smullyan's books on logic puzzles. After a review of propositional logic, the reader is gradually introduced to the subject of modal logic, where the principal notions are that of a proposition's being possibly true as opposed to being necessarily true.

Problem 19

Two prizes are offered. If you make a true statement, I will give you at least one of the two prizes and possibly both. If you make a false statement, you get no prizes. Suppose you are ambitious and wish to win both prizes. What statement would you make?

C. Oxford University Press Series on Recreations in Mathematics

This series is edited by David Singmaster of the Polytechnic of the South Bank in London. He is well known for his writing on popular mathematics and is the leading expert in the history of the subject. The four volumes published so far are new, but the series may contain translations and reprints of classic works.

Mathematical Byways: In Ayling, Beeling and Ceiling, by Hugh ApSimon, 1984.

This book contains eleven chapters, each built around a central problem, with solutions and generalizations. Each problem is in the form of an event in the three villages named in the title, though there is occasional wandering off to Dealing and beyond. The titles of the problems are ladder-box, meta-ladder-box, complete quadrilateral, bowling (as in cricket) averages, centre-point, counting sheep, transport, alley ladder, counterfeit coins, wrapping a parcel and sheepdog trials.

Problem 20

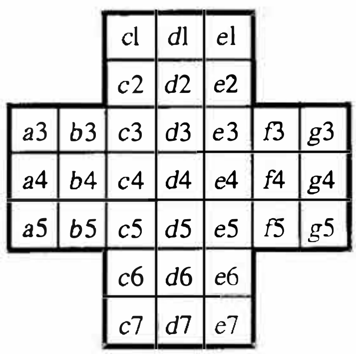
Beeling is rather a quiet village—except on the day of the annual sheep market. On that day, lines of hurdles are erected from the flagpole to each of the corners of the market place (a quadrilateral, not necessarily convex) and along three of its four sides, making three triangular pens of different sizes. Each hurdle is one metre long, there are no overlaps or gaps and no hurdle is bent or broken. Each pen is filled with sheep—one sheep to each square metre. The number of sheep in each pen is equal to the number of hurdles surrounding that pen. What is the area of the Beeling market place?

The Ins and Outs of Peg Solitaire, by John D. Beasley, 1985.

This book deals mainly with peg solitaire on the standard ''English'' board with 33 holes, each of which can hold one man. Each move consists of a jump by one man over an adjacent one onto an empty hole or a continuous sequence of such jumps. The normal objective is to convert a starting configuration into a target position with fewer men (as those jumped over are removed). The book presents a balanced treatment of the mathematical ideas behind the puzzle as well as the actual techniques in solving it.

Problem 21

A marked man is put in hole $d7$. All other holes except $d2$ are occupied by unmarked men. After many moves, only the marked man remains on the board. In which hole will he be?



Problem 21

Rubik's Cube Compendium, by E. Rubik, T. Varga, G. Kéri, G. Marx, T. Vekerdy, 1986.

This book consists of six articles and an informative update by David Singmaster. The lead article is written by the inventor of this mathematical phenomenon, giving some insight into how the idea was originally conceived. The other authors are Rubik's compatriots and their articles deal with the mathematics and the techniques of "cubing." The book has many brightly colored illustrations.

Problem 22

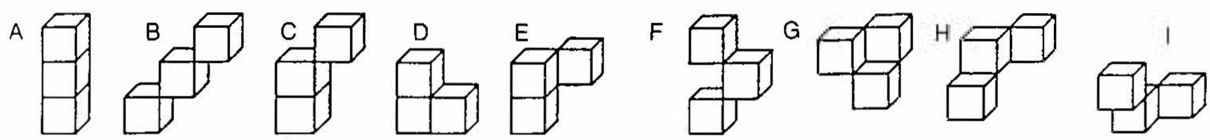
This is Rubik's version of the soma cube. Use the nine pieces (in some of them, certain cubes are only connected along edges) to construct a 3 by 3 by 3 cube. (The pieces are designated by letters for the purpose of the solution in Appendix V.)

Sliding Piece Puzzles, by L. E. Hordern, 1986.

The classic example of a sliding piece puzzle is the 14-15 puzzle of Sam Loyd. This and 271 other puzzles are catalogued by class in this book. In almost all cases, there is enough information for home-made copies to be constructed. There is also a colorful history of the subject.

Problem 23

In a 3 by 3 chessboard, two white knights occupy adjacent corner cells and two black knights occupy the other two corner cells. No captures are allowed. Each knight moves as in a normal game of chess, except that his move is not over until he chooses to stop. The white knights are to trade places with the black knights. Find a seven-move solution.



Problem 22

D. The Dolciani Mathematical Expositions Series of the Mathematical Association of America

The books in this series are selected both for their clear, informal style and stimulating mathematical content. Some are collections of articles and problems while others deal with specific topics. Each has an ample supply of exercises, many with accompanying solutions.

Mathematical Gems I, by Ross Honsberger, 1973.

This book presents 13 articles from elementary combinatorics, number theory and geometry. Topics include combinatorial geometry, recurrence relations, Hamiltonian circuits, perfect numbers, primality testing, Morley's Theorem and a story about a Hungarian prodigy, Louis Posa.

Problem 24

Prove that if you have $n + 1$ distinct positive integers less than or equal to $2n$, some pair of them are relatively prime.

Mathematical Gems II, by Ross Honsberger, 1976.

This book presents 14 articles from elementary combinatorics, number theory and geometry. Topics include combinatorial geometry, box-packing problems, Fibonacci sequence, Hamiltonian circuits, Tutte's Theorem, linear Diophantine equations, the generation of prime numbers, the harmonic series, the isosceles tetrahedron and inversion.

Problem 25

Can 250 copies of a 1 by 1 by 4 block be packed into a 10 by 10 by 10 box?

Mathematical Morsels, by Ross Honsberger, 1978.

This book contains 91 elegant problems and 25 exercises. Most are taken from the problem sections of various journals, in particular, the *American Mathematical Monthly* and the *Mathematics Magazine.*

Problem 26

There are more chess masters in New York City than in the rest of the United States combined. A chess tournament is planned to which all American masters are expected to come. It is agreed that the tournament should be held at the site which minimizes the total inter-city travelling done by the contestants. The New York masters claim that, by this criterion, the site chosen should be their city. The West Coast masters argue that a city at or near the centre of gravity of the players would be better. Where should the tournament be held?

Mathematical Plums, edited by Ross Honsberger, 1979.

This book contains two articles by Honsberger and eight more by others. The respective titles are "Some surprises in probability," "Kepler's conics," "Chromatic graphs," "How to get (at least) a fair share of the cake," "Some remarkable sequences of integers," "Existence out of chaos," "Anomalous cancellation," "A distorted view of geometry," "Convergence, divergence and the computer" and "The Skewes number."

Problem 27

Two red cards and two black cards are shuffled and dealt face down in a row. Two of them are selected at random. What is the probability that they are the same color?

Great Moments in Mathematics (Before 1650), by Howard Eves, 1980.

This is a history of mathematics before 1650 presented in 20 lectures, each highlighted by one of what the author considers a great moment. One such great moment is Euclid's Elements. Although the text is a condensed version of the author's presentation, it retains much of the vitality and smoothness of a gifted lecturer.

Problem 28

Let p and q be positive numbers with $q \leq p/2$. Segments of lengths p and q respectively are given. Construct by Euclidean means segments of lengths r and s respectively where r and s are the roots of the equation $x^2 - px + q^2 = 0$.

Maxima and Minima Without Calculus, by Ivan Niven, 1981.

The central result considered in this book is the Isoperimetric Theorem, which states that, among all figures of fixed perimeter, the circle has the greatest area. It is a problem which is not easy to handle using standard techniques in calculus. After an introduction to inequalities, in particular, the Arithmetic-Mean-Geometric-Mean Inequality and Jensen's Inequality, these elementary tools are applied to various maxima and minima problems. Several related topics are also discussed.

Problem 29

Describe the shortest path across an equilateral triangle to bisect the area.

Great Moments in Mathematics (After 1650), by Howard Eves, 1981.

This is a history of mathematics after 1650 presented in 20 lectures. As in its companion volume, each lecture is centred around a great moment in mathematics. Here, the choice of topics, by the author's own admission, is more difficult. One of the great moments chosen is the invention of differential calculus. Another is the impact of computers and the resolution of the four-color conjecture.

Problem 30

Given a point F and a line l, the locus of a point P equidistant from F and l is a parabola. Construct by Euclidean means the tangent to this parabola at a point P on it.

Map Coloring, Polyhedra and the Four-Color Problem, by D. Barnette, 1983.

The central result considered in this book is the Four-Color Theorem, but only one of eight chapters is devoted to the computer-assisted proof by Appel and Haken. After giving an early history of the problem, the book discusses many related concepts and results, including Euler's Formula, Hamiltonian circuits and convex polyhedra. Map coloring on other surfaces is also considered.

Problem 31

A convex polyhedron is said to be combinatorially regular if each face has the same number of sides and each vertex is the endpoint of the same number of sides. These two numbers may be different. Prove that there are only five types of such polyhedra.

Mathematical Gems III, by Ross Honsberger, 1985.

This book presents 18 articles from elementary combinatorics, number theory and geometry. Topics include combinatorial geometry, generating functions, Fibonacci and Lucas numbers, probability, Ramsay's Theorem, cryptography, Helly's Theorem and a selection of problems from various olympiads.

Problem 32

Given a square grid S containing 49 points in seven rows and seven columns, a subset T consisting of k points is selected. What is the maximum value of k such that no four points of T determine a rectangle with sides parallel to the sides of S?

E. The New Mathematical Library Series of the Mathematical Association of America

This series is written by professional mathematicians with the high school student in mind. The books cover topics which are not usually included in high school curricula, but are nevertheless not too far removed from classroom mathematics. Each volume contains numerous exercises with answers.

Numbers: Rational and Irrational, by Ivan Niven, 1961.

The book begins with a review of elementary number theory and the basic properties of rational numbers. It then goes on to prove that irrational numbers exist, with explicit examples. Algebraic and transcendental numbers are then introduced. There is a discussion of the impossibility of the three classical problems in geometric construction, as well as the problem of approximating irrational numbers by rational numbers.

Problem 33

Prove that sin 10° is irrational.

What Is Calculus About, by W. W. Sawyer, 1961.

The book uses a practical example to introduce the reader to the subject of calculus. From the consideration of distance, velocity and acceleration as functions of time, the concepts and techniques of differential calculus emerge. There is a brief discussion of integral calculus towards the end of the book.

Problem 34

Prove that the volume of a sphere of radius r is given by $4\pi r^3/3$.

An Introduction to Inequalities, by E. F. Beckenbach and R. Bellman, 1961.

The heart of this book lies in the fourth chapter where classical inequalities are discussed. These include the Arithmetic-Mean-Geometric-Mean Inequality, Cauchy's Inequality, Hölder's Inequality, the Triangle Inequality and Minkowski's Inequality. There is a brief discussion of basic properties of inequalities and absolute values in the earlier chapters. The classical inequalities are later applied to maxima and minima problems.

Problem 35

Prove that among all triangles with fixed perimeter, the equilateral triangle has the greatest area.

Geometric Inequalities, by N. D. Kazarinoff, 1961.

The book covers the Arithmetic-Mean-Geometric-Mean Inequality and the Isoperimetric Theorem. It emphasizes the method of reflection in solving maxima and minima problems.

Problem 36

Let P be any point inside triangle ABC. Let P_a, P_b and P_c denote the respective distances from P to BC, CA and AB. Prove that $PA + PB + PC \geq 2(p_a + p_b + p_c)$, with equality if and only if ABC is an equilateral triangle and P is its centre.

The Contest Problem Book I, by C. T. Salkind, 1961.

The book contains a reprint of the American High School Mathematics Examinations from 1950 to 1960. Each paper consists of 50 multiple choice questions except for 1960 where there are only 40 questions. An answer key is provided, followed by complete solutions.

Problem 37

For $x^2 + 2x + 5$ to be a factor of $x^4 + px^2 + q$, the values of p and q must be, respectively—
(a) $-2,5$, (b) 5, 25, (c) 10, 20, (d) 6, 25, (e) 14, 25.

The Lore of Large Numbers, by P. J. Davis, 1961.

The book presents a fascinating account of the notations and techniques in computation and approximation. The large numbers serve as a binding theme in the discussion. It starts with fairly basic material and a historical background, which later leads to glimpses of more advanced mathematics.

Problem 38

Prove that 12078521834 is not a square.

Uses of Infinity, by Leo Zippin, 1962.

Starting from an account of the popular notion of infinity, the book leads the reader on to mathematical treatments of sequences and series, limit and convergence, irrational numbers and their approximation, as well as countability and cardinal numbers. The golden ratio is featured as a detailed example.

Problem 39

Prove that the sum of any finite number of terms in the series $1/1^2 + 1/2^2 + 1/3^2 + 1/4^2 + \cdots$ is strictly less than 2.

Geometric Transformations I, by I. M. Yaglom, 1962.

The book discusses Euclidean geometry from the transformation point of view. Only distance-preserving transformations or isometries are considered, and these are classified into translations, rotations, reflections and glide reflections.

Problem 40

Suppose that two chords AB and CD are given in a circle together with a point J on the chord CD. Find a point X on the circle such that the chords AX and BX cut off on the chord CD a segment whose midpoint is J.

Continued Fractions, by C. D. Olds, 1963.

The book begins with a definition of continued fractions and shows that finite continued fractions are equivalent to rational numbers. These continued fractions are used to solve linear Diophantine equations. To represent irrational numbers, infinite continued fractions are introduced. It is then shown that periodic continued fractions are equivalent to quadratic irrationals. These continued fractions are then used to solve Pell's equation.

Problem 41

Express 68/77 as the sum of two fractions whose denominators are 7 and 11, respectively.

Graphs and Their Uses, by Oystein Ore, 1963.

This is an excellent introduction to the theory of graphs, covering connected graphs, trees, directed graphs, planar graphs, map coloring and matchings.

Problem 42

Each of four neighbors has connected his house with the other three houses by paths which do not cross. A fifth man builds a house nearby. Prove that he cannot connect his house with all the others by non-intersecting paths.

Hungarian Problem Book I, edited by G. Hajós, G. Neukomm and J. Surányi, 1963.

Hungary has probably the longest and strongest tradition in mathematics contests for high school students. This volume collects the papers from 1894 to 1905 inclusive, with detailed solutions.

Problem 43

Prove that the expressions $2x + 3y$ and $9x + 5y$ are divisible by 17 for the same set of integral values of x and y.

Hungarian Problem Book II, edited by G. Hajós, G. Neukomm and J. Surányi, 1963.

This volume collects the papers of the Hungarian contests from 1906 to 1928. Contest activities were interrupted in 1919, 1920 and 1921 as an aftermath of World War I.

Problem 44

Prove that the product of four consecutive positive integers cannot be the square of an integer.

Episodes from the Early History of Mathematics, by A. Aaboe, 1964.

The book consists of four chapters, one on Babylonian mathematics and three on Greek mathematics, the latter centred around Euclid, Archimedes and Ptolemy.

Problem 45

Let AOB be any given angle. Let $OA = OB = 1$. Extend OA to C with $OC = OA$ and draw a semicircle with diameter AC and passing through B. Draw a line through B intersecting the semicircle at D and the extension of OC at E such that $DE = 1$. Prove that $\measuredangle BEA$ is equal to one-third of $\measuredangle BOA$.

Groups and Their Graphs, by I. Grossman, 1964.

The book introduces the reader to the abstract algebraic concept of groups via many concrete examples. The graphs of the groups come into the picture when the groups are defined by generators and relations. There is a discussion of the result that there are 17 essentially different wallpaper patterns.

Problem 46

Consider the set $\{1,2,3,\ldots,p-1\}$, p a prime number. Prove that for any element x of the set, there is an element y of the set such that $xy \equiv 1 \pmod{p}$.

Mathematics of Choice, by Ivan Niven, 1965.

As its subtitle "How to Count without Counting" suggests, this book deals with counting techniques. Starting from the basic ideas of permutations, combinations and the Binomial Theorem, it leads the reader on to more sophisticated topics such as the Principle of Inclusion-Exclusion, generating functions, recurrence relations, along with many applications. There is a brief discussion of mathematical induction and the Pigeonhole Principle.

Problem 47

At formal conferences of the Supreme Court, each of the nine judges shakes hands with each of the others at the beginning of the session. How many handshakes initiate such a session?

From Pythagoras to Einstein, by K. O. Friedrichs, 1965.

The book discusses Pythagoras' Theorem and the concept of vectors in various mathematical and mechanical settings, leading eventually to the roles they play in the theory of relativity. Unlike other titles in this series, there are no exercises.

Problem 48

Let a and b be the lengths of the legs of a right triangle and let c be the length of its hypotenuse. Find a tiling of the plane using squares of sides a and b and a tiling of the plane using squares of side c. Superimpose the two tilings to obtain a proof of Pythagoras' Theorem: $a^2 + b^2 = c^2$.

The Contest Problem Book II, by C. T. Salkind, 1966.

The book contains a reprint of the American High School Mathematics Examinations from 1961 to 1965. Each paper consists of 40 multiple choice questions. An answer key is provided, followed by complete solutions.

Problem 49

If $5x + 12y = 60$, then the minimum value of $\sqrt{x^2 + y^2}$ is—
(a) 60/13, (b) 13/5, (c) 13/12, (d) 1, (e) 0.

First Concepts of Topology, by W. G. Chinn and N. E. Steenrod, 1966.

The first chapter in this book introduces the reader to point-set topology, leading to the important result in analysis: the Bolzano-Weierstrass Theorem. The second chapter involves concepts of algebraic topology from which the fundamental theorem of algebra is deduced.

Problem 50

Given a circular pancake and a second pancake which is irregular in shape, prove that there is a straight line which simultaneously cuts each pancake in half.

Geometry Revisited, by H. S. M. Coxeter and Samuel Greitzer, 1967.

This is an excellent review of Euclidean geometry, dealing with points and lines connected with a triangle, some properties of circles, collinearity and concurrence and transformations. There is also an introduction to inversive geometry and to projective geometry.

Problem 51

Prove that, if a quadrilateral is inscribed in a circle, the sum of the products of the two pairs of opposite sides is equal to the product of the diagonals.

Invitation to Number Theory, by Oystein Ore, 1967.

This is an excellent introduction to number theory, covering concepts such as divisibility, primes, greatest common divisors, congruence, Diophantine equations and numeration systems.

Problem 52

Find the smallest positive integer with exactly 100 positive divisors.

Geometric Transformations II, by I. M. Yaglom, 1968.

The book adds similarity transformations to isometries and gives many applications of the transformation approach to geometric problems.

Problem 53

Inscribe a square in a given triangle ABC so that two vertices lie on the base AB, and the other two lie on the sides AC and BC, respectively.

Elementary Cryptanalysis: A Mathematical Approach, by Abraham Sinkov, 1968.

The book gives a mathematical approach to the popular topic of secret codes. There are discussions of various systems of codes, mostly based on modular arithmetic. Methods of cracking these codes without the knowledge of the key are also given.

Problem 54

The secret message "FRZDUGV GLH PDQB WLPHV EHIRUH WKHLU GHDWKV" is obtained from the original message by shifting each letter a fixed number of places. What is the original message?

Ingenuity in Mathematics, by Ross Honsberger, 1970.

The book consists of 19 essays from elementary combinatorics, number theory and geometry. Topics include Sylvester's problem, the Isoperimetric problem, the Theorem of Barbier, probability and π, the Farey series, complementary sequences, abundant numbers, squaring the square and the construction problems of Mascheroni and Steiner.

Problem 55

A positive integer n is said to be abundant if the sum of all its positive divisors exceeds $2n$. Prove that all multiples of abundant numbers are abundant.

Geometric Transformations III, by I. M. Yaglom, 1973.

The book takes the reader beyond Euclidean geometry to affine and projective geometries, again discussed from the transformational point of view. There is a supplement on hyperbolic geometry.

Problem 56

Given a segment AB and another line parallel to AB, use a straight-edge only to construct the midpoint of AB.

The Contest Problem Book III, by C. T. Salkind and J. M. Earl, 1973.

The book contains a reprint of the American High School Mathematics Examinations from 1966 to 1972. Each of the first two papers consists of 40 multiple choice questions, the number dropping to 35 for subsequent papers. An answer key is provided, followed by complete solutions.

Problem 57

Three times Dick's age plus Tom's age equals twice Harry's age. Double the cube of Harry's age is equal to three times the cube of Dick's age added to the cube of Tom's age. Their respective ages are relatively prime to each other. The sum of the squares of their ages is—
(a) 42, (b) 46, (c) 122, (d) 290, (e) 326.

Mathematical Methods in Science, by G. Pólya, 1976.

A distinguished mathematician and scientist illustrates the applications of various mathematical concepts such as measurement, successive approximation, vectors, and differential equations in astronomy, statics and dynamics.

Problem 58

Newton's method of finding approximate values for $\sqrt{a}$ may be described as follows. If $x = \sqrt{a}$, then $x^2 = a$. If $x \neq \sqrt{a}$, then $xy = a$ for some y. One of x and y will be greater than $\sqrt{a}$ while the other is less. Thus $(x + y)/2$ will be a better approximation of $\sqrt{a}$. Use Newton's method and $x = 2$ as an initial guess to find an approximation of $\sqrt{2}$ to two decimal places.

International Mathematical Olympiads, 1959-1977, by Samuel Grietzer, 1978.

The International Mathematical Olympiad began in 1959 in Romania as an all East-European affair. It has since grown to truly international proportions. This book contains the papers of the first 19 Olympiads and their solutions.

Problem 59

Prove that the fraction $(21n + 4)/(14n + 3)$ is irreducible for every positive integer n.

The Mathematics of Games and Gambling, by E. W. Packel, 1981.

The book treats the subject of probability in a gambling setting, discussing various games of dice and cards such as backgammon, craps, poker and bridge. Basic concepts such as permutations, combinations, the binomial distribution and mathematical expectation are covered. There is also a chapter on elementary game theory.

Problem 60

Does one have an even chance of getting at least one 6 in three rolls of an honest die?

The Contest Problem Book IV, by R. A. Artino, A. M. Gaglione and N. Shell, 1982.

The book contains a reprint of the American High School Mathematics Examinations from 1973 to 1982. Each paper consists of 30 multiple choice questions except for 1973 where there are 35 questions. An answer key is provided, followed by complete solutions.

Problem 61

The units digit in the decimal expansion of $(15 + \sqrt{220})^{19} + (15 + \sqrt{220})^{82}$ is—
(a) 0, (b) 2, (c) 5, (d) 9, (e) none of these.

The Role of Mathematics in Science, by M. M. Shiffer and Leon Bowden, 1984.

The book contains seven chapters, dealing with the beginnings of mechanics, growth functions, the role of mathematics in optics, mathematics with matrices, transformations, Einstein's space-time transformation problem, relativistic addition of velocities and energy.

Problem 62

A hyperbolic mirror is in the shape of one branch of a hyperbola, with the reflecting surface on the convex side. Prove that if a light source is placed at the focus behind the other branch of the hyperbola, the reflected rays will appear to have originated from the focus behind the mirror.

International Mathematical Olympiads, 1978-1985, by M. S. Klamkin, 1986.

This book contains the International Mathematical Olympiad papers for the years indicated, except for 1980 when the Olympiad was not held. In addition to complete solutions, there are also 40 supplementary problems. Professor Klamkin of the University of Alberta is the acknowledged world authority on problem-solving.

Problem 63

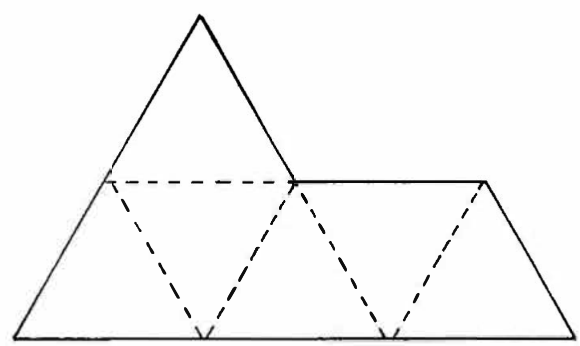
Let $1 \leq r \leq n$ and consider all subsets of r elements of the set $\{1,2,\ldots,n\}$. Each of these subsets has a smallest member. Prove that the arithmetic mean of these smallest numbers is equal to $(n + 1)/(r + 1)$.

Riddles of the Sphinx, by Martin Gardner, 1987.

This is an anthology of Martin Gardner's contributions to Isaac Asimov's *Science Fiction Magazine*. There are 36 puzzles in science fictional settings. Often, when the "first" answers are given, further questions arise, to be answered in a "second" section. "Third" answers and "fourth" answers pursue the matter even further.

Problem 64

Divide the sphinx into four smaller copies of the sphinx that are congruent to one another.



Problem 64

F. Mir Publishers' Little Mathematics Library Series

This superb series on popular mathematics is translated from Russian by Mir Publishers of Moscow. The books are paperbacks ranging from 25 to 150 pages, and may be obtained from Progress Books of Toronto at $2.50 a copy! The series is written for high school students. Each volume is a stimulating study of a particular topic, and the mathematics is of the highest quality.

Algebraic Equations of Arbitrary Degrees, by A. G. Kurosh, 1977.

High school students are familiar with polynomial equations of degrees one and two, the latter usually solved by the Quadratic Formula. This book gives a brief description of the Cubic Formula and goes on to discuss the general theory of polynomial equations.

Problem 65

Prove that the equation $x^3 - 5x^2 + 2x + 1 = 0$ has three real roots.

Areas and Logarithms, by A. I. Markushevich, 1981.

This book presents a geometric theory of logarithm, in which logarithms are introduced as various areas. Properties of logarithms are then derived from those of areas. The reader is introduced to rudimentary integral calculus without first going through differential calculus.

Problem 66

Prove that the area of the region enclosed by the graphs of $y = 1/x$, $y = 0$, $x = 1$ and $x = 3$ is equal to that enclosed by the graphs of $y = 1/x$, $y = 0$, $x = 3$ and $x = 9$.

Calculus of Rational Functions, by G. E. Shilov, 1982.

This book is an informal introduction to differential and integral calculus with sufficient rigor when attention is restricted to the rational functions, that is, functions expressible as a quotient of two polynomials. There is an illuminating preamble on graph sketching.

Problem 67

Sketch the graphs of $y = 3x^2$, $y = 3x^2 - 1$, $y = (3x^2 - 1)^2$ and $y = 1/(3x^2 - 1)^2$.

Complex Numbers and Conformal Mappings, by A. I. Markushevich, 1982.

Not assuming prior acquaintance with complex numbers, this book introduces them to the reader in geometric form as directed line segments. Functions of a complex variable are considered as geometric transformations. Of particular interest is the class of conformal mappings or angle-preserving transformations.

Problem 68

What geometric transformation corresponds to the function $f(z) = (1+i)z/\sqrt{2}$? What will be the image of the triangle with vertices at 0, $1 - i$ and $1 + i$?

Differentiation Explained, by V. G. Boltyansky, 1977.

This book introduces the reader to differential calculus by considering problems in physics, such as the problem of a free-falling body. This is followed by an informal discussion of differential equations, which are applied to tackle the problem of harmonic oscillations. Other applications of differential calculus are also given.

Problem 69

A steam boiler in the shape of a cylinder is to be built so that it will have the required volume V. It is desirable to keep the total surface area of the boiler down to a minimum. Find the best dimensions of the boiler.

Dividing a Segment in a Given Ratio, by N. M. Beskin, 1975.

This book begins with an analysis of the very elementary problem of how to divide a line segment in a given ratio. From this, the reader is led to concepts such as parallel projections, ideal points, separation, cross ratio and complete quadrilaterals. It is an excellent introduction to projective geometry.

Problem 70

Given a segment AB, construct by Euclidean means a point C on AB and a point D on AB extended such that $AC/BC = AD/BD = 4$.

Elements of Game Theory, by Ye. S. Venttsel, 1980.

Game theory deals with conflict scenarios which are resolved according to definite rules. Each party in the conflict has a finite number of options known to all others. After presenting the basic concepts, the book discusses pure and mixed strategies as well as general and approximate methods for solving games.

Problem 71

We have three kinds of weapon at our disposal, A_1, A_2 and A_3. The enemy has three kinds of aircraft, B_1, B_2 and B_3. Our goal is to hit an aircraft and our enemy's goal is to avoid that. When armament A_1 is used, aircraft B_1, B_2 and B_3 are hit with probabilities 0.4, 0.5 and 0.9, respectively. When armament A_2 is used, they are hit with probabilities 0.8, 0.4 and 0.3, respectively. When armament A_3 is used, the respective probabilities are 0.7, 0.6 and 0.8. What is our optimal strategy and what is the optimal strategy of the enemy?

Fascinating Fractions, by N. M. Beskin, 1986.

The fascinating fractions are the continued fractions. After two introductory problems, the book gives an algorithm for converting a real number into a continued fraction. Further applications follow, including the solution to Diophantine equations and the approximation of real numbers by rational numbers.

Problem 72

What real number is represented by the continued fraction $1/(1 + 1/(1 + 1/(1 + \cdots)))$?

The Fundamental Theorem of Arithmetic, by L. A. Kaluzhnin, 1979.

The Fundamental Theorem of Arithmetic states that prime factorization is unique. This is usually taken for granted, but the book argues that its importance deserves a rigorous proof. Examples of number systems in which prime factorization is not unique are given. The proof of the theorem also leads to a method of solving linear Diophantine equations.

Problem 73

Prove that every integer greater than 1 can be expressed as a product of primes in at least one way.

Geometrical Constructions with Compass Only, by A. Kostovskii, 1986.

This book presents a proof of the Mascheroni-Mohr Theorem that the ruler is a redundant tool in Euclidean constructions. Of course, the compass alone cannot draw straight lines, but we may consider a straight line constructed if at least two points on it are obtained. However, if further use of this line is made to intersect other lines and circles, those points of intersections have to be constructed explicitly. The main idea behind the proof is inversion. Later, further constraints are placed on the compass.

Problem 74

Given three points, A,B,C, and a positive number r, construct the points of intersection of the line AB and the circle with centre C and radius r, using only a compass.

Gödel's Incompleteness Theorem, by V. A. Uspensky, 1987.

Gödel's Incompleteness Theorem says roughly that, under certain very reasonable conditions in a mathematical system, there exist true but unprovable statements. This book provides the necessary background in mathematical logic and gives a formal proof of this important result.

Problem 75

A set is said to be countable if its elements can be matched one-to-one onto the set of positive integers. Prove that the union of two countable sets is countable.

Images of Geometric Solids, by N. M. Beskin, 1985.

The subject of this book is descriptive geometry, that is, the two-dimensional representation of three-dimensional objects. The concept of projection plays a central role. Numerous practical exercises are included.

Problem 76

Prove that the image of a straight line under parallel projection is either a point or a straight line.

Induction in Geometry, by L. I. Golovina and I. M. Yaglom, 1979.

This book gives an excellent account on the power of mathematical induction, applied to problems in geometry. There are problems of computation, proof, construction and locus. Induction is also used to define concepts and to generalize results to higher dimensions.

Problem 77

Prove that it is possible to cut up each of n given squares into finitely many pieces and use all pieces thus obtained to reassemble a single square.

Inequalities, by P. P. Korovkin, 1975.

This book begins with the basic Arithmetic-Mean-Geometric-Mean Inequality and uses the Bernoulli Inequality to generalize it to the Power-Means Inequality. Various applications are then given.

Problem 78

Prove that for integers n greater than 2, we have $n! < ((n + 1)/2)^n$.

The Kinematic Method in Geometrical Problems, by Yu. I. Lyubich and L. A. Shor, 1980.

When solving a geometrical problem, it is helpful to imagine what would happen to the elements of the figure under consideration if some of its points start moving. After a review of vector algebra, this book shows how kinematics, or the theory of velocities, can be applied to tackle geometric problems.

Problem 79

A treasure map gives the following directions. Go to the gallows. From there, walk in a straight line to the pine tree, turn through a right angle to the left, walk the same distance in a straight line and mark the spot. Return to the gallows. From there, walk in a straight line to the oak tree, turn through a right angle to the right, walk the same distance in a straight line and mark the spot. The treasure is buried at the point halfway between the two marked spots. The treasure hunter finds that the gallows has disappeared without a trace. Fortunately, the trees are still there. Can the treasure hunter find the treasure?

Lobachevskian Geometry, by A. S. Smogorzhevsky, 1976.

This is an introduction to hyperbolic geometry. The principal tool is the inversive transformation in Euclidean geometry. This is used to construct a model of the hyperbolic plane and various theorems in hyperbolic geometry are proved. The book ends with a discussion of the hyperbolic functions and their uses for computation in the hyperbolic plane.

Problem 80

Given a circle with centre O and radius r, the inversive image of a point $A \neq O$ with respect to this circle is the point B such that O, A and B lie on a straight line and $OA \cdot OB = r^2$. What is the inversive image of a circle passing through O?

Method of Coordinates, by A. S. Smogorzhevsky, 1980.

This is an introduction to analytic geometry, covering the most basic concepts. There is also a brief discussion of polar coordinates.

Problem 81

Find a single equation in x and y which is satisfied by every point (x, y) inside the first quadrant but no others.

The Method of Mathematical Induction, by I. S. Sominsky, 1975.

This is an excellent introduction to the important method of mathematical induction. After a detailed discussion of the basic idea, numerous examples from arithmetic, algebra and trigonometry are provided, including many proofs of identities and inequalities.

Problem 82

Prove that any amount of postage over 7¢ may be made up exactly using only 3¢ and 5¢ stamps.

Method of Successive Approximation, by N. Ya. Vilenkin, 1979.

This is an introduction to numerical analysis. Starting with the simplest form of successive approximation, the reader is led to the method of iteration, the method of chords and Newton's method which uses differential calculus.

Problem 83

The method of chords uses a straight line to approximate a small portion of a curve. Use the method of chords to find an approximate solution of the equation $x^3 + 3x - 1 = 0$.

The Monte Carlo Method, by I. M. Sobol, 1975.

This is an introduction to the theory of statistical sampling. The Monte Carlo method calculates a certain parameter by running a sequence of simulations, taking the average value and estimating the error. It is based on the probabilistic concept of a random variable. In the case of a continuous variable, calculus is used.

Problem 84

A plane figure is enclosed within a unit square. Forty points inside the square are chosen at random and the number of those inside the figure counted. The experiment is repeated ten times. If the counts are 15, 15, 14, 12, 12, 15, 14, 14, 13 and 15, respectively, what would be a good estimate of the area of the figure?

Pascal's Triangle and *Certain Applications of Mechanics to Mathematics*, by V. A. Uspensky, 1976.

This volume consists of two independent booklets by the same author. The first is an introduction to the binomial coefficients and Pascal's Formula which lead to the construction of Pascal's triangle. The second uses the mechanical principle that, in a position of equilibrium, the potential energy of a weight attains its lowest value, to solve a number of problems in geometry and number theory.

Problem 85

Give a proof based on mechanical considerations that a tangent to a circle is perpendicular to the radius at the point of tangency.

Post's Machine, by V. A. Uspensky, 1983.

The book deals with a certain abstract computing machine. Though it does not exist physically, calculations on it involve many important features inherent in the computations on computers. This machine is also known as Turing's machine.

Problem 86

Post's machine consists of an infinite tape divided into cells and a tapehead which is positioned over one cell at any time. A cell may contain either nothing or a marking. A program consists of a sequence of instructions, of which there are seven types: L instructs the tapehead to move one cell to the left, R instructs the tapehead to move one cell to the right, M instructs the tapehead to mark the cell it is over, U instructs the tapehead to unmark the cell it is over, S instructs the machine to stop, G n instructs the machine to execute statement n next, and B m n instructs the machine to execute statement m next if the tapehead is over a marked cell, and statement n if the tapehead is over an unmarked cell. The statements in a program are numbered consecutively and are executed in order unless directed otherwise by a G or B statement. Consider the following program: (1) M, (2) R, (3) B 4 2, (4) L, (5) B 6 4, (6) U, (7) S. The tapehead is initially at cell 0 and the tape is unmarked except for cell 2. At the end of this program, where would the tapehead be and what changes have been made on the tape?

Proof in Geometry, by A. I. Fetisov, 1978.

This book raises and answers the following questions. What is proof? Why is proof a necessity? What conditions should a proof satisfy for us to call it a correct one? What propositions may be accepted without proof? The discussion is illustrated with numerous examples from geometry.

Problem 87

Prove that, if two interior angle bisectors of a triangle are equal, the triangle is isosceles.

Recursion Sequences, by A. I. Markushevich, 1975.

This book deals with the counting technique based on recurrence relations. After a review of geometric progressions, the method of characteristic equations is introduced to solve recurrence relations.

Problem 88

A sequence $\{s_n\}$ is defined by $s_o = 1$ and $s_n = s_{n-1} + 3^n$. Find an explicit formula for s_n in terms of n.

Remarkable Curves, by A. I. Markushevich, 1980.

This book presents a collection of very attractive curves and their interesting properties. Starting with the basic conic sections, the reader is led on to the lemniscate of Bernoulli, the cycloid, the spiral of Archimedes, the catenary and the logarithmic spiral.

Problem 89

Find the locus of a point P such that $PA = 3PB$ where A and B are fixed points.

Shortest Lines, by L. A. Lyusternik, 1976.

This may be considered as a non-technical introduction to differential geometry. The concepts of space curve, curvature and geodesics are discussed. However, the emphasis is on the shortest-line problem on special surfaces which can be solved by elementary methods. Plenty of applications are considered.

Problem 90

A and B are diametrically opposite points on the base of a cylinder and C is the point on the top directly above B. The cylinder has height 8 and base circumference 12. Find the shortest path from A to C on the surface of the cylinder.

Solving Equations in Integers, by A. O. Gelfond, 1981.

The subject is Diophantine equations. Detailed study of the linear Diophantine equation, Pythagoras' equations and Pell's equation are presented, with continued fractions playing a central role. There is a brief discussion of other Diophantine equations.

Problem 91

Prove that the equation $x^2 - dy^2 = 1$ has no solutions in integers other than $(\pm 1, 0)$ if d is a perfect square.

Stereographic Projection, by B. A. Rosenfeld and N. D. Sergeeva, 1977.

The stereographic projection is a projection of a sphere from one of its points onto the plane tangent to the sphere at the diametrically opposite point. Applications to astronomy, geography and hyperbolic geometry are included.

Problem 92

Let S, M and N be three points on a great circle of a sphere. Let M' and N' be the respective images of M and N under the stereographic projection from S. Prove that $\measuredangle SMN = \measuredangle SN'M'$ and $\measuredangle SNM = \measuredangle SM'N'$.

Systems of Linear Inequalities, by A. S. Solodovnikov, 1979.

This is an introduction to linear programming. Systems of linear inequalities in two or three unknowns are visualized geometrically followed by a brief discussion of convexity. The simplex method is then presented, with a proof of the Duality Theorem and an application to a transportation problem.

Problem 93

Find the minimum value of the function $2x + y$ provided that $x \geq 0$, $y \geq 0$, $x - y \leq 0$, $x + y \geq 2$ and $x + 4y \geq 12$.

An Unusual Algebra, by I. M. Yaglom, 1978.

This is an introduction to Boolean algebra or the algebra of sets. Comparison with the algebra of numbers is made. Applications to propositional logic and switching circuits are discussed.

Problem 94

Let A and B be subsets of a universal set U. Prove that $A \cap (A \cup B) = A$.

G. Books from W. H. Freeman & Company, Publishers

W. H. Freeman has a relatively small selection of titles for a major publisher, but what a selection! Among the more advanced books are the acclaimed *The Fractal Geometry of Nature* by B. B. Mandelbrot and the long-awaited *Tilings and Patterns* by B. Grünbaum and G. C. Shephard. We will concentrate on books at the high school and beginning college levels. We point out that three of the books in Martin Gardner's *Scientific American* series are published by Freeman.

aha! Insight, by Martin Gardner, 1978.

This is the book form of six filmstrips titled *Combinatorial aha! Geometry aha! Number aha! Logic aha! Procedural aha!* and *Word aha!* Selected sequences of frames featuring problems and paradoxes are shown with accompanying text.

Problem 95

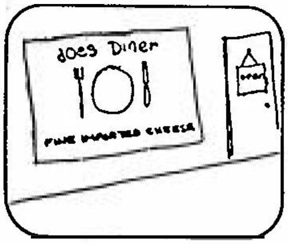
See the sequence of frames (next page).

aha! Gotcha, by Martin Gardner, 1982.

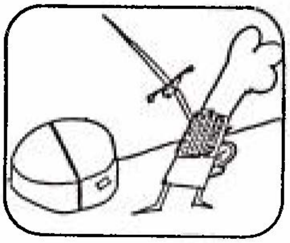
This is the book form of six filmstrips titled *aha Logic! aha Number! aha Geometry! aha Probability! aha Statistics!* and *aha Time!* Selected sequences of frames featuring problems and paradoxes are shown with accompanying text.

Problem 96

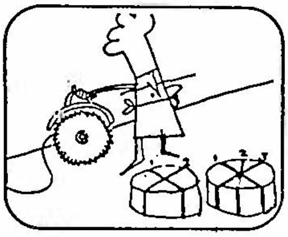
See the sequence of frames (next page).



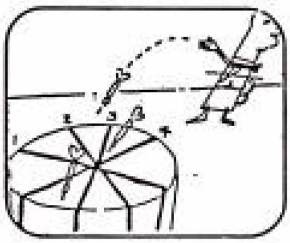
The food at Joe's Diner may not be the best, but the place is famous for its delicious cheese.



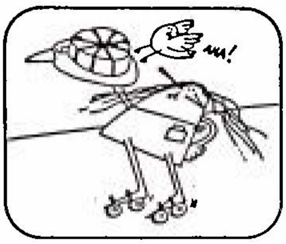
You can have a lot of fun with the cylindrical pieces of cheese. With one straight cut it's easy to divide one piece into two identical pieces.



With two straight cuts it's easy to cut it into four identical pieces. And three cuts will make six identical pieces.

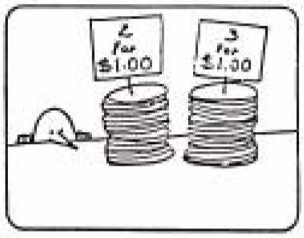


One day, Rosie, the waitress, asked Joe to slice the cheese into eight identical pieces.
Joe: Okay, Rosie. That's simple enough. I can do it with four straight cuts like this.

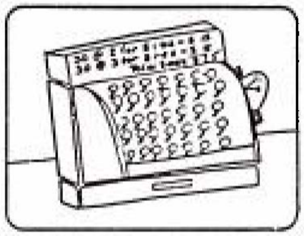


While Rosie was carrying the slices to the table, she suddenly realized that Joe could have gotten the eight identical pieces with only three straight cuts. What insight did Rosie have?

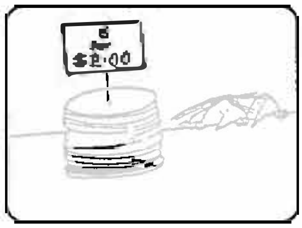
Problem 95



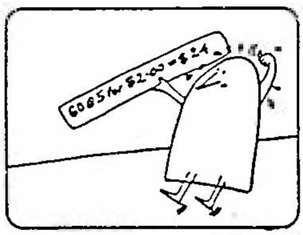
A record store put 30 old rock records on sale at two for a dollar, and another 30 on sale at three for a dollar. All 60 were gone by the end of the day.



The 30 two-for-a-dollar disks brought in $15. The 30 three-for-a-dollar disks brought in $10. Altogether—$25.



The next day the store manager put another 60 records on the counter.
Clerk: Why bother to sort them? If 30 sell at two for a dollar, and 30 at three for a dollar, why not put all 60 in one pile and sell that at five for $2? It's the same thing.



When the store closed, all 60 records had been sold at five for $2. But when the manager checked the cash, he was surprised to find that proceeds from the sale were only $24, not $25.

What do you think happened to that missing dollar? Did the clerk steal it? Did a customer get the wrong change?

Problem 96

Geometry, by H. R. Jacobs, 1987.

The second edition of this outstanding textbook covers standard Euclidean plane and solid geometry in a way which students will find enjoyable. The serious mathematics is interlaced with cartoons, anecdotes and practical problems. A Teacher's Guide provides specific lesson plans.

Problem 97

Shipwrecked on an island which is in the shape of an equilateral triangle, a sailor builds a hut so that the total of its distances to the three sides of the triangle is a minimum. Where is the best place on the island for the hut?

Elementary Algebra, by H. R. Jacobs, 1979.

This textbook covers functions and graphs, number systems, equations, polynomials, exponents and radicals, inequalities and number sequences. It is done in a lucid and refreshing manner, much in the style of its companion volume *Geometry*. The Teacher's Guide provides specific lesson plans.

Problem 98

Solve the system of simultaneous equations $55x + 45y = 520$ and $45x + 55y = 480$.

Mathematics: A Human Endeavor, by H. R. Jacobs, 1982.

The subtitle of this book is ''A Book for Those Who Think They Don't Like the Subject.'' It is a stimulating survey of number theory, algebra, geometry, combinatorics, probability, statistics and topology.

Problem 99

There are three boxes, one containing two red marbles, one containing two white marbles and one containing a red marble and a white marble. The labels telling the contents of the boxes have been switched, however, so that the label on each box is wrong. You are allowed to choose one of the three boxes, draw out at random one marble from inside and deduce from its color the contents of each box. How can this be done?

Mathematics: A Man-Made Universe, by S. K. Stein, 1976

The third edition of this classic contains 19 chapters, covering number theory, geometry, graph theory, modern algebra, number systems, constructibility problems and infinite sets. Its outstanding feature is a large collection of exercises that urge the reader to explore and discover. Despite the ease with which various topics are handled, the book has tremendous depth.

Problem 100

We outline a way of multiplying any two positive integers that uses only multiplication and division by 2. We illustrate it by computing 35×56. First, find the quotient when 2 is divided into 35, namely 17. Repeat the same process on 17, obtaining 8. Continue till you reach 1. Pair off with these numbers those that you obtain by repeatedly multiplying by 2, in this manner:

35	56
17	112
8	224(X)
4	448(X)
2	896(X)
1	1792
	1960

Next cross out all entries in the right-hand column that correspond to even entries in the left-hand column. Add the remaining numbers, 56, 112 and 1792, in the right-hand column. Their sum, 1960, is the product 35×56. Prove that this method always works.

Mathematics: Problem Solving Through Recreational Mathematics, by B. Averbach and O. Chein, 1980.

This book presents a minimum of theory and plenty of problems. After a general discussion on problem solving techniques, problems in logic, algebra, number theory, graph theory, games and puzzles are posed and solved.

Problem 101

A college student sent a postcard to her parents with the message

$$\begin{array}{r} \text{SEND} \\ +\ \text{MORE} \\ \hline \text{MONEY} \end{array}$$

If each letter represents a digit, with different letters representing different digits and the same letter representing the same digit each time it occurs, how much money is being requested?

Mathematics: An Introduction to Its Spirit and Use, edited by M. Kline, 1979.

This book contains 40 articles reprinted from the *Scientific American*, with 14 from Martin Gardner's ''Mathematical Games.'' They are classified into six categories: history, number and algebra, geometry, statistics and probability, symbolic logic and computers and applications. Note that W. H. Freeman also handles offprints of other *Scientific American* articles.

Problem 102

A cone of light casts the shadow of a ball onto the level surface on which the ball is resting. Prove that the shadow is elliptical.

H. Books from Dover Publications, Inc.

The majority of Dover's publications are reprints of excellent (otherwise, why do it?) books that are no longer available in other formats. The new editions are usually paperbound and inexpensive (averaging about $5 U.S. each). Often, errors in the original versions are corrected and new material is appended. While Dover has a large selection of titles in mainstream mathematics (as well as in many other areas, academic or otherwise), we will focus on the best of its line on popular mathematics.

Challenging Mathematical Problems with Elementary Solutions I, by A. M. Yaglom and I. M. Yaglom, 1987.

This is one of the finest collections of problems in elementary mathematics. The 100 problems in combinatorial analysis and probability theory are all easy to understand, but some are not easy to solve, even though no advanced mathematics is required.

Problem 103

Evaluate $\binom{n}{0} + \binom{n}{3} + \binom{n}{6} + \binom{n}{9} + \cdots$, where n is a positive integer and $\binom{n}{k}$ denotes the binomial coefficient, with $\binom{n}{k} = 0$ if $n<k$.

Challenging Mathematical Problems with Elementary Solutions II, by A. M. Yaglom and I. M. Yaglom, 1987.

In this second volume, 74 problems are selected from various branches of mathematics, in particular, number theory and combinatorial geometry. This book and the earlier volume are a must for every school library.

Problem 104

A certain city has 10 bus routes. Is it possible to arrange the routes and the bus stops so that, if one route is closed, it is still possible to get from any one stop to any other (possibly changing buses along the way) but, if any two routes are closed, there are at least two stops such that it is impossible to get from one to the other?

Mathematical Bafflers, edited by A. F. Dunn, 1980.

The bafflers in this book originally appeared as a most successful weekly corporate advertisement in technical publications. They are contributed by the readers, with a consequent diversity in their levels of sophistication. Some require almost no mathematics while others are quite demanding. However, there is a beautiful idea behind each baffler, which is compactly stated and accompanied by a cartoon.

Problem 105

Two similar triangles with integral sides have two of their sides the same. The third sides differ by 387. What are the lengths of the sides?

Second Book of Mathematical Bafflers, edited by A. F. Dunn, 1983.

This second collection of bafflers, like the earlier volume, is organized by chapters, each dealing with one area of mathematics. These include algebra, geometry, Diophantine problems and other number theory problems, logic, probability and "insight."

Problem 106

Only two polygons can have a smallest interior angle of 120° with each successive angle 5° greater than its predecessor. One is a nonagon with angles 120°, 125°, 130°, 135°, 140°, 145°, 150°, 155° and 160°. What is the other?

Ingenious Mathematical Problems and Methods, by L. A. Graham, 1959.

The 100 problems in the book originally appeared in the *Graham Dial*, a publication circulated among engineers and production executives. They are selected from areas not commonly included in school curricula and have new and unusual twists that call for ingenious solutions.

Problem 107

Given three unequal disjoint circles, prove that the three points of intersection of the external common tangents of the three pairs of circles lie on a straight line.

The Surprise Attack in Mathematical Problems, by L. A. Graham, 1968.

These 52 problems are selected from the *Graham Dial* on the criterion that the best solutions are not the ones the original contributors had in mind. The reader will enjoy the elegance of the unexpected approach. Like the earlier volume, the book includes a number of illustrated Mathematical Nursery Rhymes.

Problem 108

Construct an isosceles triangle given the base and the bisector of one of the base angles.

One Hundred Problems in Elementary Mathematics, by H Steinhaus, 1979.

The 100 problems cover the more traditional areas of number theory, algebra, plane and solid geometry, as well as a host of practical and non-practical problems. There are also 13 problems without solutions; some but not all of these actually have known solutions. The unsolved problems are not identified in the hope that the reader will not be discouraged from attempting them.

Problem 109

We construct a sequence of numbers as follows. The first term is 2 and the next is 3. Since $2 \times 3 = 6$, the third term is 6. Since $3 \times 6 = 18$, the fourth term is 1 and the fifth is 8. The product of each pair of consecutive terms is computed in turn and appended digit by digit to the sequence. Prove that the number 5 never appears in this sequence.

Fifty Challenging Problems in Probability with Solutions, by F. Mosteller, 1987.

This book actually contains 56 problems, each with an interesting story line. There are the familiar ''gambler's ruin'' and ''birthday surprises'' scenarios, but with new twists. Others are unconventional, including one which turns out tc be a restatement of Fermat's Last Theorem.

Problem 110

A three-person jury has two members each of whom independently has probability p of making the correct decision and a third member who flips a coin for each decision. A one-person jury has probability p of making the correct decision. Which jury has the better probability of making the correct decision if majority rules in the three-person jury?

Mathematical Quickies, by C. Trigg, 1985.

This book contains 270 problems. Each is chosen because there is an elegant solution. Classification by subject is deliberately avoided, nor are the problems graduated in increasing level of difficulty. This encourages the reader to explore each problem with no preconceived idea of how it should be approached.

Problem 111

How many negative roots does the equation $x^4 - 5x^3 - 4x^2 - 7x + 4 = 0$ have?

Entertaining Mathematical Puzzles, by Martin Gardner, 1986.

The master entertains with 39 problems and 28 quickies, covering arithmetic, geometry, topology, probability and mathematical games. There is a brief introduction to the basic ideas and techniques in each section.

Problem 112

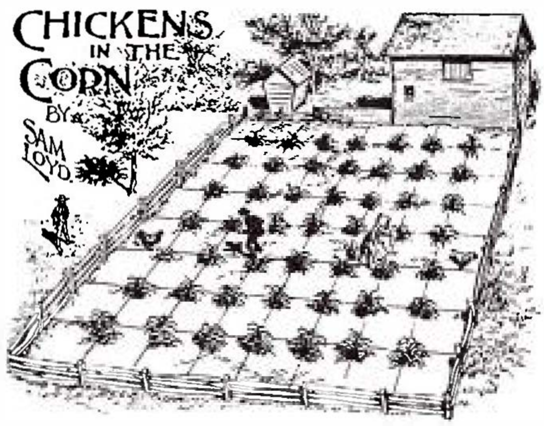
A straight line is called self-congruent because any portion of the line can be exactly fitted to any other portion of the same length. The same is true of the circle. There is a third type of curve that is self-congruent. What is it?

Mathematical Puzzles of Sam Loyd I, edited by Martin Gardner, 1959.

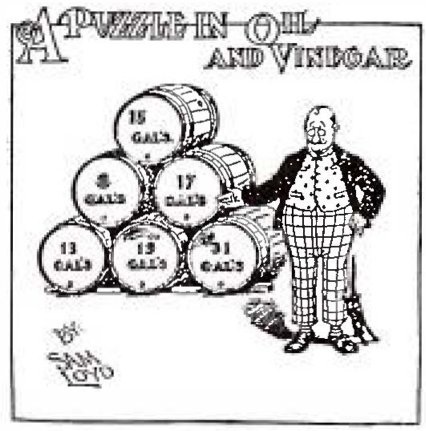
Sam Loyd is generally considered the greatest American puzzlist. He had a knack of posing problems in a way which attracted the eye of the public. Many of the 117 problems in this book, the first of two volumes, had been used as novelty advertising give-aways.

Problem 113

The game of chicken-catching is played in a garden divided into 64 square patches as shown in the illustration (next page). There are the farmer and his wife, as well as a rooster and a hen. The humans move first, followed by the chickens, and alternately thereafter. In a move, each of the two members of the team must move to an adjacent square (not diagonally). A chicken is caught if a human moves into the square it occupies. Can the humans catch the rooster and the hen?



Problem 113



Problem 114

Mathematical Puzzles of Sam Loyd II, edited by Martin Gardner, 1960

This book contains 166 problems, most of which are accompanied by Loyd's own illustrations, as is the case with the earlier volume. The two books represent the majority of the mathematical problems in the mammoth *Cyclopedia* by Sam Loyd, published after his death.

Problem 114

Each barrel in the illustration contains either oil or vinegar. The oil sells for twice as much per gallon as the vinegar. A customer buys $14 worth of each, leaving one barrel. Which barrel did he leave?

Amusements in Mathematics, by H. E. Dudeney, 1970.

Henry Ernest Dudeney, a contemporary of Sam Loyd, is generally considered the greatest English puzzlist and a better mathematician than Loyd. This book contains 430 problems, representing only part of Dudeney's output. There are plenty of illustrations in the book.

Problem 115

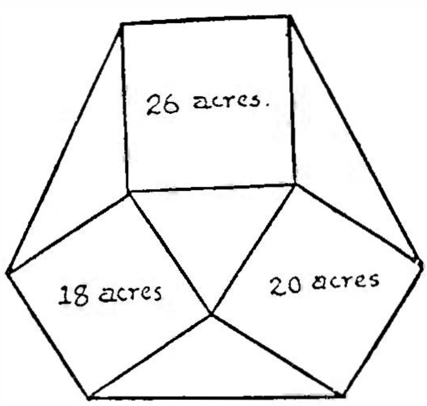
Farmer Wurzel owned the three square fields shown in the illustration. In order to get a ring-fence round his property he bought the four intervening triangular fields. What is the total area of his estate?

Mathematical Puzzles for Beginners and Enthusiasts, by G. Mott-Smith, 1954.

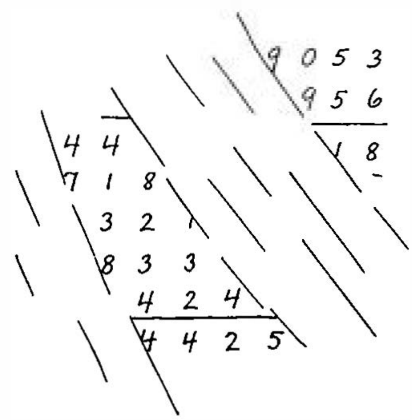
This book contains 189 problems in arithmetic, logic, algebra, geometry, combinatorics, probability and mathematical games. They are both instructive and entertaining.

Problem 116

The janitor inadvertently wiped out a good share of Miss Gates' multiplication. What are the erased digits?



Problem 115



Problem 116

Mathematical Recreations and Essays, by W. R. Ball and H. S. M. Coxeter, 1988.

This is the foremost single-volume classic of popular mathematics. Written by two distinguished mathematicians, it covers a variety of topics in great detail. After arithmetical and geometrical recreations, it moves on to polyhedra, chessboard recreations, magic squares, map-coloring problems, unicursal problems, Kirkman's schoolgirls problem, the three classical geometric construction problems, calculating prodigies, cryptography and cryptanalysis.

Problem 117

A schoolmistress was in the habit of taking her girls for a daily walk. The girls were 15 in number and were arranged in five rows of three each, so that each girl might have two companions. How is it possible that for seven consecutive days no girl will walk with any of her school-fellows in any triplet more than once?

Mathematical Recreations, by M. Kraitchik, 1953.

This is a revision of the author's original work in French. It covers more or less the same topics as *Mathematical Recreations and Essays*. There is a chapter on ancient and curious problems from various sources.

Problem 118

For their common meal, Caius provided seven dishes and Sempronius eight. But Titus arrived unexpectedly, so all shared the food equally. Titus paid Caius 14 denarii and Sempronius 16. The latter cried out against this division and the matter was referred to a judge. What should his decision be, granting that the 30 denarii is the correct total amount?

The Master Book of Mathematical Recreation, by F. Schuh, 1968.

This is a translation of the author's original work in German. Four of the fifteen chapters are devoted to the analysis of mathematical games. The remaining ones deal with puzzles of various kinds. General hints for solving puzzles are given in the introductory chapter. The last chapter is on puzzles in mechanics.

Problem 119

Two players start out with two piles of matches, say, with 19 and 89, respectively. They take turns removing 1, 2, 3, 4 or 5 matches, but all from one pile. Taking the last match means a win. Which player wins this game and what is a winning strategy?

Puzzles and Paradoxes, by T. H. O'Beirne, 1984.

Like Martin Gardner's series, this book is an anthology of the author's column in *New Scientist.* It consists of 12 largely independent articles.

Problem 120

There are twelve coins that look identical, but there may be one which has a different weight from the others. It may be either be slightly heavier or slightly lighter. Using a beam balance three times, how can one determine if there is a "false" coin and, if so, which coin is "false" and whether it is heavier or lighter?

I. Individual Titles

In this penultimate section, we present some outstanding work that is not in a continuing series. Regrettably, we have to be content with two dozen titles. There is simply not enough room to mention many other worthwhile books.

How to Solve It, by G. Pólya, Princeton University Press, 1973.

This is the second edition of the first of five books by an outstanding scientist and educator on his theory and methods of problem solving. Here, numerous examples illustrate the Pólya method which divides the task into four phases, understanding the problem, devising a plan, carrying out the plan and looking back.

Problem 121

Construct a triangle by Euclidean means, given one angle, the altitude from the vertex of the given angle and the perimeter of the triangle.

Mathematics and Plausible Reasoning I, by G. Pólya, Princeton University Press, 1954.

The subtitle of this volume is "Induction and

Analogy in Mathematics.'' It is a continuation and elaboration of the ideas propounded in *How To Solve It*. It discusses inductive reasoning and making conjectures, with examples mainly from number theory and geometry. The transition to deductive reasoning is via mathematical induction.

Problem 122

Guess an expression for $(1 - 1/4)(1 - 1/9)(1 - 1/16) \cdots (1 - 1/n^2)$ valid for $n \geq 2$ and prove it by mathematical induction.

Mathematics and Plausible Reasoning II, by G. Pólya, Princeton University Press, 1954.

The subtitle of this volume is ''Patterns of Plausible Inference.'' It is primarily concerned with the role of plausible reasoning in the discovery of mathematical facts. Two chapters in probability provide most of the illustrative examples.

Problem 123

Of nine patients treated the old way, six died. Of eleven patients treated the new way, two died. If there is really no difference between the two treatments, what is the probability that the observed results are as favorable as, or more favorable than, the above for the new way?

Mathematical Discovery I, by G. Pólya, Wiley, 1965.

This volume contains part one of the work, titled ''Patterns'' and the first two chapters of the second part of the work, titled ''Toward a General Method.'' The first part gives practices for pattern recognition in geometric loci, analytic geometry, recurrence relations and interpolation. The two chapters in the second part discuss general philosophy in problem solving.

Problem 124

A farmer has chickens and rabbits. These animals have 50 heads and 140 feet. How many chickens and how many rabbits has the farmer?

Mathematical Discovery II, by G. Pólya, Wiley, 1965.

This volume contains the remaining nine chapters of the second part of this work by Pólya. More specific advice on the art and science of problem solving is offered. There is a chapter on learning, teaching and learning teaching.

Problem 125

Three circles have the same radius r and pass through the same point O. Let A, B and C be the other points of intersection of the three pairs of these circles, respectively. Prove that A, B and C lie on a circle of radius r.

Discovering Mathematics, by A. Gardiner, Oxford University Press, 1987.

This is a do-it-yourself package through which the reader can learn and develop methods of problem solving. The first part of the book contains four short investigations and the second part two extended ones. Each investigation is conducted via a structured sequence of questions.

Problem 126

Find all five-digit numbers such that, when multiplied by 9, the product is given by writing the five digits of the number in reverse order.

Mathematical Puzzling, by A. Gardiner, Oxford University Press, 1987.

This book contains 31 chapters. The first 29 are groups of related puzzles. The chapters are independent except for three on counting and three on primes. In each chapter, commentaries follow the statements of the puzzles. Answers are given at the end of the book. Chapter 30 re-examines four of the earlier puzzles while Chapter 31 discusses the role of puzzles in mathematics.

Problem 127

Every digit in a multiplication has been copied down wrongly, but each digit is only one out. It now reads $16 \times 4 = 64$. What should it have been?

Selected Problems and Theorems in Elementary Mathematics, by D. O. Shklyarsky, N. N. Chentsov and I. M. Yaglom, Mir Publications, 1979.

This excellent book contains 350 problems in arithmetic and algebra, many from papers of the U.S.S.R. Olympiads. It is the first of three volumes but, unfortunately, the volumes on plane geometry and solid geometry have not been translated into English.

Problem 128

Find all three-digit numbers such that, when each is raised to any integral power, the result is a number whose last three digits form the original number.

All the Best from the Australian Mathematics Competition, edited by J. D. Edwards, D. J. King and P. J. O'Halloran, Australian Mathematics Competition, 1986.

This book contains 463 multiple choice questions taken from one of the world's most successful mathematics competitions. They are grouped by subject area to facilitate the study of specific topics.

Problem 129

For all numbers a and b, the operation $a \cdot b$ is defined by $a \cdot b = ab - a + b$. The solution of the equation $5 \cdot x = 17$ is—
(a) 17/5, (b) 2, (c) −2, (d) 3, (e) 11/3.

The First Ten Canadian Mathematics Olympiads, 1969-1978, edited by E. Barbeau and W. Moser, Canadian Mathematical Society, 1978.

This booklet contains the questions and solutions of the first ten Canadian Mathematics Olympiads. Each of the first five papers consists of ten questions. The number of questions in the remaining five varies between six and eight.

Problem 130

Let n be an integer. If the tens digit of n^2 is 7, what is the units digit of n^2?

The Canadian Mathematics Olympiads, 1979-1985, edited by C. M. Reis and S. Z. Ditor, Canadian Mathematical Society, 1987.

This booklet contains the questions and solutions of the Canadian Mathematics Olympiads from 1979 to 1985. Each paper consists of five questions.

Problem 131

$P(x)$ and $Q(x)$ are two polynomials that satisfy the identity $P(Q(x)) = Q(P(x))$ for all real numbers x. If the equation $P(x) = Q(x)$ has no real solutions, show that the equation $P(P(x)) = Q(Q(x))$ also has no real solutions.

1001 Problems in High School Mathematics, edited by E. Barbeau, M. S. Klamkin and W. Moser, Canadian Mathematical Society, 1985.

To date, half of this work has appeared in a preliminary version in the form of five booklets. In addition to problems and solutions, a mathematical "tool chest" is appended to each booklet.

Problem 132

The number 3 can be expressed as an ordered sum of one or more positive integers in four ways, namely, as 3, 1 + 2, 2 + 1 and 1 + 1 + 1. Show that the positive integer n can be so expressed in 2^{n-1} ways.

Winning Ways I, by E. R. Berlekamp, J. H. Conway and R. K. Guy, Academic Press, 1982.

This is the definitive treatise on mathematical games, As the subtitle "Games in General" suggests, the general theory of mathematical games is presented in this first volume, but there are also plenty of specific games to be analyzed, played and enjoyed. The book is written with a great sense of humor and is profusely illustrated, often in bright colors.

Problem 133

The army has been in disarray and the general has reduced all officers to the ranks and made everyone directly responsible to him. He now intends, on the alternate recommendation of his two military advisers, to recruit from outside the army a new hierarchy of officers. Each adviser, in turn, recommends that some officer (possibly the general himself) currently in direct charge of four or more officers and men should recruit a new subordinate. The new officer will be directly responsible to the one who appointed him and will, until further notice, take over direct responsibility for three or more, but not all, of those officers and men presently directly responsible to his appointer. Of course, no further appointment will be possible when every officer has either two or three direct subordinates. Whichever adviser makes the last recommendation retains the confidence of the general. In a seven-man army, does the first adviser or the second adviser win the confidence game?

Winning Ways II, by E. R. Berlekamp, J. H. Conway and R. K. Guy, Academic Press, 1982.

The subtitle of this volume is "Games in Particular." Here, all kinds of mathematical games, classical as well as brand new, are presented attractively. Most of them are two-player games. There are two chapters devoted to one-player games or solitaire puzzles and the book concludes with a chapter on a zero-player game!

Problem 134

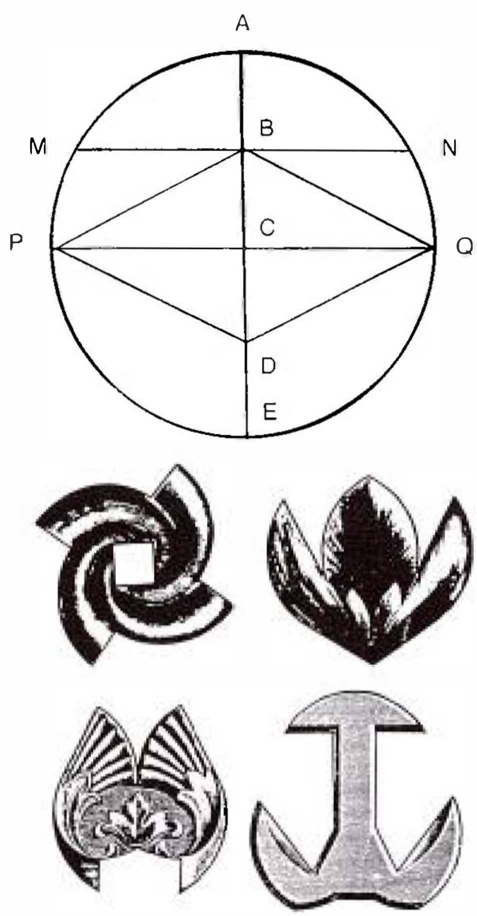
Pack one 2 by 2 by 2 block, one 2 by 2 by 1 block, three 3 by 1 by 1 blocks and thirteen 4 by 2 by 1 blocks into a 5 by 5 by 5 box.

Puzzles Old and New, by J. Slocum and J. Botermans, distributed by University of Washington Press, 1986.

Jerry Slocum has probably the largest puzzle collection in the world. This book features a small subset of his mechanical puzzles, that is, puzzles made of solid pieces that must be manipulated by hand to obtain a solution. They are classified into ten broad categories, with enough information to make most of them and to solve some of them. The book is full of striking full-color plates.

Problem 135

The Circular Puzzle, which dates back to 1891, consists of ten pieces of a circle as follows. AE is a diameter of a circle with centre C. B and D are the midpoints of AC and CE, respectively. MN and PQ are perpendicular to AE and passing through B and C, respectively. P and Q are then joined to B and D. Use the resulting ten pieces to construct each of the following four figures.



Problem 135

The Mathematical Gardner, edited by D. A. Klarner, Wadsworth International, 1981.

This book contains 30 articles dedicated to Martin Gardner for his sixty-fifth birthday. They reflect part of his mathematical interest and are classified under the headings, Games, Geometry, Two-Dimensional Tiling, Three-Dimensional Tiling, Fun and Problems, and Numbers and Coding Theory.

Problem 136

For what values of n can an n by n square be tiled using 2 by 2 squares and 3 by 3 squares?

Mathematical Snapshots, by H. Steinhaus, Oxford University Press, 1983.

This is an outstanding book on significant mathematics presented in puzzle form. Topics include dissection theory, the golden ratio, numeration systems, tessellations, geodesics, projective geometry, polyhedra, Platonic solids, mathematical cartography, spirals, ruled surfaces, graph theory and statistics.

Problem 137

An 8 by 8 square is to be divided into three rectangles with equal diagonal. What is the minimum common value of the length of the diagonal?

Mathematics Can Be Fun, by Y. I. Perelman, Mir Publishers, 1979.

This is a translation of two books in Russian, *Figures for Fun* and *Algebra Can Be Fun*. The former is an excellent collection of simple puzzles. The latter is a general discourse of algebra with quite a few digressions into number theory.

Problem 138

It is claimed that a tripod always stands firmly on a level surface, even when its three legs are of different length. Is that right?

Fun with Maths and Physics, by Y. I. Perelman, Mir Publishers, 1984.

The first half of this beautiful book describes a large number of interesting experiments in physics. The second half consists of a large collection of mathematical puzzles.

Problem 139

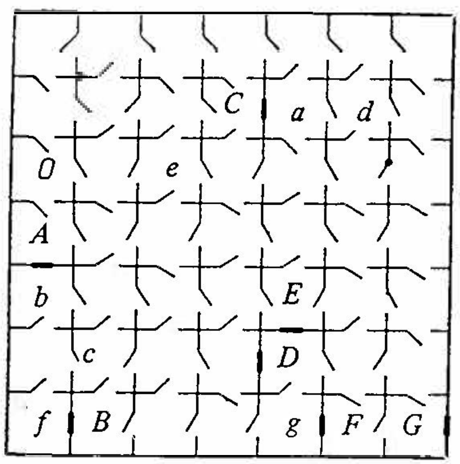
A hundred nuts are to be divided (not necessarily equally) among 25 people. Is it possible to arrange it so that each gets an odd number of nuts?

The Moscow Puzzles, by B. A. Kordemsky, Charles Scribner's Sons, 1972.

This is the translation of the outstanding single-volume puzzle collection in the history of Soviet mathematics. Many of the 359 problems are presented in amusing and charming story form, often with illustrations.

Problem 140

The dungeon has 49 cells. In each of cells *A* to *G*, there is a locked door (black bar). The keys are in cells *a* to *g*, respectively. The other doors open only from one side. How does a prisoner in cell *O* escape? The doors need not be unlocked in any special order and an open door can be passed through any number of times.



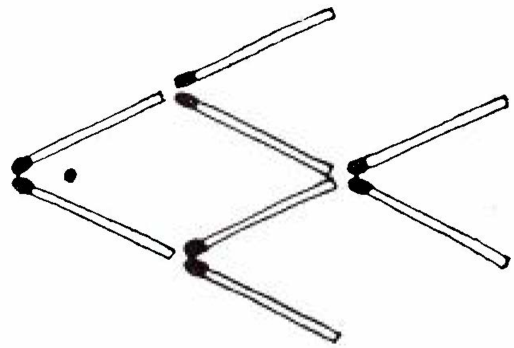
Problem 140

The Tokyo Puzzles, by K. Fujimura, Charles Scribner's Sons, 1978.

This is the translation of one of many books from the leading puzzlist of modern-day Japan. It contains 98 problems, most of them previously unfamiliar to the western world.

Problem 141

A tropical fish is swimming in a tank. The fish faces west. Make it face east by changing the positions of only three matches, in addition to the dot that represents the eye.



Problem 141

536 Puzzles and Curious Problems, by H. E. Dudeney, Charles Scribner's Sons, 1967.

This book is a combination of two out-of-print works of the author, *Modern Puzzles* and *Puzzles and Curious Problems*. Together with Dover's *Amusements in Mathematics*, they constitute a substantial portion of Dudeney's mathematical problems. Those in this book are classified under three broad headings, arithmetic and algebra, geometry, and combinatorics and topology.

Problem 142

A man entered a store and spent one-half of the money on him. When he came out, he found that he had just as many cents as he had dollars when he went in and half as many dollars as he had cents when he went in. How much money did he have on him when he entered?

Science Fiction Puzzle Tales, by Martin Gardner, Clarkson N. Potter, 1981.

This is the first of three collections of Martin Gardner's contribution to Isaac Asimov's *Science Fiction Magazine*. The book contains 36 mathematical puzzles in science fiction settings. When solutions are presented, related questions are often raised.

Problem 143

Dr. Moreau III produced a new type of microbe which multiplies at an alarming rate. Every hour, a single microbe replicates into seven microbes. One day, he put a single microbe, just "born," into a large and empty glass container. Fifty hours later, the container was completely filled. How many hours had elapsed when the container was 1/7 full?

Puzzles From Other Worlds, by Martin Gardner, Vintage Books, 1984.

This is the sequel to *Science Fiction Tales* and the predecessor of New Mathematical Library's *Riddles of the Sphinx*. It contains 37 mathematical puzzles plus further questions raised in the answer sections.

Problem **144**

VOZ, the computer on the spaceship Bagel, was getting bored. So it began to search for palindromic primes, that is, prime numbers which read the same in either direction. The first few VOZ found (not counting single-digit primes) are 11, 101, 131, 151, 181, 191, 313, 353, 373, 383, 727, 757, 787, 797, 919, 929 and 10301. There are no four-digit palindromic primes. Apart from 11, can there be a palindromic prime with an even number of digits?

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