Mathematical Modeling Using Spreadsheets

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In 1979, Ken Iverson, born and raised in Camrose, received the ACM Turing Award, one of the highest honors in computing science. This award was presented to Dr. Iverson for his development of a computer language called APL. The title of his Turing address was "Notation as a Tool of Thought." In the early 1980s, concomitant with the emergence of the microcomputer, were two other notational developments. One was Logo, a computer language developed by Seymour Papert and his colleagues at MIT. Papert has referred to the Logo environment as "objects to think with." Another notational development was the spreadsheet, originally developed by Dan Bricklin and Bob Frankston and marketed as Visi-Calc. Many people believe that it was the spreadsheet that gave impetus to the micro revolution. Although the spreadsheet was first used within business environments, its more general use is now beginning to be appreciated.

The developments are milestones, not only in their ostensible domain of computing science, but, even more importantly, in the evolution of human thought. Their true value is not in what they are, but in what they permit. Quite simply, they permit ideas and ways of thought that are not possible otherwise. Notational conventions both enhance and restrict ideas. Consider the advances in mathematics and in civilization that have accrued as a result of switching from Roman numerals to the present Arabic system.

This paper will attempt to show that spreadsheets deserve serious consideration by educators as a notational system for exploring ideas. Spreadsheets should not be viewed as a topic in computing science or business courses, but as a tool like cursive writing which transcends the curriculum. This paper should be viewed as the opening of a door rather than as a comprehensive survey of possible applications.

The paper is divided into three sections. The first section examines an example on equations and graphing, taken from the Grade 10 mathematics curriculum. The second section looks at a section from the Grade 12 syllabus that discusses exponential functions, although the intent is to show the potential of the topic in science and economics. Finally, a reflective section examines the pedagogical and philosophic implications in what has been presented.

The substantive impact of computers on education will not be at the technological level. It will be at the tacit level where we come to recognize and reflect on the underlying assumptions of schools and education. The purpose of this paper is not to extoll the virtues of computers or even spreadsheets, although both will be done. Rather the purpose is to suggest ways in which our classrooms can better honor the integrity of the educational process.

Equations and Graphing

The following question is typical of those found in a unit on equations and graphing at the Grade 10 level: Solve $2(4x - 7) = 5x + 10$.

Usually the pedagogical emphasis is on deriving equivalent equations by applying the same arithmetic operation to both sides of the equation. Fundamentally the emphasis is on skills. Certainly there is concern for understanding, but the assumption is that such understanding will emerge from having the student properly solve a reasonably large number of similar problems. There may be efforts to encourage back substitution to verify an answer, but the critical criterion tends to be the answer at the back of the book or the answer that the teachers have on their desks. The basic pedagogical paradigm is some small variation from the model of doing a few problems in front of the class and then asking the class to do 10 or 20 more "for practice."

Graphical interpretations may be mentioned once or twice but the labor involved militates against extensive use of such an approach. This is a legitimate instructional decision—until recently. A number of software packages are now available that make it relatively easy for the student to obtain the graph of a specified mathematical expression. Spreadsheets are one such family of packages, as we shall see.

What does it mean to solve an expression like $2(4x - 7) = 5x + 10$? When you obtain a solution, what do you have? Could there be more than one solution? Why or why not? Could there be no solution? Why or why not? There would appear to be room for social interaction and discussion here. Opportunities abound for students to reflect on their understanding and to share it with their peers. Precision in language and thought becomes important for communication.

A Spreadsheet Approach

Perhaps the curriculum objective that currently reads: "Maintain skills in solving first degree equations with rational coefficients," which current textbooks are well designed to facilitate, should be rephrased, "Develop an understanding of the relationships inherent in a first degree equation," which textbooks are not well designed for but which emerging software programs are designed to support.

One of the difficulties at present is that many students are still relatively unfamiliar with computers and particularly with spreadsheets. This is likely to become less of a problem in the future as computers continue to become more powerful and cheaper, but the present classroom teacher is faced with the dual problems of teaching the tool as well as the application. A suggestion is to focus on the application and introduce spreadsheet concepts and commands as they are needed.

Let's reexamine the problem, Solve $2(4x - 7) = 5x + 10$, using a spreadsheet approach. First, try to get a better understanding of the left side of the equation. If we write this as $y = 2(4x - 7)$, then we can see how the value of the expression changes as x changes. One could try a few values of x to get a feel for the substitution process. Thus, keeping it simple, if x is 1 then y is $2(4 - 7)$ which is $2(-3)$ which gives -6. To save time and effort, we can use spreadsheets to do most of the tedious work for us. However, we must now give the proper instructions to the spreadsheet program. The particular spreadsheet program used while preparing this paper was Microsoft's Excelf on a MacIntosh Plus computer, but many of the ideas should transfer to other spreadsheet programs.

Spreadsheets are essentially a tabular structure of rows and columns. The resulting cells may contain information such as words or phrases and numbers. In addition, cells may contain formulas for computing new values from the values stored in other cells.

In column A, row 5 (leave the first four rows blank for possible titles later) have the students type in the expression x. In column B, row 5 type in the expression $y = 2(4x - 7)$. Emphasize that so far they have simply typed in a couple of labels for their benefit. In fact, this might be a good opportunity to add a title for this activity: "Solve $2(4x - 7) = 5x + 10$" is a possible title (place this in column A, row 1).

Now begin to type in numeric values. At this point, students may have little idea of an appropriate range of values, so you may suggest they try some conventional guesses such as integers in the range 0 to 10 or 0 to 100 or -100 to 100. They can always be changed later so having a good guess is not all that important. Judgment comes from an awareness of context. The context here is "textbook problems," so small integral values are likely useful. I hope we will begin to see a loosening of this restriction as computers become more widespread.

Thus, in column A, row 6 students would type in the lower boundary of their domain.

Now come the key spreadsheet commands.

In column B, row 6 have the students type in the formula =2*(4*A6 − 7). When they press return, the value in the cell B6 is automatically computed and displayed. Similarly, the right hand side of the original expression may be computed in column C. The spreadsheet can display either the formulas or the computed values. Thus we now have, displaying the values:

	A	B	C
1	Solve $2(4x - 7) = 5x + 10$		
2			
3			
4			
5	x	$y = 2(4x - 7)$	$y = 5x + 10$
6	1	−6	15
7			

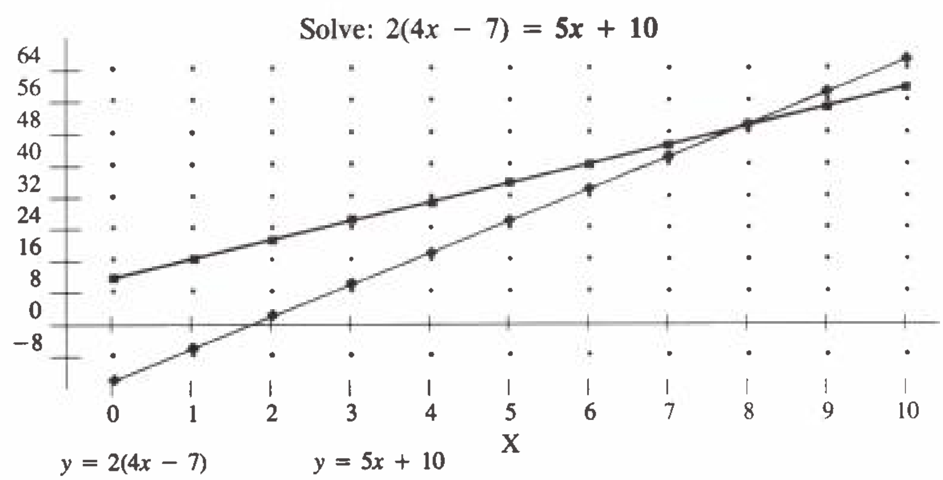
Using the Fill Down command, you can quickly get a large table of computed values. This is where the computational power of the spreadsheet first becomes apparent. Thus selecting rows 6 through 15 and using the Fill Down feature yields:

	A	B	C
1	Solve $2(4x - 7) = 5x + 10$		
2			
3			
4			
5	x	$y = 2(4x - 7)$	$y = 5x + 10$
6	1	−6	15
7	2	2	20
8	3	10	25
9	4	18	30
10	5	26	35
11	6	34	40
12	7	42	45
13	8	50	50
14	9	58	55
15	10	66	60

The labor involved in producing this table is truly trivial—just a few keystrokes and mouse clicks. However, much is going on. Paramount, particularly in the mind of the user, is the issue of notation. When we are typing in an expression to look like it does in the text, we type a string of characters such as $2(4x - 7)$. When we type a formula into a cell to indicate how the spreadsheet is to compute the value for that cell we type =2*(4*A6 − 7). Here the conventions are different from normal usage. The = symbol at the beginning signifies that what follows is a formula. The * is the explicit symbol for multiplication. Instead of a variable name represented by a letter, it is now represented by a cell label (for example, A6). Questions related to order of operations and use of parentheses are likely to come up. Flexibility with notation may be one of the hidden benefits from using spreadsheet approaches. How many of us are familiar with the students who can solve for x but not for w? The resulting table provides an explicit representation of the different values for two expressions as x changes. The students can see that as x gets larger, so do each of the expressions $2(4x - 7)$ and $5x + 10$. What patterns are noticeable? One column increases in increments of 8, the other in increments of 5. Why? Row 13 (where x is 8) is also noteworthy. Why?

At this stage, you may wish to take time out from the original problem to explore expression. What is the relative impact of doubling any of the original coefficients? In what ways can the original equations be modified?

Returning to the original problem, we can also ask for a graph of this table. Here the spreadsheet approach becomes exciting! Once again, only a few commands give us the following picture:



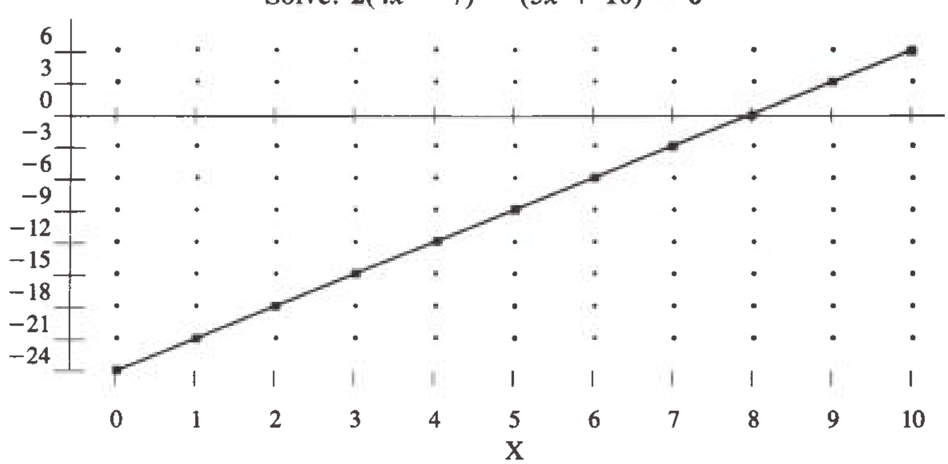
We now see that the original question may be reworded to say, "For what value(s) of x do the equations of the two straight lines have the same value?" This appears to have more inherent meaning than the original question, where the point about straight lines is not likely to even occur. One should also note the parallels between the tabular and graphical representations of the same idea.

Some students may realize that the original equation is equivalent to Solve $2(4x - 7) - (5x + 10) = 0$. The table and graph now look like:

	1	2	3	4	5	6	7
1	Solve: $2(4x - 7) = 5x + 10$			equivalent to: $2(4x - 7) - (5x + 10) = 0$			
2							
3	x	$2(4x - 7)$	$5x + 10$		$(4x - 7) - (5x + 10) = 0$		
4							
5	0	−14	10		−24		
6	1	1−6	15		−21		
7	2	12	20		−18		
8	3	10	25		−15		
9	4	18	30		−12		
10	5	26	35		−9		
11	6	34	40		−6		
12	7	42	45		−3		
13	8	50	50		0		
14	9	58	55		3		
15	−10	66	60		6		

and

Solve: $2(4x - 7) - (5x + 10) = 0$



$2(4x - 7) - (5x + 10) = 0$

The student should now compare the two quite different representations of the same problem (two intersecting lines versus one line which crosses the x-axis). Are these different representations compatible?

From here the student could be encouraged to play with the spreadsheet formulation. To change one or both of the equations and to see the effect graphically is an easy matter. In fact, it is not a large step to equations that are not straight lines. A couple of weeks of exploration and helpful hints may well cover a large component of the entire high school curriculum! The following quotation is from a recent article by Gordon (1987, 5) on cultural comparisons among schools: "A Japanese teacher might spend one whole day on just one problem, working and reworking it from every angle, until every student understood."

Learning is a complex process. Thus the behaviorist tradition that spawned many of our current educational practices has had some success. In the absence of alternatives, we tend to stay with approaches that have had some success over those that have had no success or even those that have an unknown rate of success. To some extent, this is highly defensible and lauditory. We do not want to "fool around with our students" while experimenting with new educational procedures. On the other hand, this guideline can become a shackle. We do use new materials and textbooks from time to time. What are the appropriate next steps for the mathematics education community, given recent developments in computer technology and software?

Mathematical Modeling

As indicated, spreadsheets permit one to explore higher order polynomials. Other mathematical functions, such as those found in trigonometry and economics as well as exponential and logarithmic functions, are provided within most spreadsheet programs. Topics in calculus, such as the exploration of the concepts of derivative and integral from first principles, are possible. I am confident that many more topics are waiting for the teacher or student who has the time (say, an hour or two) to explore various ideas.

Problem solving was the theme of the 1980 NCTM Yearbook. Yet, often the cry has a hollow ring to it. Too often it is equated with the rote memorization of "type problems." The existence of a problem, and the existence of an algorithm for solving the problem, and even the correct application of the algorithm to the problem is not a sufficient condition for one to say that problem solving has occurred. Problem solving refers to a situation where there is a discrepancy between your present state and a future state, and you don't know how to close the gap. Problem solving has a hueristic tone to it, not an algorithmic one. It has a sense of urgency and action, rather than passivity and autonomicity; a sense of excitement rather than drudgery; a feeling of play rather than work. True problem solving often implies that even the teacher is not too sure what to do next. This is not to be feared but welcomed.

The following example is taken from *Holt Mathematics 6* (p. 153):

An advertising company, asked to market a new product, estimates that within *t* days of the commencement of an advertising campaign the percentage of the total market which will buy that product is given by the exponential function $f(t) = 1 - 2.8^{-0.04t}$. Determine

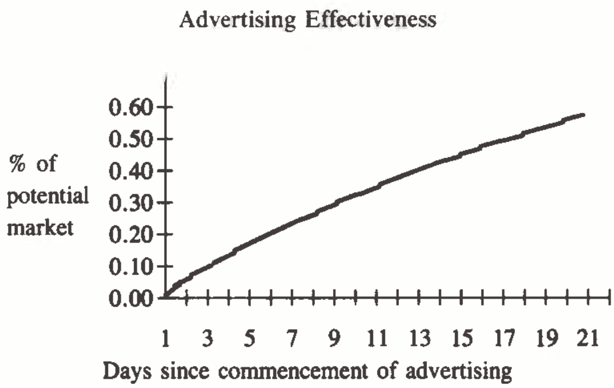
1. the percentage of the potential market which will buy within 20 days of the campaign,
2. the number of days required before 90 percent of the potential market will buy.

In my mind, the real question is not 1 or 2, but the nature of the exponential function. How many people can glance at this equation and visualize the basic shape of the resulting curve? Try it before reading further.

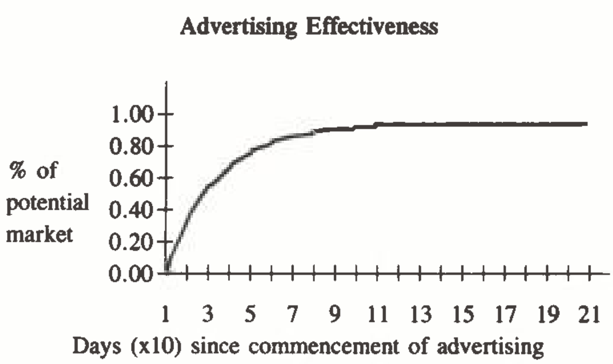
Using a spreadsheet, we have:

	A	B
1	Advertising Campaign	
2		
3	t	$f(t)$
4	Days	% of potential market
5	0	0.00
6	1	0.04
7	2	0.08
8	3	0.12
9	4	0.15
10	5	0.19
11	6	0.22
12	7	0.25
13	8	0.28
14	9	0.31
15	10	0.34
16	11	0.36
17	12	0.39
18	13	0.41
19	14	0.44
20	15	0.46
21	16	0.48
22	17	0.50
23	18	0.52
24	19	0.54
25	20	0.56

and the corresponding graph is:



By changing the scale, it is easy to see the effect of over 200 days.



The danger with topics such as this, without computer support, is that one may be able to arrive at desired answers but still have very little idea of what is really going on. Yet the underlying relationships can be intrinsically interesting if they are revealed. One can also discuss the effect of scale on the shape of the curve. Students should realize that they are always "writing on rubber."

An extension to the question is on the following page of the textbook:

> Unfortunately, the advertising company in the example overlooked the "turn-off factor," which is people reacting negatively to advertising. When this element is included, the function becomes $f(t) = 1 - 2.8^{0.04t} - 0.008t$.

Without going into the posed questions, what is the shape of this new curve? This is where meaning and understanding lie. A spreadsheet is useful here, not just to save labor (although it certainly does that) but to enhance understanding.

Pedagogy, Philosophy and People

If one views this paper as a teacher-developed handout for students then one can see a partial answer to the perennial question, "What do you do when you have one computer and a class of students?" It is a mistake to assume that using computers in education means that the material should always be presented on the computer. Until we reach the day when we all (student and teacher alike) have easy (home and school) access to a very powerful computing environment (even more powerful than the MacIntosh II or the IBM PS/2 model 80), various intermediate approaches continue to merit serious consideration.

Philosophy and pedagogy are currents in the same breeze (a Lethbridge metaphor). Our pedagogical principles are usually predicated on a set of implicit societal and cultural beliefs and assumptions. Speaking personally, my 15-plus years of working with students and teachers has lead me to the position that individuals are capable of responsible and intelligent thought when they are truly given some degree of autonomy for their own learning. I would like to suggest that much of our school system is predicated on a principle of cod-liver oil: "You may not like this now but take it anyway, it is good for you." The fact that many students have difficulty with school is not that they are immature or stupid, but that the overarching system is fundamentally an insult to their emerging maturity and intelligence. To build on it rather than to suppress it is far better. An analogous statement applies to teachers. Professionals deserve the respect of being capable of making sound pedagogical and educational decisions. I do not view a teacher as a disseminator of the curriculum but as a highly trained professional who is continually attempting to match curricular goals with individual learners.

I am happier when I visit classrooms where this respect for the individual is practised. And nowhere is this respect manifested more clearly than when the teacher relinquishes some control to the learner.

As with Logo, so too with spreadsheets. The principal task of the teacher is to set up a learning environment that permits, even encourages, learners to come to grips with the material in their own personal way. The task of the teacher is primarily to act as a resource and a support rather than as a "teacher" and a "drill sergeant." Classroom management becomes one of structuring the day so that the students have an opportunity to learn, rather than to keep a tight lid on everything. Classroom management is planning and motivating, not disciplining and ordering. The same attitude and approach that I take when working with Logo applies to working with spreadsheets. In both cases, the languages (be it Logo or the spreadsheet commands) are surface features of minimal importance. Learning Logo (as a programming language) or learning how to use a spreadsheet is not important and certainly should not be taught as an end in its own right. Both should be viewed as tools, permitting one to explore ideas within other domains, be it mathematics, ecology or advertising.

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