Essential Mathematics for the 21st Century

The National Council of Supervisors of Mathematics

Introduction

Students who enter kindergarten in 1988 can expect to graduate from high school in the year 2001. Yet these students who will graduate in the 21st century still frequently face a computation-dominated curriculum more suitable for the 19th century. To address this inconsistency, the National Council of Supervisors of Mathematics (NCSM) now updates its 1977 basic skills position statement to describe the essential mathematical competencies that citizens will need to begin adulthood in the next millenium. This position by NCSM is intended to complement and support positions on mathematics education by the National Council of Teachers of Mathematics and other professional groups.

Our technological world is changing at an ever-increasing rate, and our responsibilities in international affairs continue to increase. As the demands of society change, so do the essential competencies needed by individuals to live productively in that society. *All* students, including those of all races and both sexes, will need competence in essential areas of mathematics.

What Is Essential?

The NCSM views as essential those competencies that are necessary for the doors to employment and further education to remain open. Essential mathematics, as described in this paper, represents the mathematical competence students will need for responsible adulthood.

The students we educate today can expect to change jobs many times during their lifetimes. The jobs which they hold will develop and change around them. Often, specific job skills will not transfer from one position to another. To prepare for mobility, students must develop a thorough understanding of mathematical concepts and principles; they must reason clearly and communicate effectively; they must recognize mathematical applications in the world around them; and they must approach mathematical problems with confidence. Individuals will need the fundamental skills that will enable them to apply their knowledge to new situations and to take control of their own lifelong learning.

Skill in whole number computation is not an adequate indicator of mathematical achievement. Nor is it sufficient to develop skills apart from their applications or to memorize rules without understanding the concepts on which they are based. Students must understand mathematical principles; they must know when and how to use computation; and they must develop proficiency in problem solving and higher order thinking.

The NCSM position statement of 1977 responded to the "Back-to-Basics" movement with its overly narrow conception of basic skills. Now, as we look to the future, we recognize that the use of calculators and computers and the application of statistical methods will continue to expand. Creative problem solving, precise reasoning and effective communication will grow in importance. To function effectively in the next century, students will need proficiency in an enriched body of essential mathematics. The list that follows identifies 12 critical areas of mathematical competence for all students. It does not imply an instructional sequence or a priority among topics. In fact, the 12 essential mathematics areas are interrelated; competence in each area requires competence in other areas.

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Twelve Components of Essential Mathematics

Problem Solving

Learning to solve problems is the principal reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem solving, but students also should be faced with nontext problems. Problem solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams and using trial and error. Students should see alternate solutions to problems; they should experience problems with more than a single solution.

Communicating Mathematical Ideas

Students should learn the language and notation of mathematics. For example, they should understand place value and scientific notation. They should learn to receive mathematical ideas through listening, reading and visualizing. They should be able to present mathematical ideas by speaking, writing, drawing pictures and graphs, and demonstrating with concrete models. They should be able to discuss mathematics and ask questions about mathematics.

Mathematical Reasoning

Students should learn to make independent investigations of mathematical ideas. They should be able to identify and extend patterns and use experiences and observations to make conjectures (tentative conclusions). They should learn to use a counter example to disprove a conjecture, and they should learn to use models, known facts and logical arguments to validate a conjecture. They should be able to distinguish between valid and invalid arguments.

Applying Mathematics to Everyday Situations

Students should be encouraged to take everyday situations, translate them into mathematical representations (graphs, tables, diagrams or mathematical expressions), process the mathematics and interpret the results in light of the initial situation. They should be able to solve ratio, proportion, percent, direct variation and inverse variation problems. Not only should students see how mathematics is applied in the real world, but they should observe how mathematics grows from the world around them.

Alertness to the Reasonableness of Results

In solving problems, students should question the reasonableness of a solution or conjecture in relation to the original problem. Students must develop the number sense to determine if results of calculations are reasonable in relation to the original numbers and the operations used. With the increase in the use of calculating devices in society, this capability is more important than ever.

Estimation

Students should be able to carry out rapid approximate calculations through the use of mental arithmetic and a variety of computational estimation techniques. When computation is needed in a problem or consumer setting, an estimate can be used to check reasonableness, examine a conjecture or make a decision. Students should acquire simple techniques for estimating measurements such as length, area, volume and mass (weight). They should be able to decide when a particular result is precise enough for the purpose at hand.

Appropriate Computational Skills

Students should gain facility in using addition, subtraction, multiplication and division with whole numbers and decimals. Today, long, complicated computations should be done with a calculator or computer. Knowledge of single digit number facts is essential, and using mental arithmetic is a valuable skill. In learning to apply computation, students should have practice in choosing the appropriate computational method: mental arithmetic, paper-pencil algorithm or calculating device. Moreover, everyday situations demand recognition of, and simple computation with, common fractions. In addition, the ability to recognize, use and estimate with percents must also be developed and maintained.

Algebraic Thinking

Students should learn to use variables (letters) to represent mathematical quantities and expressions; they should be able to represent mathematical functions and relationships using tables, graphs and equations. They should understand and correctly use positive and negative numbers, order of operations, formulas, equations and inequalities. They should recognize the ways in which one quantity changes in relation to another.

Measurement

Students should learn the fundamental concepts of measurement through concrete experiences. They

should be able to measure distance, mass (weight), time, capacity, temperature and angles. They should learn to calculate simple perimeters, areas and volumes. They should be able to perform measurement in both metric and customary systems using the appropriate tools and levels of precision.

Geometry

Students should understand the geometric concepts necessary to function effectively in the three-dimensional world. They should have knowledge of concepts such as parallelism, perpendicularity, congruence, similarity and symmetry. Students should know properties of simple plane and solid geometric figures. Students should visualize and verbalize how objects move in the world around them using terms such as slides, flips and turns. Geometric concepts should be explored in settings that involve problem solving and measurement.

Statistics

Students should plan and carry out the collection and organization of data to answer questions in their everyday lives. Students should know how to construct, read and draw conclusions from simple tables, maps, charts and graphs. They should be able to present information about numerical data such as measures of central tendency (mean, median, mode) and measures of dispersion (range, deviation). Students should recognize the basic uses and misuses of statistical representation and inference.

Probability

Students should understand elementary notions of probability to determine the likelihood of future events. They should identify situations where immediate past experience does not affect the likelihood of future events. They should become familiar with how mathematics is used to help make predictions such as election results, business forecasts and outcomes of sporting events. They should learn how probability applies to research results and to the decision making process.

Climate for Learning

To learn the essential mathematics needed for the 21st century, students need a nonthreatening environment in which they are encouraged to ask questions and take risks. The learning climate should incorporate high expectations for all students, regardless of sex, race, handicapping condition or socioeconomic status. Students need to explore mathematics using manipulatives, measuring devices, models, calculators and computers. They need to have opportunities to talk to each other about mathematics.

Students need modes of instruction that are suitable for the increased emphasis on problem solving, applications and higher order thinking skills. For example, cooperative learning allows students to work together in problem solving situations to pose questions, analyze solutions, try alternative strategies and check for reasonableness of results.

To implement the new instructional strategies, extensive professional development opportunities as well as new learning materials will be needed.

Technology

Calculators should be used by students throughout the mathematics program, beginning in the primary grades. As adults, students will use calculators or computers to do difficult computations. They will need facility with single digit facts, estimation skills and mental arithmetic, and they must be able to determine if results obtained from calculators or computers are reasonable. Students will need practice in deciding if calculations should be done mentally, with paper and pencil or with a computing device.

The use of computers should be incorporated throughout the K-12 mathematics curriculum. Mathematics classrooms should be equipped with computers and projection devices or multiple large-screen monitors for classroom demonstrations. In addition, computer laboratories should be readily available to all students. Computer use should move away from drill and practice on isolated low level skills and toward meaningful involvement by students in problem solving and concept development.

Telecommunication technology should be used to ensure opportunities to learn mathematics are available to all students in all locations.

Evaluation

Evaluation at each administrative level, from the state or province level to the classroom, should be aligned with the objectives of the curriculum. At this time, caution should be exercised in using standardized tests to monitor student progress and evaluate the effectiveness of instruction. Existing standardized tests could perpetuate the domination of the mathematics curriculum by lower order skills. To prepare for the 21st century, there is a need for new tests that shift the focus from computation to problem solving and reasoning. Use of calculators

should be allowed on tests. In addition to paper and pencil tests, evaluation should involve other means such as teacher observations, interviews, student projects and presentations.

Extending the Essentials

As we move from the industrial society of the 20th century to the information society of the 21 century, knowledge of mathematics is becoming increasingly important for individuals who wish to have options for careers and higher education. Almost all careers require a background in mathematics. Most college majors require elementary algebra, advanced algebra and geometry as prerequisites. In addition, students specializing in fields like engineering, the sciences or business will need a precalculus course that includes trigonometry. Today most majors involve some statistics. Opportunities to learn statistics, probability, discrete mathematics and computer courses should be available in high school. Students who have completed a precalculus course by their senior year should have the opportunity to take a college level calculus or a discrete mathematics course in high school. Students should plan to take mathematics every year, avoiding gaps in their mathematics education that could require remediation when they resume their study of mathematics. The more mathematics students take in high school, the more options they will have for the future.

Today our society has inequities by sex and race in employment, income and participation in mathematics. To move toward an equitable society in the next century, we must address the disparities in mathematics education in this century. Working together with high expectations for all, we will offer our students education that will provide pathways to opportunity in the 21st century.