

From Theory into Practice

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Preamble

“Let problem solving be the focus for the '80s.” The NCTM's Agenda for Action clearly calls for reform in the ways and on the emphasis mathematics teachers have traditionally given to problem solving.

As a result of the Secondary Education Review, Alberta Education highlights the maximization of individual critical, creative and conceptual thinking processes. Further, curriculum proposals for implementation from 1989 to 1991 include heavy emphasis on problem solving skills throughout the high school mathematics curriculum.

With these points as background, the purpose of this paper is to report on methods implemented at Bishop Carroll High School to prepare for the inevitable influence of this “new” curriculum on all future students of mathematics.

As a further result of the Secondary Education Review, teachers in the math department at Bishop Carroll High School began a problem solving project in September 1986. The project focused on students' abilities to think and to solve problems. The target group was the Grade 10 student mathematics population. The emphasis was to change the focus, at the Grade 10 level, from one directed exclusively at finding answers to traditional word problems, to one of acquiring problem solving skills and strategies. The goal, ultimately, was to enable students to be better able to apply these skills to a wide variety of unfamiliar problem solving situations.

One year later, in September 1987, with several revisions and modifications, a commitment has been

made to the teaching and learning of problem solving skills not only at the Grade 10 level but also in the Grade 11 mathematics program. The enthusiasm of students and teachers and the stimulation that began in project form now makes the foundation on which new ideas and challenges continue to build.

Bishop Carroll High School is based entirely on a continuous progress, individualized instruction mode of learning. Nevertheless, the objectives and application of the methods of instruction and learning related to problem solving are easily transferred to a typical high school mathematics classroom.

Our department goals, derived from those suggested by Randall Charles and Frank Lester (1982), are

1. to develop students' awareness of several different problem solving strategies,
2. to improve students' abilities to select and appropriately use these strategies,
3. to acquaint students with Polya's four-step model as a framework they can use to approach problems in a systematic manner,
4. to foster students' willingness and perseverance in solving problems,
5. to better students' self-concept with respect to their ability to solve problems and their use of mathematical skills.

Math 10 Problem Solving

Students entering the program are introduced to the four topics involving problem solving and the way they are integrated throughout the Math 10 course, as follows.

Topic I: An Introduction—Expectations

Students are presented with the rationale for becoming better problem solvers and are introduced to Polya's framework.

The following problem solving format illustrates the expectations we have for students to solve and then document their attempts and efforts.

Problem # _____

Problem:

Understand
(Clarify key words, relate in own words)

Make a plan
(Name heuristics/strategies)

Carry out the plan

Look back
(Check reasonableness of your answer, state complete answer)

Comments

Emphasis is placed on each of the steps of Polya's model with the intent that students who may normally give up will at least be able to progress through the *understanding* step (where they identify key words for phrases, restate the problem in their own words, clarify wanted, needed and given data) to the next step, *making a plan*. Very often by the time students draw a diagram or make a table, a method of solution will appear where no immediate solution had been evident.

Next, students are shown a list of six strategies or "heuristics" that they are expected to become more proficient at using. The six strategies emphasized this year are as follows:

1. Draw a picture or diagram.
2. Find a pattern.
3. Make an organized list.
4. Make a table.
5. Solve a simpler problem.
6. Work backwards.

Topic II: Solving Word Problems

This unit is constructed around solving traditional age and number types of word problems, translating English into algebra, and making and solving equations.

Topic III and IV: Flexibility in Selecting and Choosing Problems

A potpourri of problem solving activities offered throughout the year form the basis for these additional problem solving requirements.

Approximately 10 additional hours of student problem solving experiences are expected and may be fulfilled through participation in one or more of the following.

1. Weekly, teacher-facilitated, hour-long problem solving sessions. Teachers select several suitable nonroutine problems which may be solved using a specific strategy. The students have the option of working alone or in small groups toward presenting a solution to the problems. Students may share ideas and present their solutions to the group.
2. "Problems of the Week" are posted on a bulletin board each week. Students may hand in their solutions to any of these, using the standardized format sheets.
3. Creating extension problems. A paper and pencil assignment asks for five original problems modeled on dimension, coin, mixture and investment type problems. Here is an example submitted by one student:

The Calgary Olympic Organization is selling 2,000 Hidy and Howdy mascot souvenirs. The Hidy souvenirs cost \$1.75 each and Howdy's cost \$2.25 each. If the total is \$3,950.00, how many of each are there?
4. Large group presentations feature biweekly problem solving motivational sessions prepared by a teacher and delivered to up to 200 students at one time.

5. Other activities vary throughout the year. A highly successful one offered as an option was a "3-on-3 Problem Solving Tournament." Ten percent of the Grade 10 students signed up on teams of three students to compete in a timed competition during Mathematics Education Week.

To meet the needs of those students with exceptional interest and/or mathematical aptitude, regular biweekly honors problem solving sessions are offered. Honors Math 10 students may attend these in place of the regular weekly Grade 10 sessions. Mathematics contests and competitions also form a part of the Mathematics Honors Club problem solving experiences.

As mentioned earlier, the focus on problem solving has mushroomed into the Grade 11 program at Bishop Carroll High School. Grade 11 students experience a more integrated, curriculum specific exposure to problem solving based on knowledge acquired throughout their Grade 11 mathematics year. Packages of problems have been prepared which require students to choose four of eight problems per package to work through on their own or within weekly group problem solving sessions.

The continued emphasis in Grade 11 is on the "Carry Out the Plan" and "Look Back" stage of Polya's model. Solutions must be verified or problems extended. Answers appear next to each problem and it is suggested that students experiencing difficulty explore the strategy of "Working Backwards." An expanded list of 10 former and new strategies from which to choose is presented at this level.

Honors packages of more challenging course-related problems have been prepared for students continuing to pursue an honors mathematics program in Grade 11.

I believe it is highly desirable to share positive experiences with fellow colleagues in an effort to further the cause of teaching in today's complex and ever changing environment. I hope that any ideas gleaned from this paper will be useful to other educators in the mathematics community. I gratefully acknowledge the contributions and dedication of colleagues Gerry Fijal, Patricia McManus, Susan Osterkamp and Joseph Shenher.

Reference

Charles, Randall, and Frank Lester. *Teaching Problem Solving: What, Why and How*. Palo Alto, California: Dale Seymour Publications, 1982.