Some More on the Formulation of Mathematical Problems

John G. Heuver

John Heuver teaches mathematics at Grande Prairie Composite High School, Grande Prairie.

George A. Calder's article, "Algebra Can Be a Language," in the September 1988 issue of *delta-K,* has a number of errors in it. German and French words have been anglicized. However, the use of a dictionary is quite legitimate as a tool for teaching word problems since it quickly puts the task at hand into focus. Even the late Marc Kac (1959) used it.

The article also raises some thought-provoking questions that are worthwhile formulating:

1. How do mathematical theories come about? In the preface to his book, Thomas M. Thompson (1983) quotes from an interview with Richard M. Hamming, who in 1947 had access to a computer:

> Two weekends in a row I came in and found that all my stuff had been dumped and nothing was done. . . . And so I said, " . . . if the machine can detect an error, why can't it locate the position of the error and correct it?"

This formulates pretty well the origin of a present day flourishing branch of mathematics, namely, coding theory with spin-offs in many other branches of mathematics.

2. Some problems do not lend themselves to easy mathematical solutions. Richard K. Guy (1983) from the University of Calgary is editor of the "Unsolved Problems" section in *The American Mathematical Monthly.* He speaks of the difficulty in posing certain problems which he illustrates by giving an example.

> Jose M. Bayod, University of Santander, Santander, Spain, notes that in some parts of his country, a farmer owns a part of the mountains that surround his farm, according to the "flowing water law" which can be stated thus: A spot of rain lands at A on the mountain and flows downhill until it reaches a point B on the farm. Then the owner of point B in the valley also owns point A on the mountain. The main problem is to build a mathematical model for the problem: given an area in the valley, what part of the mountain is associated with it?

One can just imagine the amount of detailed information that would be required to attempt a mathematical solution. A general solution that would work under all circumstances seems even more difficult. Luckily, we are informed in the article that the real-life situation causes very few problems.

Solving and posing problems is the lifeblood of mathematics. To give the student an opportunity to learn how to solve problems, we need carefully stated and well-posed problems.

3. Following are examples from past Grade 12 mathematics exams on how not to state problems. The January 1987 Mathematics 30 exam, question 6 states:

> An automobile tire has a mean life of 64,000 km with a standard deviation of 3,200 km. In a purchase of 1,500 tires, the number lasting less than 54,000 km would be. . . .

The masses must have quoted a pleasing answer to the satisfaction of Alberta Education. The question has no answer without information about the distribution function, if such exists.

The January 1988 exam contains the following treasure in question 40:

> Three coins are tossed simultaneously. The probability that the coins will land with two tails and one head showing is. . . .

The trouble here is that no mention is made of the probability that the coins show head or tail. Providing the department with a detailed analysis of the situation resulted in a cold shoulder. It would have been satisfactory to state that the coins were fair.

Almost every departmental exam has errors of this type.

References

Guy, Richard K. "An Olla-podrida of the Open Problems Often Oddly Posed." *The American Mathematical Monthly.* Mathematical Association of America, Washington, D.C., March 1983, 196.

Kac, Marc. *Statistical Independence in Probability, Analysis and Number Theory.* The Carus Mathematical Monographs, Mathematical Association of America, 1959, 8.

Thompson, Thomas M. *From Error Correcting Codes Through Sphere Packings to Simple Groups.* The Carus Mathematical Monographs, Mathematical Association of America, 1983, vii.