PERMANENT.

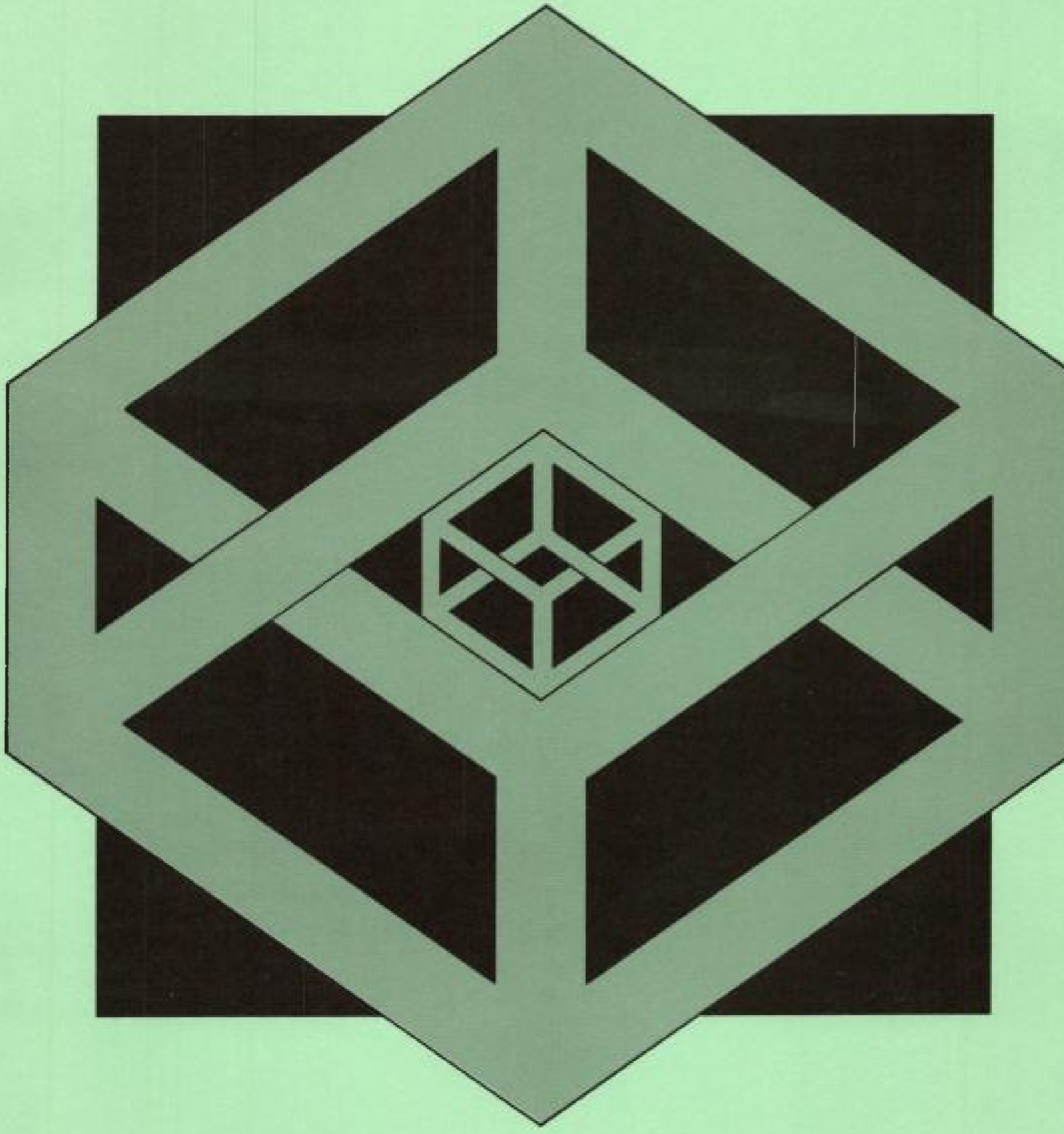
delta-k

JOURNAL OF THE MATHEMATICS COUNCIL OF THE ALBERTA TEACHERS' ASSOCIATION

123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789

Volume 27, Number 4 June 1989

123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789



delta-k

JOURNAL OF THE MATHEMATICS COUNCIL OF THE ALBERTA TEACHERS' ASSOCIATION

Volume 27, Number 4 June 1989

CONTENTS

EDPRESS

delta-K is published by The Alberta Teachers' Association (ATA) for the Mathematics Council (MCATA). EDITOR: John B. Percevault, 2510 22 Avenue South, Lethbridge, Alberta T1K 1J5. EDITORIAL AND PRODUCTION SERVICES: Central Word Services staff, ATA. Please address all correspondence regarding this publication to the editor. *delta-K* is indexed in the Canadian Education Index. ISSN 0319-8367

Achievement Testing Program

On two occasions during September, I attended meetings in which Alberta Education presented the Achievement Testing Program to teachers. In each instance, I recognized two distinct but mutually exclusive views. I am certain that Alberta Education was saying that an Achievement Testing Program would be implemented. Teachers were saying that the program was educationally ill-advised, that the results were poorly used and that the cost/educational benefit ratio of the program was questionable.

The document *Proposed Enhancements to the Achievement Testing Program,* distributed by Alberta Education at the September meeting of the MCATA executive, states:

> In order to address the weaknesses in the current program, it is proposed that the Achievement Testing Program shift in emphasis from that of collecting information for monitoring and program evaluation purposes to collecting information that can be used to serve the needs of students more directly.

Certainly, this is a goal that The Alberta Teachers' Association (ATA) and Alberta Education must support. Both must be concerned with providing the best educational experience for students. Despite this admirable goal, an attitude of confrontation prevailed.

What, then, may be the cause of concern of interested teachers? The two major concerns appear to be

1. the educational justification of an external testing program, and
2. the misuse of results of such testing programs in rating teachers.

The use of standardized or actual authority tests is difficult to justify. In principle, such tests must be invalid. There is no recognition that classrooms are fundamentally unique. No one test has validity across the normal level of diversity. Consider the effect of mainstreaming on the test results attained in certain classrooms. Such tests shift the curriculum implementation away from the needs of the students to teaching for students to do well on the tests. I suggest that centralized tests measure objectives that are easily tested while most worthwhile educational objectives are not easily tested. I suggest that Alberta Education is still trying to determine how to test the problem solving process, an essential ingredient of all mathematics curriculum. I doubt if Alberta Education has even begun to think of testing the desirable objective of teaching students to think critically. Finally, student performance on tests is measured under strict timelines. Compare results of the students' performance on test items when adequate time is given for the students to show their mastery of concepts to those when a time limit is imposed.

I am confident that this brief case against centralized testing is well known to Alberta Education officials. However, teachers must recognize that Alberta Education not only has the right but the responsibility to monitor educational achievement in the province. Also recognize that the teacher who "teaches to the test" is abrogating professional standards and responsibility to the student. Such action is unprofessional as is the misuse of test results.

The misuse of results of centralized testing programs is of primary concern to teachers and apparently of little concern to Alberta Education. At the meetings I attended, the following statements were made:

1. An assistant superintendent of schools, in discussing his school authority's results on a provincial test, stated that his objective was to have every student achieve beyond the provincial average.

2. The test results were listed by school and in comparison to provincial averages.
3. Principals have reassigned teachers to teach at grade levels other than those grade levels subjected to the provincial testing program. The reason? You know it!

Other statements could be made to add to the political misuse of test results. Unfortunately, such misuse does not enhance the teacher's value and demeans the profession.

The unfortunate irony of the situation is that the administrators (described above), at whatever position in the hierarchy they occupy, deem themselves as professionals. Their actions belie the claim. Eventually they must realize that their peers view such statements as more political self-aggrandizement than professional.

Concerns are easily identified. The remediation process is harder to specify. The ATA should be able to discipline its members if it becomes aware of a grievance. Alberta Education must become involved at the political level with school authorities and administrators. If the Achievement Testing Program is to proceed (I suggest that it is more a reality for the future than a proposal), Alberta Education may be well advised to judiciously distribute the tests to districts that do not misuse the results.

Alberta Education and the ATA must analyze the cost/benefit ratio if the needs of students are to be served. Alternate methods of monitoring achievement could be designed. Test item banks could be developed allowing teachers to choose from the items at their discretion. The money now budgeted could be better utilized in effective inservice programs for teachers who have recognized a need for improving instruction. The goal "to serve the needs of the student more effectively" may then be achieved. It must be a cooperative effort of Alberta Education and the ATA.

Teachers must become more proactive in identifying the misuse of test results. The place to start is internal to the profession. Those responsible for the education of our children must cease compounding each other's mistakes and work toward cooperative resolution of the confrontational attitude to achieve the goal of serving students.

John B. Percevault

Essential Mathematics for the 21st Century

The National Council of Supervisors of Mathematics

Introduction

Students who enter kindergarten in 1988 can expect to graduate from high school in the year 2001. Yet these students who will graduate in the 21st century still frequently face a computation-dominated curriculum more suitable for the 19th century. To address this inconsistency, the National Council of Supervisors of Mathematics (NCSM) now updates its 1977 basic skills position statement to describe the essential mathematical competencies that citizens will need to begin adulthood in the next millenium. This position by NCSM is intended to complement and support positions on mathematics education by the National Council of Teachers of Mathematics and other professional groups.

Our technological world is changing at an ever-increasing rate, and our responsibilities in international affairs continue to increase. As the demands of society change, so do the essential competencies needed by individuals to live productively in that society. *All* students, including those of all races and both sexes, will need competence in essential areas of mathematics.

What Is Essential?

The NCSM views as essential those competencies that are necessary for the doors to employment and further education to remain open. Essential mathematics, as described in this paper, represents the mathematical competence students will need for responsible adulthood.

The students we educate today can expect to change jobs many times during their lifetimes. The jobs which they hold will develop and change around them. Often, specific job skills will not transfer from one position to another. To prepare for mobility, students must develop a thorough understanding of mathematical concepts and principles; they must reason clearly and communicate effectively; they must recognize mathematical applications in the world around them; and they must approach mathematical problems with confidence. Individuals will need the fundamental skills that will enable them to apply their knowledge to new situations and to take control of their own lifelong learning.

Skill in whole number computation is not an adequate indicator of mathematical achievement. Nor is it sufficient to develop skills apart from their applications or to memorize rules without understanding the concepts on which they are based. Students must understand mathematical principles; they must know when and how to use computation; and they must develop proficiency in problem solving and higher order thinking.

The NCSM position statement of 1977 responded to the "Back-to-Basics" movement with its overly narrow conception of basic skills. Now, as we look to the future, we recognize that the use of calculators and computers and the application of statistical methods will continue to expand. Creative problem solving, precise reasoning and effective communication will grow in importance. To function effectively in the next century, students will need proficiency in an enriched body of essential mathematics. The list that follows identifies 12 critical areas of mathematical competence for all students. It does not imply an instructional sequence or a priority among topics. In fact, the 12 essential mathematics areas are interrelated; competence in each area requires competence in other areas.

Reprinted with permission of the National Council of Supervisors of Mathematics, Volume XVII, Number 4, June 1988.

Twelve Components of Essential Mathematics

Problem Solving

Learning to solve problems is the principal reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem solving, but students also should be faced with nontext problems. Problem solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams and using trial and error. Students should see alternate solutions to problems; they should experience problems with more than a single solution.

Communicating Mathematical Ideas

Students should learn the language and notation of mathematics. For example, they should understand place value and scientific notation. They should learn to receive mathematical ideas through listening, reading and visualizing. They should be able to present mathematical ideas by speaking, writing, drawing pictures and graphs, and demonstrating with concrete models. They should be able to discuss mathematics and ask questions about mathematics.

Mathematical Reasoning

Students should learn to make independent investigations of mathematical ideas. They should be able to identify and extend patterns and use experiences and observations to make conjectures (tentative conclusions). They should learn to use a counter example to disprove a conjecture, and they should learn to use models, known facts and logical arguments to validate a conjecture. They should be able to distinguish between valid and invalid arguments.

Applying Mathematics to Everyday Situations

Students should be encouraged to take everyday situations, translate them into mathematical representations (graphs, tables, diagrams or mathematical expressions), process the mathematics and interpret the results in light of the initial situation. They should be able to solve ratio, proportion, percent, direct variation and inverse variation problems. Not only should students see how mathematics is applied in the real world, but they should observe how mathematics grows from the world around them.

Alertness to the Reasonableness of Results

In solving problems, students should question the reasonableness of a solution or conjecture in relation to the original problem. Students must develop the number sense to determine if results of calculations are reasonable in relation to the original numbers and the operations used. With the increase in the use of calculating devices in society, this capability is more important than ever.

Estimation

Students should be able to carry out rapid approximate calculations through the use of mental arithmetic and a variety of computational estimation techniques. When computation is needed in a problem or consumer setting, an estimate can be used to check reasonableness, examine a conjecture or make a decision. Students should acquire simple techniques for estimating measurements such as length, area, volume and mass (weight). They should be able to decide when a particular result is precise enough for the purpose at hand.

Appropriate Computational Skills

Students should gain facility in using addition, subtraction, multiplication and division with whole numbers and decimals. Today, long, complicated computations should be done with a calculator or computer. Knowledge of single digit number facts is essential, and using mental arithmetic is a valuable skill. In learning to apply computation, students should have practice in choosing the appropriate computational method: mental arithmetic, paper-pencil algorithm or calculating device. Moreover, everyday situations demand recognition of, and simple computation with, common fractions. In addition, the ability to recognize, use and estimate with percents must also be developed and maintained.

Algebraic Thinking

Students should learn to use variables (letters) to represent mathematical quantities and expressions; they should be able to represent mathematical functions and relationships using tables, graphs and equations. They should understand and correctly use positive and negative numbers, order of operations, formulas, equations and inequalities. They should recognize the ways in which one quantity changes in relation to another.

Measurement

Students should learn the fundamental concepts of measurement through concrete experiences. They should be able to measure distance, mass (weight), time, capacity, temperature and angles. They should learn to calculate simple perimeters, areas and volumes. They should be able to perform measurement in both metric and customary systems using the appropriate tools and levels of precision.

Geometry

Students should understand the geometric concepts necessary to function effectively in the three-dimensional world. They should have knowledge of concepts such as parallelism, perpendicularity, congruence, similarity and symmetry. Students should know properties of simple plane and solid geometric figures. Students should visualize and verbalize how objects move in the world around them using terms such as slides, flips and turns. Geometric concepts should be explored in settings that involve problem solving and measurement.

Statistics

Students should plan and carry out the collection and organization of data to answer questions in their everyday lives. Students should know how to construct, read and draw conclusions from simple tables, maps, charts and graphs. They should be able to present information about numerical data such as measures of central tendency (mean, median, mode) and measures of dispersion (range, deviation). Students should recognize the basic uses and misuses of statistical representation and inference.

Probability

Students should understand elementary notions of probability to determine the likelihood of future events. They should identify situations where immediate past experience does not affect the likelihood of future events. They should become familiar with how mathematics is used to help make predictions such as election results, business forecasts and outcomes of sporting events. They should learn how probability applies to research results and to the decision making process.

Climate for Learning

To learn the essential mathematics needed for the 21st century, students need a nonthreatening environment in which they are encouraged to ask questions and take risks. The learning climate should incorporate high expectations for all students, regardless of sex, race, handicapping condition or socioeconomic status. Students need to explore mathematics using manipulatives, measuring devices, models, calculators and computers. They need to have opportunities to talk to each other about mathematics.

Students need modes of instruction that are suitable for the increased emphasis on problem solving, applications and higher order thinking skills. For example, cooperative learning allows students to work together in problem solving situations to pose questions, analyze solutions, try alternative strategies and check for reasonableness of results.

To implement the new instructional strategies, extensive professional development opportunities as well as new learning materials will be needed.

Technology

Calculators should be used by students throughout the mathematics program, beginning in the primary grades. As adults, students will use calculators or computers to do difficult computations. They will need facility with single digit facts, estimation skills and mental arithmetic, and they must be able to determine if results obtained from calculators or computers are reasonable. Students will need practice in deciding if calculations should be done mentally, with paper and pencil or with a computing device.

The use of computers should be incorporated throughout the K-12 mathematics curriculum. Mathematics classrooms should be equipped with computers and projection devices or multiple large-screen monitors for classroom demonstrations. In addition, computer laboratories should be readily available to all students. Computer use should move away from drill and practice on isolated low level skills and toward meaningful involvement by students in problem solving and concept development.

Telecommunication technology should be used to ensure opportunities to learn mathematics are available to all students in all locations.

Evaluation

Evaluation at each administrative level, from the state or province level to the classroom, should be aligned with the objectives of the curriculum. At this time, caution should be exercised in using standardized tests to monitor student progress and evaluate the effectiveness of instruction. Existing standardized tests could perpetuate the domination of the mathematics curriculum by lower order skills. To prepare for the 21st century, there is a need for new tests that shift the focus from computation to problem solving and reasoning. Use of calculators should be allowed on tests. In addition to paper and pencil tests, evaluation should involve other means such as teacher observations, interviews, student projects and presentations.

Extending the Essentials

As we move from the industrial society of the 20th century to the information society of the 21 century, knowledge of mathematics is becoming increasingly important for individuals who wish to have options for careers and higher education. Almost all careers require a background in mathematics. Most college majors require elementary algebra, advanced algebra and geometry as prerequisites. In addition, students specializing in fields like engineering, the sciences or business will need a precalculus course that includes trigonometry. Today most majors involve some statistics. Opportunities to learn statistics, probability, discrete mathematics and computer courses should be available in high school. Students who have completed a precalculus course by their senior year should have the opportunity to take a college level calculus or a discrete mathematics course in high school. Students should plan to take mathematics every year, avoiding gaps in their mathematics education that could require remediation when they resume their study of mathematics. The more mathematics students take in high school, the more options they will have for the future.

Today our society has inequities by sex and race in employment, income and participation in mathematics. To move toward an equitable society in the next century, we must address the disparities in mathematics education in this century. Working together with high expectations for all, we will offer our students education that will provide pathways to opportunity in the 21st century.

From Theory into Practice

Barbara J. Morrison

This paper was prepared by Barbara J. Morrison, and it was presented jointly with Joseph Shenher at the MCATA Conference in Calgary, October 1987. Barbara and Joseph teach at Bishop Carroll High School, Calgary.

Preamble

"Let problem solving be the focus for the '80s." The NCTM's Agenda for Action clearly calls for reform in the ways and on the emphasis mathematics teachers have traditionally given to problem solving.

As a result of the Secondary Education Review, Alberta Education highlights the maximization of individual critical, creative and conceptional thinking processes. Further, curriculum proposals for implementation from 1989 to 1991 include heavy emphasis on problem solving skills throughout the high school mathematics curriculum.

With these points as background, the purpose of this paper is to report on methods implemented at Bishop Carroll High School to prepare for the inevitable influence of this "new" curriculum on all future students of mathematics.

As a further result of the Secondary Education Review, teachers in the math department at Bishop Carroll High School began a problem solving project in September 1986. The project focused on students' abilities to think and to solve problems. The target group was the Grade 10 student mathematics population. The emphasis was to change the focus, at the Grade 10 level, from one directed exclusively at finding answers to traditional word problems, to one of acquiring problem solving skills and strategies. The goal, ultimately, was to enable students to be better able to apply these skills to a wide variety of unfamiliar problem solving situations.

One year later, in September 1987, with several revisions and modifications, a commitment has been made to the teaching and learning of problem solving skills not only at the Grade 10 level but also in the Grade 11 mathematics program. The enthusiasm of students and teachers and the stimulation that began in project form now makes the foundation on which new ideas and challenges continue to build.

Bishop Carroll High School is based entirely on a continuous progress, individualized instruction mode of learning. Nevertheless, the objectives and application of the methods of instruction and learning related to problem solving are easily transferred to a typical high school mathematics classroom.

Our department goals, derived from those suggested by Randall Charles and Frank Lester (1982), are

1. to develop students' awareness of several different problem solving strategies,
2. to improve students' abilities to select and appropriately use these strategies,
3. to acquaint students with Polya's four-step model as a framework they can use to approach problems in a systematic manner,
4. to foster students' willingness and perseverance in solving problems,
5. to better students' self-concept with respect to their ability to solve problems and their use of mathematical skills.

Math 10 Problem Solving

Students entering the program are introduced to the four topics involving problem solving and the way they are integrated throughout the Math 10 course, as follows.

Topic I: An Introduction—Expectations

Students are presented with the rationale for becoming better problem solvers and are introduced to Polya's framework.

The following problem solving format illustrates the expectations we have for students to solve and then document their attempts and efforts.

Problem #________

Problem:

Understand
(Clarify key words, relate in own words)

Make a plan
(Name heuristics/strategies)

Carry out the plan

Look back
(Check reasonableness of your answer, state complete answer)

Comments

Emphasis is placed on each of the steps of Polya's model with the intent that students who may normally give up will at least be able to progress through the *understanding* step (where they identify key words for phrases, restate the problem in their own words, clarify wanted, needed and given data) to the next step, *making a plan.* Very often by the time students draw a diagram or make a table, a method of solution will appear where no immediate solution had been evident.

Next, students are shown a list of six strategies or "heuristics" that they are expected to become more proficient at using. The six strategies emphasized this year are as follows:

1. Draw a picture or diagram.
2. Find a pattern.
3. Make an organized list.
4. Make a table.
5. Solve a simpler problem.
6. Work backwards.

Topic II: Solving Word Problems

This unit is constructed around solving traditional age and number types of word problems, translating English into algebra, and making and solving equations.

Topic III and IV: Flexibility in Selecting and Choosing Problems

A potpourri of problem solving activities offered throughout the year form the basis for these additional problem solving requirements.

Approximately 10 additional hours of student problem solving experiences are expected and may be fulfilled through participation in one or more of the following.

1. Weekly, teacher-facilitated, hour-long problem solving sessions. Teachers select several suitable nonroutine problems which may be solved using a specific strategy. The students have the option of working alone or in small groups toward presenting a solution to the problems. Students may share ideas and present their solutions to the group.

2. "Problems of the Week" are posted on a bulletin board each week. Students may hand in their solutions to any of these, using the standardized format sheets.

3. Creating extension problems. A paper and pencil assignment asks for five original problems modeled on dimension, coin, mixture and investment type problems. Here is an example submitted by one student:

   The Calgary Olympic Organization is selling 2,000 Hidy and Howdy mascot souvenirs. The Hidy souvenirs cost $1.75 each and Howdy's cost $2.25 each. If the total is $3,950.00, how many of each are there?

4. Large group presentations feature biweekly problem solving motivational sessions prepared by a teacher and delivered to up to 200 students at one time.

5. Other activities vary throughout the year. A highly successful one offered as an option was a "3-on-3 Problem Solving Tournament." Ten percent of the Grade 10 students signed up on teams of three students to compete in a timed competition during Mathematics Education Week.

To meet the needs of those students with exceptional interest and/or mathematical aptitude, regular biweekly honors problem solving sessions are offered. Honors Math 10 students may attend these in place of the regular weekly Grade 10 sessions. Mathematics contests and competitions also form a part of the Mathematics Honors Club problem solving experiences.

As mentioned earlier, the focus on problem solving has mushroomed into the Grade 11 program at Bishop Carroll High School. Grade 11 students experience a more integrated, curriculum specific exposure to problem solving based on knowledge acquired throughout their Grade 11 mathematics year. Packages of problems have been prepared which require students to choose four of eight problems per package to work through on their own or within weekly group problem solving sessions.

The continued emphasis in Grade 11 is on the "Carry Out the Plan" and "Look Back" stage of Polya's model. Solutions must be verified or problems extended. Answers appear next to each problem and it is suggested that students experiencing difficulty explore the strategy of "Working Backwards." An expanded list of 10 former and new strategies from which to choose is presented at this level.

Honors packages of more challenging course-related problems have been prepared for students continuing to pursue an honors mathematics program in Grade 11.

I believe it is highly desirable to share positive experiences with fellow colleagues in an effort to further the cause of teaching in today's complex and ever changing environment. I hope that any ideas gleaned from this paper will be useful to other educators in the mathematics community. I gratefully acknowledge the contributions and dedication of colleagues Gerry Fijal, Patricia McManus, Susan Osterkampf and Joseph Shenher.

Reference

Charles, Randall, and Frank Lester. *Teaching Problem Solving: What, Why and How.* Palo Alto, California: Dale Seymour Publications, 1982.

Some More on the Formulation of Mathematical Problems

John G. Heuver

John Heuver teaches mathematics at Grande Prairie Composite High School, Grande Prairie.

George A. Calder's article, "Algebra Can Be a Language," in the September 1988 issue of *delta-K*, has a number of errors in it. German and French words have been anglicized. However, the use of a dictionary is quite legitimate as a tool for teaching word problems since it quickly puts the task at hand into focus. Even the late Marc Kac (1959) used it.

The article also raises some thought-provoking questions that are worthwhile formulating:

1. How do mathematical theories come about? In the preface to his book, Thomas M. Thompson (1983) quotes from an interview with Richard M. Hamming, who in 1947 had access to a computer:

> Two weekends in a row I came in and found that all my stuff had been dumped and nothing was done. . . . And so I said, " . . . if the machine can detect an error, why can't it locate the position of the error and correct it?"

This formulates pretty well the origin of a present day flourishing branch of mathematics, namely, coding theory with spin-offs in many other branches of mathematics.

2. Some problems do not lend themselves to easy mathematical solutions. Richard K. Guy (1983) from the University of Calgary is editor of the "Unsolved Problems" section in *The American Mathematical Monthly*. He speaks of the difficulty in posing certain problems which he illustrates by giving an example.

> Jose M. Bayod, University of Santander, Santander, Spain, notes that in some parts of his country, a farmer owns a part of the mountains that surround his farm, according to the "flowing water law" which can be stated thus: A spot of rain lands at A on the mountain and flows downhill until it reaches a point B on the farm. Then the owner of point B in the valley also owns point A on the mountain. The main problem is to build a mathematical model for the problem: given an area in the valley, what part of the mountain is associated with it?

One can just imagine the amount of detailed information that would be required to attempt a mathematical solution. A general solution that would work under all circumstances seems even more difficult. Luckily, we are informed in the article that the real-life situation causes very few problems.

Solving and posing problems is the lifeblood of mathematics. To give the student an opportunity to learn how to solve problems, we need carefully stated and well-posed problems.

3. Following are examples from past Grade 12 mathematics exams on how not to state problems. The January 1987 Mathematics 30 exam, question 6 states:

> An automobile tire has a mean life of 64,000 km with a standard deviation of 3,200 km. In a purchase of 1,500 tires, the number lasting less than 54,000 km would be. . . .

The masses must have quoted a pleasing answer to the satisfaction of Alberta Education. The question has no answer without information about the distribution function, if such exists.

The January 1988 exam contains the following treasure in question 40:

Three coins are tossed simultaneously. The probability that the coins will land with two tails and one head showing is. . . .

The trouble here is that no mention is made of the probability that the coins show head or tail. Providing the department with a detailed analysis of the situation resulted in a cold shoulder. It would have been satisfactory to state that the coins were fair.

Almost every departmental exam has errors of this type.

References

Guy, Richard K. "An Olla-podrida of the Open Problems Often Oddly Posed." *The American Mathematical Monthly.* Mathematical Association of America, Washington, D.C., March 1983, 196.

Kac, Marc. *Statistical Independence in Probability, Analysis and Number Theory.* The Carus Mathematical Monographs, Mathematical Association of America, 1959, 8.

Thompson, Thomas M. *From Error Correcting Codes Through Sphere Packings to Simple Groups.* The Carus Mathematical Monographs, Mathematical Association of America, 1983, vii.

Mathematical Modeling Using Spreadsheets

J. Dale Burnett

J. Dale Burnett is associate professor in the Faculty of Education, University of Lethbridge. This paper was presented at the MCATA Conference in Calgary, October 1987.

In 1979, Ken Iverson, born and raised in Camrose, received the ACM Turing Award, one of the highest honors in computing science. This award was presented to Dr. Iverson for his development of a computer language called APL. The title of his Turing address was "Notation as a Tool of Thought." In the early 1980s, concomitant with the emergence of the microcomputer, were two other notational developments. One was Logo, a computer language developed by Seymour Papert and his colleagues at MIT. Papert has referred to the Logo environment as "objects to think with." Another notational development was the spreadsheet, originally developed by Dan Bricklin and Bob Frankston and marketed as Visi-Calc. Many people believe that it was the spreadsheet that gave impetus to the micro revolution. Although the spreadsheet was first used within business environments, its more general use is now beginning to be appreciated.

The developments are milestones, not only in their ostensible domain of computing science, but, even more importantly, in the evolution of human thought. Their true value is not in what they are, but in what they permit. Quite simply, they permit ideas and ways of thought that are not possible otherwise. Notational conventions both enhance and restrict ideas. Consider the advances in mathematics and in civilization that have accrued as a result of switching from Roman numerals to the present Arabic system.

This paper will attempt to show that spreadsheets deserve serious consideration by educators as a notational system for exploring ideas. Spreadsheets should not be viewed as a topic in computing science or business courses, but as a tool like cursive writing which transcends the curriculum. This paper should be viewed as the opening of a door rather than as a comprehensive survey of possible applications.

The paper is divided into three sections. The first section examines an example on equations and graphing, taken from the Grade 10 mathematics curriculum. The second section looks at a section from the Grade 12 syllabus that discusses exponential functions, although the intent is to show the potential of the topic in science and economics. Finally, a reflective section examines the pedagogical and philosophic implications in what has been presented.

The substantive impact of computers on education will not be at the technological level. It will be at the tacit level where we come to recognize and reflect on the underlying assumptions of schools and education. The purpose of this paper is not to extoll the virtues of computers or even spreadsheets, although both will be done. Rather the purpose is to suggest ways in which our classrooms can better honor the integrity of the educational process.

Equations and Graphing

The following question is typical of those found in a unit on equations and graphing at the Grade 10 level: Solve $2(4x - 7) = 5x + 10$.

Usually the pedagogical emphasis is on deriving equivalent equations by applying the same arithmetic operation to both sides of the equation. Fundamentally the emphasis is on skills. Certainly there is concern for understanding, but the assumption is that such understanding will emerge from having the student properly solve a reasonably large number of similar problems. There may be efforts to encourage back substitution to verify an answer, but the critical criterion tends to be the answer at the back of the book or the answer that the teachers have on their desks. The basic pedagogical paradigm is some small variation from the model of doing a few problems in front of the class and then asking the class to do 10 or 20 more "for practice."

Graphical interpretations may be mentioned once or twice but the labor involved militates against extensive use of such an approach. This is a legitimate instructional decision—until recently. A number of software packages are now available that make it relatively easy for the student to obtain the graph of a specified mathematical expression. Spreadsheets are one such family of packages, as we shall see.

What does it mean to solve an expression like $2(4x - 7) = 5x + 10$? When you obtain a solution, what do you have? Could there be more than one solution? Why or why not? Could there be no solution? Why or why not? There would appear to be room for social interaction and discussion here. Opportunities abound for students to reflect on their understanding and to share it with their peers. Precision in language and thought becomes important for communication.

A Spreadsheet Approach

Perhaps the curriculum objective that currently reads: "Maintain skills in solving first degree equations with rational coefficients," which current textbooks are well designed to facilitate, should be rephrased, "Develop an understanding of the relationships inherent in a first degree equation," which textbooks are not well designed for but which emerging software programs are designed to support.

One of the difficulties at present is that many students are still relatively unfamiliar with computers and particularly with spreadsheets. This is likely to become less of a problem in the future as computers continue to become more powerful and cheaper, but the present classroom teacher is faced with the dual problems of teaching the tool as well as the application. A suggestion is to focus on the application and introduce spreadsheet concepts and commands as they are needed.

Let's reexamine the problem, Solve $2(4x - 7) = 5x + 10$, using a spreadsheet approach. First, try to get a better understanding of the left side of the equation. If we write this as $y = 2(4x - 7)$, then we can see how the value of the expression changes as x changes. One could try a few values of x to get a feel for the substitution process. Thus, keeping it simple, if x is 1 then y is $2(4 - 7)$ which is $2(-3)$ which gives -6. To save time and effort, we can use spreadsheets to do most of the tedious work for us. However, we must now give the proper instructions to the spreadsheet program. The particular spreadsheet program used while preparing this paper was Microsoft's Excelf on a MacIntosh Plus computer, but many of the ideas should transfer to other spreadsheet programs.

Spreadsheets are essentially a tabular structure of rows and columns. The resulting cells may contain information such as words or phrases and numbers. In addition, cells may contain formulas for computing new values from the values stored in other cells.

In column A, row 5 (leave the first four rows blank for possible titles later) have the students type in the expression x. In column B, row 5 type in the expression $y = 2(4x - 7)$. Emphasize that so far they have simply typed in a couple of labels for their benefit. In fact, this might be a good opportunity to add a title for this activity: "Solve $2(4x - 7) = 5x + 10$" is a possible title (place this in column A, row 1).

Now begin to type in numeric values. At this point, students may have little idea of an appropriate range of values, so you may suggest they try some conventional guesses such as integers in the range 0 to 10 or 0 to 100 or -100 to 100. They can always be changed later so having a good guess is not all that important. Judgment comes from an awareness of context. The context here is "textbook problems," so small integral values are likely useful. I hope we will begin to see a loosening of this restriction as computers become more widespread.

Thus, in column A, row 6 students would type in the lower boundary of their domain.

Now come the key spreadsheet commands.

In column B, row 6 have the students type in the formula =2*(4*A6 − 7). When they press return, the value in the cell B6 is automatically computed and displayed. Similarly, the right hand side of the original expression may be computed in column C. The spreadsheet can display either the formulas or the computed values. Thus we now have, displaying the values:

	A	B	C
1	Solve $2(4x - 7) = 5x + 10$		
2			
3			
4			
5	x	$y = 2(4x - 7)$	$y = 5x + 10$
6	1	−6	15
7			

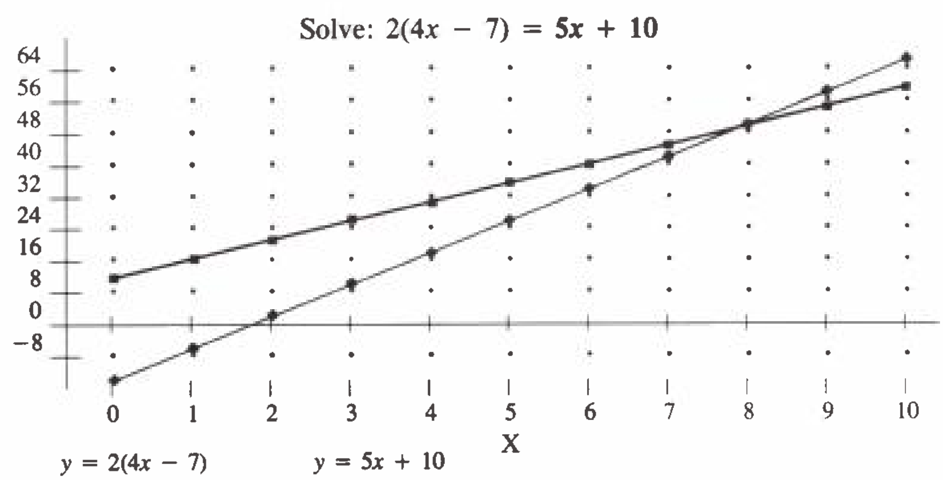
Using the Fill Down command, you can quickly get a large table of computed values. This is where the computational power of the spreadsheet first becomes apparent. Thus selecting rows 6 through 15 and using the Fill Down feature yields:

	A	B	C
1	Solve $2(4x - 7) = 5x + 10$		
2			
3			
4			
5	x	$y = 2(4x - 7)$	$y = 5x + 10$
6	1	−6	15
7	2	2	20
8	3	10	25
9	4	18	30
10	5	26	35
11	6	34	40
12	7	42	45
13	8	50	50
14	9	58	55
15	10	66	60

The labor involved in producing this table is truly trivial—just a few keystrokes and mouse clicks. However, much is going on. Paramount, particularly in the mind of the user, is the issue of notation. When we are typing in an expression to look like it does in the text, we type a string of characters such as $2(4x - 7)$. When we type a formula into a cell to indicate how the spreadsheet is to compute the value for that cell we type =2*(4*A6 − 7). Here the conventions are different from normal usage. The = symbol at the beginning signifies that what follows is a formula. The * is the explicit symbol for multiplication. Instead of a variable name represented by a letter, it is now represented by a cell label (for example, A6). Questions related to order of operations and use of parentheses are likely to come up. Flexibility with notation may be one of the hidden benefits from using spreadsheet approaches. How many of us are familiar with the students who can solve for x but not for w? The resulting table provides an explicit representation of the different values for two expressions as x changes. The students can see that as x gets larger, so do each of the expressions $2(4x - 7)$ and $5x + 10$. What patterns are noticeable? One column increases in increments of 8, the other in increments of 5. Why? Row 13 (where x is 8) is also noteworthy. Why?

At this stage, you may wish to take time out from the original problem to explore expression. What is the relative impact of doubling any of the original coefficients? In what ways can the original equations be modified?

Returning to the original problem, we can also ask for a graph of this table. Here the spreadsheet approach becomes exciting! Once again, only a few commands give us the following picture:



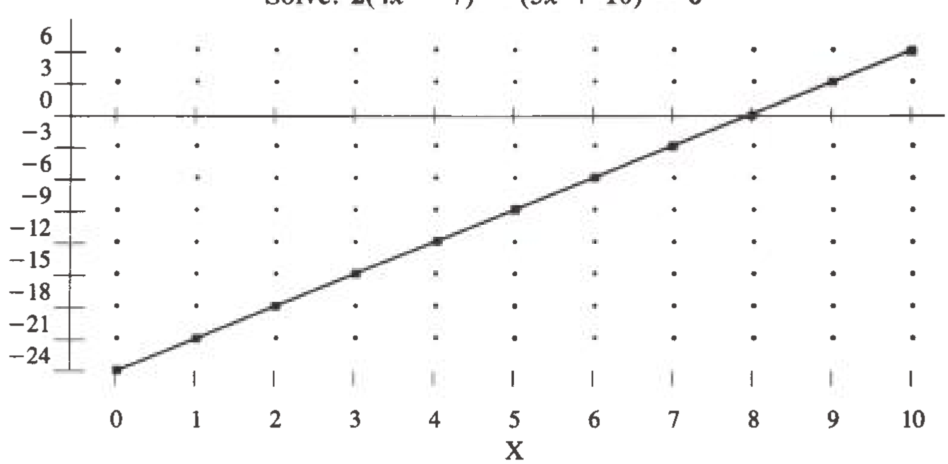
We now see that the original question may be reworded to say, "For what value(s) of x do the equations of the two straight lines have the same value?" This appears to have more inherent meaning than the original question, where the point about straight lines is not likely to even occur. One should also note the parallels between the tabular and graphical representations of the same idea.

Some students may realize that the original equation is equivalent to Solve $2(4x - 7) - (5x + 10) = 0$. The table and graph now look like:

	1	2	3	4	5	6	7
1	Solve: $2(4x - 7) = 5x + 10$			equivalent to: $2(4x - 7) - (5x + 10) = 0$			
2							
3	x	$2(4x - 7)$	$5x + 10$		$(4x - 7) - (5x + 10) = 0$		
4							
5	0	−14	10		−24		
6	1	1−6	15		−21		
7	2	12	20		−18		
8	3	10	25		−15		
9	4	18	30		−12		
10	5	26	35		−9		
11	6	34	40		−6		
12	7	42	45		−3		
13	8	50	50		0		
14	9	58	55		3		
15	−10	66	60		6		

and

Solve: $2(4x - 7) - (5x + 10) = 0$



$2(4x - 7) - (5x + 10) = 0$

The student should now compare the two quite different representations of the same problem (two intersecting lines versus one line which crosses the x-axis). Are these different representations compatible?

From here the student could be encouraged to play with the spreadsheet formulation. To change one or both of the equations and to see the effect graphically is an easy matter. In fact, it is not a large step to equations that are not straight lines. A couple of weeks of exploration and helpful hints may well cover a large component of the entire high school curriculum! The following quotation is from a recent article by Gordon (1987, 5) on cultural comparisons among schools: "A Japanese teacher might spend one whole day on just one problem, working and reworking it from every angle, until every student understood."

Learning is a complex process. Thus the behaviorist tradition that spawned many of our current educational practices has had some success. In the absence of alternatives, we tend to stay with approaches that have had some success over those that have had no success or even those that have an unknown rate of success. To some extent, this is highly defensible and lauditory. We do not want to "fool around with our students" while experimenting with new educational procedures. On the other hand, this guideline can become a shackle. We do use new materials and textbooks from time to time. What are the appropriate next steps for the mathematics education community, given recent developments in computer technology and software?

Mathematical Modeling

As indicated, spreadsheets permit one to explore higher order polynomials. Other mathematical functions, such as those found in trigonometry and economics as well as exponential and logarithmic functions, are provided within most spreadsheet programs. Topics in calculus, such as the exploration of the concepts of derivative and integral from first principles, are possible. I am confident that many more topics are waiting for the teacher or student who has the time (say, an hour or two) to explore various ideas.

Problem solving was the theme of the 1980 NCTM Yearbook. Yet, often the cry has a hollow ring to it. Too often it is equated with the rote memorization of "type problems." The existence of a problem, and the existence of an algorithm for solving the problem, and even the correct application of the algorithm to the problem is not a sufficient condition for one to say that problem solving has occurred. Problem solving refers to a situation where there is a discrepancy between your present state and a future state, and you don't know how to close the gap. Problem solving has a hueristic tone to it, not an algorithmic one. It has a sense of urgency and action, rather than passivity and autonomicity; a sense of excitement rather than drudgery; a feeling of play rather than work. True problem solving often implies that even the teacher is not too sure what to do next. This is not to be feared but welcomed.

The following example is taken from *Holt Mathematics 6* (p. 153):

An advertising company, asked to market a new product, estimates that within t days of the commencement of an advertising campaign the percentage of the total market which will buy that product is given by the exponential function $f(t) = 1 - 2.8^{-0.04t}$. Determine

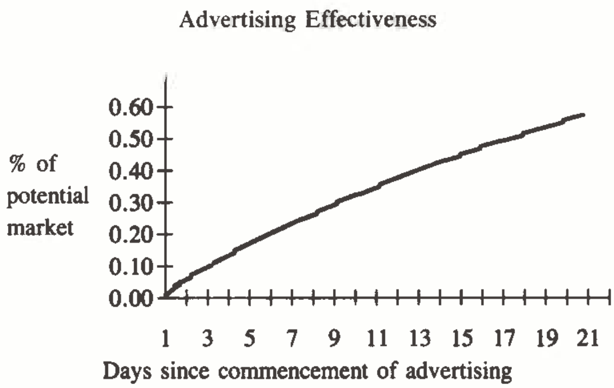
1. the percentage of the potential market which will buy within 20 days of the campaign,
2. the number of days required before 90 percent of the potential market will buy.

In my mind, the real question is not 1 or 2, but the nature of the exponential function. How many people can glance at this equation and visualize the basic shape of the resulting curve? Try it before reading further.

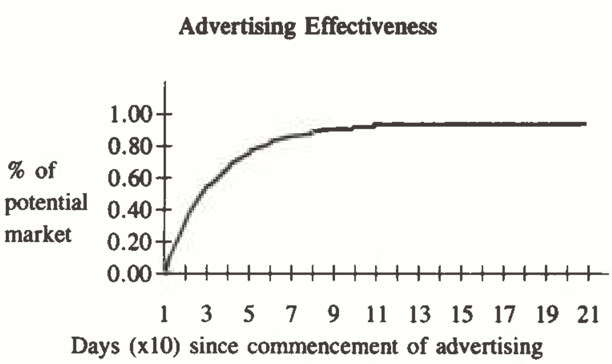
Using a spreadsheet, we have:

	A	B
1	Advertising Campaign	
2		
3	t	$f(t)$
4	Days	% of potential market
5	0	0.00
6	1	0.04
7	2	0.08
8	3	0.12
9	4	0.15
10	5	0.19
11	6	0.22
12	7	0.25
13	8	0.28
14	9	0.31
15	10	0.34
16	11	0.36
17	12	0.39
18	13	0.41
19	14	0.44
20	15	0.46
21	16	0.48
22	17	0.50
23	18	0.52
24	19	0.54
25	20	0.56

and the corresponding graph is:



By changing the scale, it is easy to see the effect of over 200 days.



The danger with topics such as this, without computer support, is that one may be able to arrive at desired answers but still have very little idea of what is really going on. Yet the underlying relationships can be intrinsically interesting if they are revealed. One can also discuss the effect of scale on the shape of the curve. Students should realize that they are always "writing on rubber."

An extension to the question is on the following page of the textbook:

> Unfortunately, the advertising company in the example overlooked the "turn-off factor," which is people reacting negatively to advertising. When this element is included, the function becomes $f(t) = 1 - 2.8^{0.04t} - 0.008t$.

Without going into the posed questions, what is the shape of this new curve? This is where meaning and understanding lie. A spreadsheet is useful here, not just to save labor (although it certainly does that) but to enhance understanding.

Pedagogy, Philosophy and People

If one views this paper as a teacher-developed handout for students then one can see a partial answer to the perennial question, "What do you do when you have one computer and a class of students?" It is a mistake to assume that using computers in education means that the material should always be presented on the computer. Until we reach the day when we all (student and teacher alike) have easy (home and school) access to a very powerful computing environment (even more powerful than the MacIntosh II or the IBM PS/2 model 80), various intermediate approaches continue to merit serious consideration.

Philosophy and pedagogy are currents in the same breeze (a Lethbridge metaphor). Our pedagogical principles are usually predicated on a set of implicit societal and cultural beliefs and assumptions. Speaking personally, my 15-plus years of working with students and teachers has lead me to the position that individuals are capable of responsible and intelligent thought when they are truly given some degree of autonomy for their own learning. I would like to suggest that much of our school system is predicated on a principle of cod-liver oil: "You may not like this now but take it anyway, it is good for you." The fact that many students have difficulty with school is not that they are immature or stupid, but that the overarching system is fundamentally an insult to their emerging maturity and intelligence. To build on it rather than to suppress it is far better. An analogous statement applies to teachers. Professionals deserve the respect of being capable of making sound pedagogical and educational decisions. I do not view a teacher as a disseminator of the curriculum but as a highly trained professional who is continually attempting to match curricular goals with individual learners.

I am happier when I visit classrooms where this respect for the individual is practised. And nowhere is this respect manifested more clearly than when the teacher relinquishes some control to the learner.

As with Logo, so too with spreadsheets. The principal task of the teacher is to set up a learning environment that permits, even encourages, learners to come to grips with the material in their own personal way. The task of the teacher is primarily to act as a resource and a support rather than as a "teacher" and a "drill sergeant." Classroom management becomes one of structuring the day so that the students have an opportunity to learn, rather than to keep a tight lid on everything. Classroom management is planning and motivating, not disciplining and ordering. The same attitude and approach that I take when working with Logo applies to working with spreadsheets. In both cases, the languages (be it Logo or the spreadsheet commands) are surface features of minimal importance. Learning Logo (as a programming language) or learning how to use a spreadsheet is not important and certainly should not be taught as an end in its own right. Both should be viewed as tools, permitting one to explore ideas within other domains, be it mathematics, ecology or advertising.

References

Fryer, K.D., et al. *Holt Mathematics* 6. Toronto: Holt, Rinehart and Winston of Canada, 1981.

Gordon, B. "Cultural Comparisons of Schooling." *Educational Researcher* 16, no. 6 (August–September 1987): 4–7.

Iverson, K.E. "Notation as a Tool of Thought." *Communications of the ACM* 23, no. 8, (1980): 444–65.

Krulik, S., and R.E. Reys, eds. *Problem Solving in School Mathematics.* 1980 Yearbook of the National Council of Teachers of Mathematics. Reston, Virginia: NCTM, 1980.

Papert, S. *Mindstorms.* New York: Basic Books, 1980.

Psychology in Teaching Mathematics

Marlow Ediger

Marlow Ediger is professor of Mathematics Education, Northeast Missouri State University, Kirksville, Missouri. Dr. Ediger is a frequent contributor to delta-K.

Numerous reputable psychologies are provided to assist mathematics teachers in guiding students to achieve optimally. The teacher of mathematics should study the diverse psychologies of education to implement the best teaching strategy possible. The lay public focuses on student achievement in the three Rs or the basics. Mathematics represents a highly salient basic. Students need to do well in mathematics to do well in school and in society. Teachers need to select objectives, learning opportunities and appraisal procedures that assist learners to achieve as well as is possible.

Behaviorism in the Mathematics Curriculum

Precise, measurably stated objectives and their use is the heart of behaviorism. Objectives are selected prior to being implemented in the classroom. Generally, no student participation has been emphasized in selecting these goals. Behaviorism can be emphasized with state mandated objectives in terms of core competencies and key skills. At the state level, precise measurably stated objectives have been chosen. The department of education of each state selects a cross section of educators to agree upon the stated ends. The mathematics teacher then plans learning opportunities to help students attain each objective.

A second example of behaviorism emphasizes instructional management systems (IMS) at the district level. The central office selects a cross section of teachers within the district to select salient objectives in mathematics. Again, the classroom teacher must emphasize each objective in teaching-learning situations.

The mathematics teacher, without stated mandated objectives or IMS, may write and implement specific ends for student attainment.

An early pioneer in measurably stated objectives and their use was B.F. Skinner. Dr. Skinner emphasized programmed learning in either textbook or software form. The ingredients of programmed learning include

1. sequential items of small amounts of information acquired by students in each step of learning;
2. students responding to a test item, such as a multiple-choice question based on information presented in book or software form;
3. learners receiving feedback based on the response made;
4. reinforcement being rather common with high frequency of correct responses made.

Behaviorism, in its diverse manifestations, emphasizes that a student either does or does not achieve an objective as a result of instruction. If an objective is not attained, the mathematics teacher needs to try a different teaching strategy.

Behaviorism appears to be a dominant psychology of education emphasized in the teaching of mathematics. With the popularity of behaviorism, the writer recommends

1. that each objective in mathematics be carefully selected in terms of being useful in school as well as in society;
2. that students achieve success in attaining sequential objectives;
3. that a variety of challenging learning opportunities be provided for learners to attain each end;

4. that students experience meaning, interest and purpose in achieving desired ends;
5. that critical and creative thinking, as well as problem solving, receive ample attention in the mathematics curriculum;
6. that appraisal procedures be varied, valid and reliable to evaluate learner progress.

Humanism in the Math Curriculum

Humanism, as a psychology of learning, emphasizes students being heavily involved in determining objectives, learning opportunities and evaluation procedures. Each student is guided to attain self-realization. The late A.H. Maslow (1954), humanist psychologist, listed five sequential levels where individuals need assistance to achieve realization of self:

1. assisting students to meet physiological needs, such as adequate food, clothing and proper shelter;
2. helping learners to feel safe and secure in their environment;
3. guiding students in meeting love and belonging needs;
4. developing situations in which esteem needs of students are being met;
5. assisting learners to achieve self-actualization.

Only after the above sequential needs of students have been met can students achieve optimally, according to humanism as a psychology of learning. Meeting the needs of students to increase achievement behooves any school system.

Input from learners in selecting objectives, learning opportunities and appraisal procedures is highly important. There are several excellent ways of emphasizing humanism in the mathematics curriculum. One plan is to utilize learning centres. More tasks than any one student can complete would be at the diverse centres. Students learn to make decisions. They choose, sequentially, which tasks to complete and which to omit. Each learner then selects what is perceived to be of interest, meaning and purpose. Sequence in selecting ordered tasks resides within the student. A psychological curriculum is then evident. Internally, the student makes choices in terms of tasks to pursue.

A second plan of humanism as a psychology of education is to use a contract system. In a contract, the students and their teachers together plan specific learning opportunities for the former to complete. There must be considerable input from the students in the contract for humanistic psychology to be evident. The due date of the contract is indicated with the students' and the teachers' signatures.

A third plan of humanism is when the teacher lists, for example, 10 activities for students to consider to complete in mathematics. Each student may choose 5 or more to complete. The student here has input as to what to pursue and what to omit.

Humanism emphasizes a humane mathematics curriculum. Humanness is defined as students being able to decide from among alternatives which learning activities have value and need to be completed satisfactorily.

The writer, in evaluating humanism in teaching mathematics, recommends that

1. worthwhile tasks be developed for students to pursue sequentially (trivia is to be omitted for learners to pursue);
2. students be guided to stay on task and not digress from achieving relevant objectives;
3. tasks be written on diverse levels of achievement to challenge each student to achieve as much as possible.

The Structure of Knowledge

During the 1960s and early 1970s, much emphasis was placed upon mathematicians on the higher education level identifying structural ideas for public school students to attain. The structure of knowledge emphasized underlying principles that provided a framework for an academic discipline. Thus, in the academic discipline of mathematics, selected broad generalizations provided a structure for students in ongoing lessons and units. The key ideas included the commutative property of addition and multiplication, the distributive property of multiplication over addition, the property of closure and the identity elements.

The structure of knowledge approach, as identified by Jerome Bruner of Harvard University and his associates (1960), emphasized that public school students utilize methods of learning used by mathematicians on the higher education level. An inductive procedure is then evident. Students are guided by the teacher to learn by discovery in moving from the specific to the general to achieve structural ideas. Materials to use in teaching students to acquire content inductively include inactive (manipulative items), iconic (pictures, drawings, slides and filmstrips emphasizing main ideas) and symbolic (abstract content such as printed words and numerals).

The structure of knowledge approach has much to recommend itself. The writer recommends that

1. teachers emphasize structural ideas in a spiral curriculum. However, the spiral curriculum should not be excessively repetitious. The structure has a built-in review when these key generalizations receive attention at more complex levels in mathematics curriculum;
2. induction receive adequate attention in teaching-learning situations. However, continued use of inductive methods is time-consuming. The mathematics teacher needs to inject meaningful explanations also at definite points in ongoing lessons and units;
3. creative teaching in using diverse methodologies be emphasized thoroughly. Methods and subject matter have to be adjusted to the present achievement level of each student as students differ in interests, purposes and present levels of achievement.

Diagnosis in Mathematics

Mathematics teachers must utilize the concept of diagnosis in teaching-learning situations. To diagnose means to pinpoint specific difficulties students experience in computation, concept development and problem solving. Students need assistance to overcome errors made.

Robert Gagné (1985) advocates a hierarchy of objectives be stated in measurable terms for student attainment. If a learner cannot achieve a specific end, the teacher needs to move to an easier sequential objective. Reversing to easier ends is necessary until the student's present attainment level is found. The last three levels of Gagné's hierarchy are especially important to know when teachers diagnose difficulties students experience in mathematics. The three in sequence are concept learning, rule learning and problem solving. Thus, if students cannot solve a problem in mathematics, the teachers need to assist the former to determine if they understand the involved rules. For example, if the problem involves finding the volume of a cylinder, the students must understand the involved formula: $r^2\pi h$. If the learners do not understand the rule to determine the volume of a cylinder, they need assistance in attaching meaning to concepts. The separate concepts are radius, radius times radius, pi and height.

Diagnosis is involved when the mathematics teacher assists the student to pinpoint specific weaknesses in a lesson or unit. Gagné provides a quality model for mathematics teachers to follow in helping learners to progress sequentially.

In using diagnostic-remediation procedures in the teaching of mathematics, the writer recommends that

1. students attach meaning to each sequential step of learning;
2. learners be assisted to perceive holism and sequence in the subject matter learned. Diagnosis is available if a student fails to attach meaning to ongoing rules (generalizations) and concepts to solve problems in mathematics.

Conclusion

Relevant principles of learning from the psychology of education need to be implemented in teaching-learning situations. The teacher of mathematics must assist each student to attain in an optimal manner.

Four schools of thought were discussed in the psychology of education: behaviorism, humanism, the structure of knowledge and diagnosis based on a hierarchy of objectives.

The writer recommends that teachers of mathematics

1. implement tenets of behaviorism with its measurably stated objectives. Higher levels of cognition must not be hindered with the use of behaviorism in teaching-learning situations;
2. provide ample opportunities for students to engage in decision making. Learners need to have chances to select sequential learning opportunities as advocated by humanism;
3. stress the structure of knowledge so that students may perceive that subject matter is related;
4. adequately diagnose and remediate students' problems in lessons and units. Students need to perceive mathematics as being holistic and not isolated specifics in diagnostic/remediation situations.

References

Ashlock, Robert B. *Error Patterns in Computation.* Columbus, Ohio: Charles E. Merrill Publishing Company, 1982.

Bley, Nancy S., and Carol A. Thornton. *Teaching Mathematics to the Learning Disabled.* Rockville, Maryland: An Aspen Publication, 1981.

Ballew, Hunter. *Teaching Children Mathematics.* Columbus, Ohio: Charles E. Merrill Publishing Company, 1973.

Bruner, Jerome S. *The Process of Education.* New York: Vintage Books, 1960.

Cawley, John F. *Cognitive Strategies and Mathematics for the Learning Disabled.* Rockville, Maryland: An Aspen Publication, 1985.

Fehr, Howard F., and Jo McKeeby Phillips. *Teaching Modern Mathematics in the Elementary School.* 2d ed. Reading, Massachusetts: Addison-Wesley, 1972.

Gagné, Robert. *The Conditions of Learning*. New York: Holt, Rinehart and Winston, 1985.

Higley, Joan. *Activities Deskbook for Teaching Arithmetic Skills.* West Nyack, New York: Parker Publishing Company, 1983.

Jenson, Rosalie. *Exploring Mathematical Concepts and Skills in the Elementary School.* Columbus, Ohio: Charles E. Merrill Publishing Company, 1973.

Kennedy, Larry G. *Guiding Children's Learning of Mathematics.* 4th ed. Belmont, California: Wadsworth Publishing Company, 1984. Chapter One.

Lesh, Richard, and Marshall Landau, eds. *Acquisition of Mathematics Concepts and Processes.* New York: Academic Press, 1983.

Maslow, A.H. *Motivation and Personality.* New York: Harper and Row, 1954.

Stern, Catherine, and Margaret B. Stern. *Children Discover Arithmetic.* New York: Harper and Row, 1971.

Building a Bird Feeder Station

A. Craig Loewen

A. Craig Loewen is assistant professor in the Faculty of Education at the University of Lethbridge and is a doctoral student at the University of Alberta.

Many students have developed the opinion that mathematics is not very useful. It is not surprising in a way, as mathematics is rarely taught with a specific purpose in mind. When pushed by our students, many of us find ourselves giving trite answers to questions such as, "When am I ever going to use this?" Yet, mathematics is useful. All of that which surrounds us is rooted in mathematics, and much of our world would be different if someone had not started looking for patterns, postulating theorems and generalizing principles to new situations. The "useful" nature of mathematics is reflected in the *Junior High Mathematics Curriculum Guide*: "An application is the process of using a mathematical skill to arrive at a solution to a real life or practical situation. This aspect of mathematics is extremely important for all students to experience" (Alberta Education 1978, 7). As can be seen, the concept of applications has existed for many years. Mathematics teachers simply need to learn how to identify and adapt applications to their own teaching situations, and need to start collecting a variety of possible projects which are useful for teaching specific mathematical concepts. This paper presents the building of a bird feeder station as one possible application which would enhance the teaching of much of the junior high mathematics curriculum.

Defining and Evaluating Applications

The definition above states that applications must somehow relate specific mathematical principles and properties to the world which surrounds the students. More generally, applications should provide the intuitive sense that mathematics is real, valuable and useful.

A publication of the National Council of Teachers of Mathematics (Bushaw et al. 1980, vii) delineates three different kinds of applications:

1. Short problems similar in size and difficulty (but not in quality!) to traditional "story problems."
2. Medium-length problems that might serve as topics for a full class meeting.
3. More time-consuming problems that could be used as bases for individual study projects and the like.

The major difference in each of these types of applications is the time that is required to introduce the application, and the time required to develop and generalize from the experiences the application provides. This paper is intended to provide an example of a project application.

Bushaw et al. also list some standards by which applications may be identified and evaluated. First, the data must be realistic. If the numbers or examples within a problem do not simulate some aspect of the real world or the manner in which events unfold in the real world, then it is not an acceptable application.

Second, real data are preferable to that which the teacher creates. Wherever it is possible or reasonable for students to do so, they should be asked to research and collect data themselves. The process of collecting data allows students to fully realize that mathematics is an inherent element of our world, and it also allows the students to invest a part of themselves in the project. The greater the degree of personal investment, the greater the benefit in terms of student motivation, cooperation and interest.

Third, the application must have a sense of relevance. If the students cannot fathom why anyone would want to engage in a particular project, then that project is not appropriate for those students.

Fourth, some mathematical approach must be necessary. It is not an acceptable application if the students are not challenged by the project. Furthermore, it is not an acceptable application if the students need not apply any of their mathematics skills or knowledge to resolve any of the problems inherent within the project.

Fifth, the project must not rely upon the repetitive use of a memorized algorithm. To use the context of the application to introduce, develop and present mathematical concepts and principles is acceptable and desirable, but to simply ask the students to use a formula not previously discussed or to use an equation which was developed elsewhere is not acceptable. The purpose of applications is to bring to the surface the meaningful nature of mathematics, not simply to provide another source or form of drill and practice exercises.

Building the Bird Feeder Station

To assemble the feeder, follow these instructions:

1. Cut the bottom piece and seed retainer pieces (see ① in Diagram 1). Attach these pieces to the bottom of the feeder station using small nails (see Diagram 2).

2. Cut the back piece and attach to the bottom piece (see ② in Diagram 1) with two screws which pass through the back piece into the bottom piece.

3. Cut the internal tin slider and bend it into the correct shape (see ③ in Diagram 1). Attach this slider to the back and bottom using four small nails (see Diagram 3).

4. Cut the side pieces and the four glass guides to fit to the inside fronts of the side pieces (see Diagram 4). Attach both sets of sliders to the side pieces; each glass guide will require three small nails (see ④ in Diagram 1).

5. Attach the sides to the back and the bottom (see ⑤ in Diagram 1). Two screws on each side will pass through the side piece into the bottom piece. Two screws on each side of the back piece will pass through the back piece and into the side pieces.

6. Cut the glass front and insert between the glass guides. The glass front is held in place by the glass guides and is held up from the bottom piece by resting on the seed retainer pieces.

7. Cut the top piece and lid to fit. The top piece should be large enough to leave a 2 cm eave around each side of the bird feeder. Attach the top piece with four screws (see ⑥ in Diagram 1).

8. Paint the exterior and lid of the bird feeder station.

9. Once the paint is dry, attach the lid to the top of the bird feeder station using a single screw (see ⑦ in Diagram 1).

10. Fill the bird feeder station with birdseed and attach it outside the classroom window, or attach it to a backyard fence or pole.

Note: Not all dimensions of each individual piece are given. Many of the specifications are deliberately minimized to promote active discussion and problem solving. Students will need to visualize precisely how certain pieces will fit together (for example, the back piece to the top piece) to make decisions regarding exact dimensions.

Correspondence to the Mathematics Curriculum

A project as large as this relates to many different components of the junior high curriculum. A few are identified below.

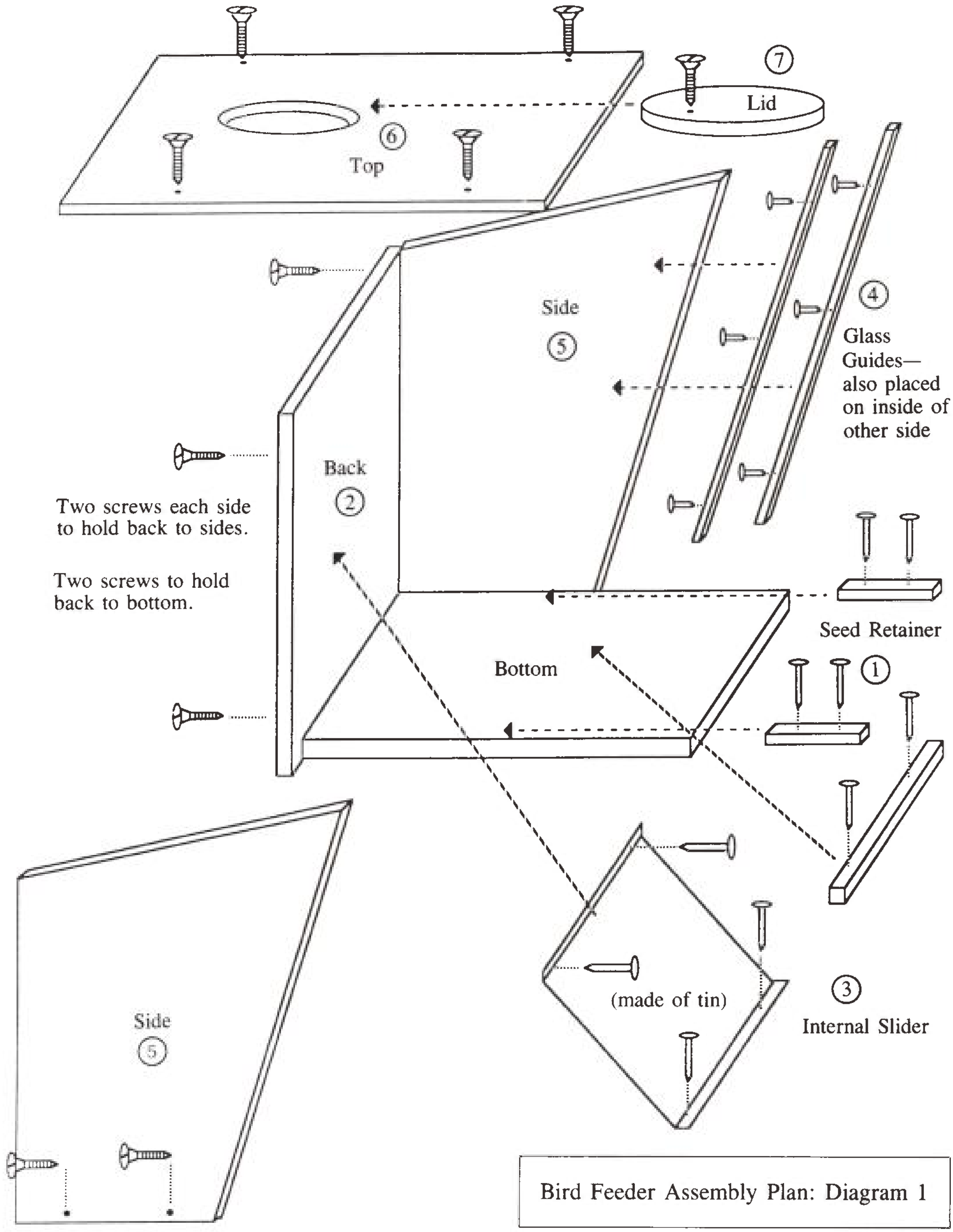
1. Pythagorean theorem. There are several points at which the Pythagorean theorem may be implemented in the completion of this project. In the construction of the side pieces, two right angled triangles are removed from a rectangular piece of wood (see Diagram 4). The lengths of the hypotenuses of these triangles relates to the length of the top of the feeder station and to the length of the glass front and glass guides. For example, once the triangle that measures 16 cm along one side and 4 cm along a second side is removed, the length of the hypotenuse is needed to calculate the dimensions for the top piece as shown below:

Length of Top = Lengths of Eaves + Length of Hypotenuse

$= (2 \text{ cm} + 2 \text{ cm}) + \sqrt{(16 \text{ cm})^2 + (4 \text{ cm})^2}$

$= 4 \text{ cm} + 16.5 \text{ cm}$

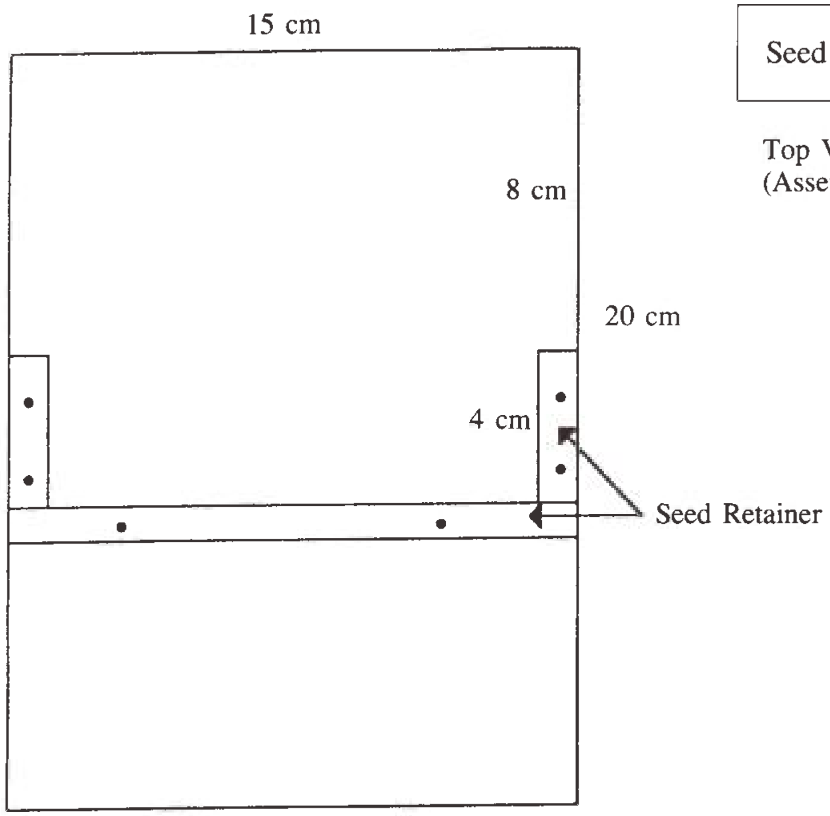
$= 20.5 \text{ cm}$



Bird Feeder Assembly Plan: Diagram 1

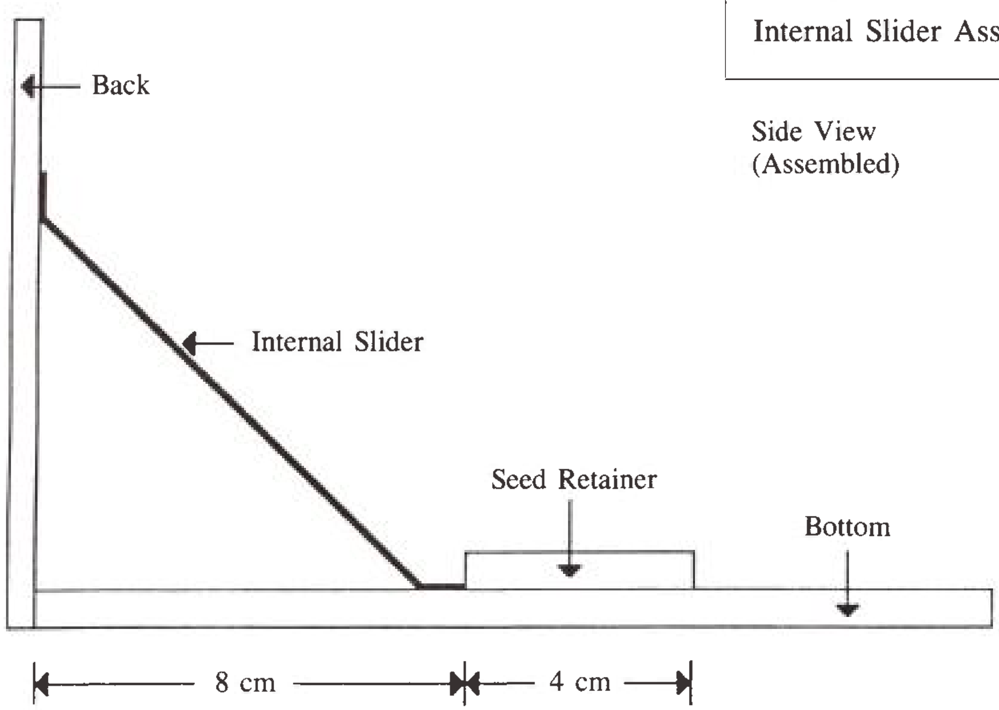
Seed Retainer Assembly: Diagram 2

Top View
(Assembled)

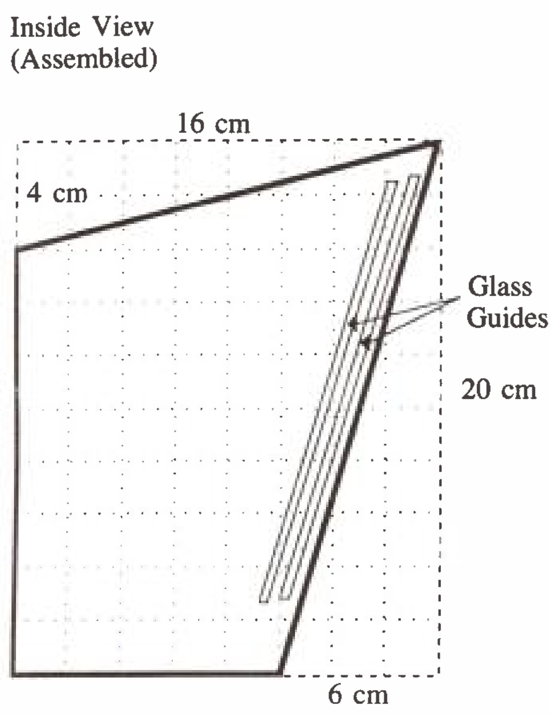


Internal Slider Assembly: Diagram 3

Side View
(Assembled)



Side Piece Assembly: Diagram 4



The Pythagorean theorem could also be employed to calculate the length of the internal tin slider. This problem is a little more complicated as the exact length is variable, although the tin piece must be at least 11 cm in length.

2. Problem Solving. To calculate the dimensions of certain pieces prior to cutting them from the wood, many multistep problems must be completed. These problems often require (and allow students to practise) much higher order thinking skills such as visualizing, generalizing, estimating, rounding and evaluating. As the teacher works through these problems with the students, the teacher has the opportunity to point out the different approaches which students adopt and thus identify the variety of strategies which are available in problem solving. Some students will want to try guessing and checking, others will try to list needed information, others will look for simpler or alternate questions. All of these approaches could lead to correct answers in the variety of specific tasks which are a part of this project. The problem solving nature of this project is a direct function of the amount of information which is given, thus teachers should be careful not to give away too many answers. A better teaching strategy would be to encourage students to ask alternate questions of themselves and their peers. In general, as the problems associated with this project are solved, the students will have the opportunity to demonstrate interest in problem solving and experiment with a variety of heuristics, and the teacher will have an opportunity to discuss the stages and strategies of problem solving.

3. Surface area. The surface area must be calculated in deciding how much wood and other materials need to be purchased for one station or for the entire class set. External surface area will also be needed to calculate the amount of paint needed to finish the project.

4. Volume. The volume of the feeder is important when filling the stations with birdseed. This problem is certainly not trivial, as several triangular prisms must be subtracted from the basic rectangular shape of the interior. The teacher will also have the opportunity to discuss the need for appropriate units as these calculations are completed.

5. Geometry. This pattern for a bird feeder station contains many different shapes, including circles, rectangles and triangles. Each of these shapes may be discussed in terms of the vocabulary that they support. For example, the lid must be cut in such a way that it is larger than the hole it covers. This fact provides the opportunity to talk about the concepts of radius, diameter and area of circles. Also, the simple fact that one side piece is a translation of the other and that the lid rotates about the screw provides a context in which transformations may be introduced and discussed.

6. Money. Once all the bird feeders have been constructed, they could be entered into a garage sale to raise money for some school project. The process of selling the bird feeders provides the opportunity to relate surface area to cost, and provides the opportunity to discuss percentages in terms of percent profit and loss.

7. Ratio and proportion. The concept of ratio and proportion can be introduced after the volume of the bird feeder stations has been calculated. This concept is needed to determine the least number of bags of birdseed which must be purchased to fill each feeder a given number of times (you may wish to provide extra seed to the individual who purchases your bird feeder station).

8. Number systems. Concepts from this portion of the curriculum which students will encounter include whole numbers, decimals, fractions and rounding. Students will also have the opportunity to work with calculators, and the project itself may serve as a form of manipulative around which many teacher- and student-derived problems may be created.

Issues in Classroom Implementation

Many junior high mathematics concepts are inherent within the structure of this bird feeder station. Identifying which topics are inherent within each piece and thus pacing the assembly of the bird feeder station with the development of the corresponding concepts becomes the teacher's task. However, the teacher may choose to save this activity as a type of extension and build the feeder station with the students at the end of the semester, or the teacher may prefer to use this project as a form of review. In either case, the feeder station will serve the function of providing a context in which mathematical concepts may be discussed.

To make these concepts explicit, the teacher must be certain that key vocabulary terms are used and noted for the students' benefit. The learning and teaching context which surrounds a project application is that of class discussions. For discussions to occur, students need the vocabulary to communicate and share their ideas and to make their experiences meaningful. The teacher's job is to make sure that in all the activities which surround this project, the mathematical knowledge which the project embodies is brought to the students' attention. It cannot be assumed that by simply building this bird feeder station the students will learn mathematics any more than it can be assumed that because they complete a page of equations they understand the nature of algebra. In both instances, the discussions that the teacher leads and the manner in which focus is brought to the principles involved determines the amount of learning and understanding which will evolve. Teachers must be willing to make explicit the concepts and vocabulary which students are to learn, and teachers must be constantly ready to build upon the insights which students provide to lead these students forward to constantly greater levels of comprehension.

Teachers must carefully analyze their roles in activities such as this. When should the teacher interrupt the proceedings to present theorems or major concepts? When and how should the teacher assist the students in generalizing from one problem situation to another? That the role that the teacher plays in these project applications is somewhat different than the traditional role immediately becomes obvious. Teachers must learn to be more patient and to ask probing questions that allow students to realize that their present knowledge level is not sufficient for all tasks. When students realize that a knowledge gap is present and that this gap prevents them from solving a given problem, then these students will be more motivated to listen and learn to eliminate that gap (Posamentier 1986). For example, the teacher may ask the following question (or perhaps this question may be asked by one of the students): "Is there any way that we can determine the length of the top piece without measuring the top edge of a side piece?" This probing question establishes the context in which the Pythagorean theorem can be introduced. The teacher needs to be able to recognize this question as the beginning of a teachable moment to help students develop and generalize mathematical knowledge.

Teachers must realize that their responsibilities are to empower students to ask questions of themselves and of each other, to clarify their thinking and to apply their realizations toward the completion of a greater task. Project applications thus lead students to higher levels of cognitive activity while learning the prescribed curriculum. In short, the advantages that project applications provide include a motivating learning environment, a context in which principles can be discussed and applied, an opportunity to extend and enhance mathematical vocabularies, as well as means to inculcate and practise higher order cognitive skills.

Students will probably never stop asking, "But when will I use this?" And perhaps there will always be teachers who will respond, "You'll need it next year," or "You'll need it to pass Grade 8." Teachers need not be satisfied with answers such as this. Project applications provide an alternative.

Bibliography

Alberta Education. Curriculum Branch. *Junior High School Mathematics Curriculum Guide.* Alberta Education, 1978.

Bushaw, D., et al. *A Sourcebook of Applications of School Mathematics.* National Council of Teachers of Mathematics: Reston, Virginia, 1980.

Posamentier, A.S., and J. Stepelman. *Teaching Secondary School Mathematics: Techniques and Enrichment Units.* 2d ed. Charles E. Merrill Publishing Company: Toronto, Ontario, 1986.

Activities on the Natural Number Triangle: Sums and Quotients

Bonnie H. Litwiller and David R. Duncan

Dr. Bonnie Litwiller and Dr. David Duncan are professors in the Department of Mathematics and Computer Science at the University of Northern Iowa, Cedar Falls, Iowa. Dr. Duncan is chairman of the Department. In addition to teaching responsibilities within the Department, both teach methods courses to elementary and secondary preservice teachers.

Consider the natural number triangle as shown in Figure I. Each row of the triangle, after the first, consists of one more natural number than the row above it. In general, the n^{th} row consists of the natural numbers 1, 2, 3, . . ., n. If the triangle were extended indefinitely, all columns consist of the natural numbers.

Figure I

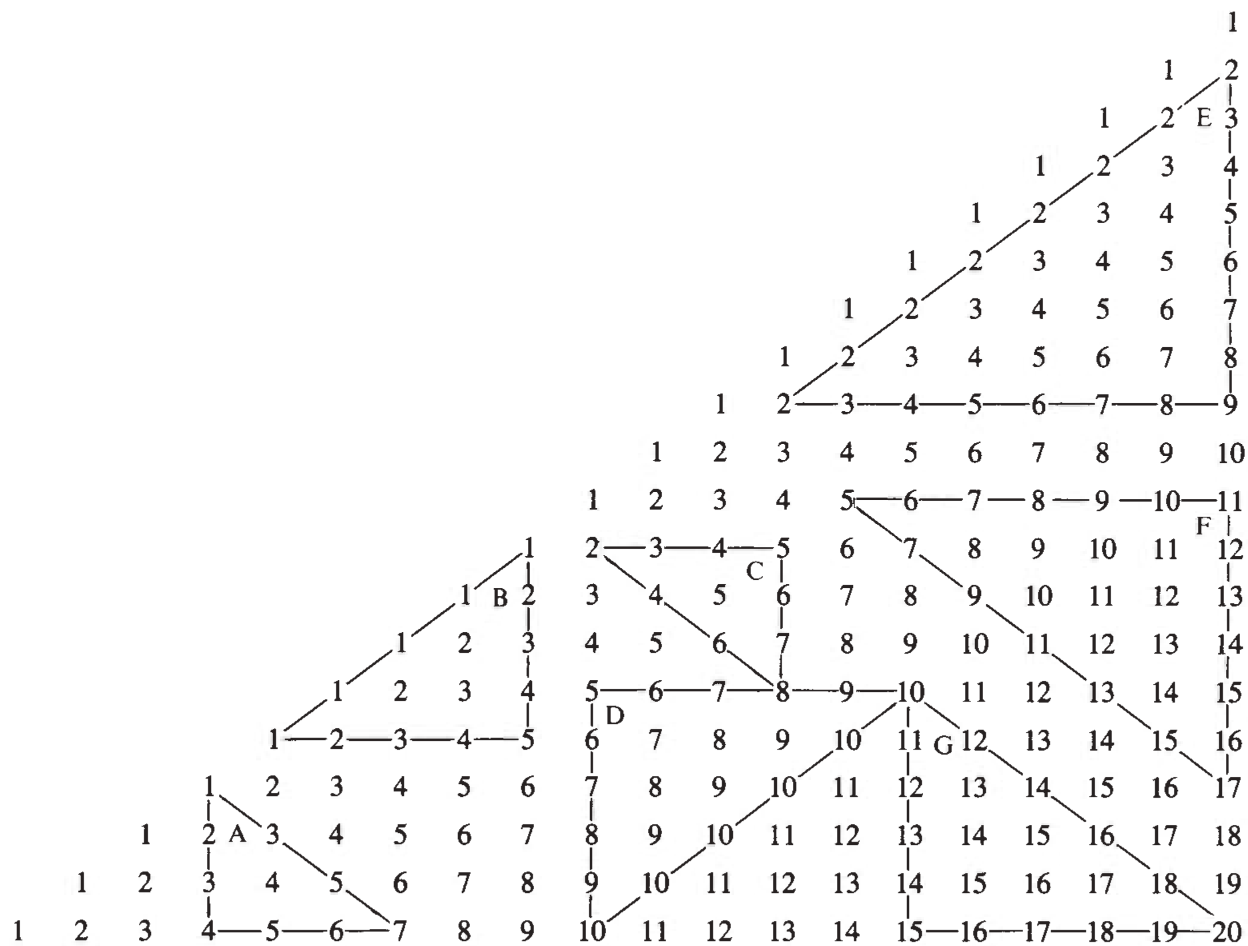
																			1
																		1	2
																	1	2	3
																1	2	3	4
															1	2	3	4	5
														1	2	3	4	5	6
													1	2	3	4	5	6	7
												1	2	3	4	5	6	7	8
											1	2	3	4	5	6	7	8	9
										1	2	3	4	5	6	7	8	9	10
									1	2	3	4	5	6	7	8	9	10	11
								1	2	3	4	5	6	7	8	9	10	11	12
							1	2	3	4	5	6	7	8	9	10	11	12	13
						1	2	3	4	5	6	7	8	9	10	11	12	13	14
					1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Note that the row sums generate the set of triangular numbers: 1, 3, 6, 10, 15, 21, 18. . . .
We shall draw selected geometric figures on the natural number triangle and compute sums and ratios.

Activity 1

Isosceles right triangles have been drawn on Figure II. The perpendicular sides lie on the rows and columns of the triangle.

Figure II



For each triangle, compute the following:

1. The sum of the numbers on the three vertices of the isosceles triangle. Call this sum V.

2. The sum of the numbers which lie in the interior of the triangle. Call this sum I. Also, count the number of interior numbers of each triangle and call this number N.

3. Compute:

   A $\frac{V}{I}$ B $\frac{3}{N}$

   (Recall that a triangle has three vertices.)

Table 1 reports the results of our computation for triangles A through G.

Table 1

Triangle	V	I	N	$\frac{V}{I}$	$\frac{3}{N}$
A	12	4	1	$\frac{12}{4}$	$\frac{3}{1}$
B	7	7	3	$\frac{7}{7}$	$\frac{3}{3}$
C	15	5	1	$\frac{15}{5}$	$\frac{3}{1}$
D	25	50	6	$\frac{25}{50}$	$\frac{3}{6}$
E	13	65	15	$\frac{13}{65}$	$\frac{3}{15}$
F	33	110	10	$\frac{33}{110}$	$\frac{3}{10}$
G	45	90	6	$\frac{45}{90}$	$\frac{3}{6}$

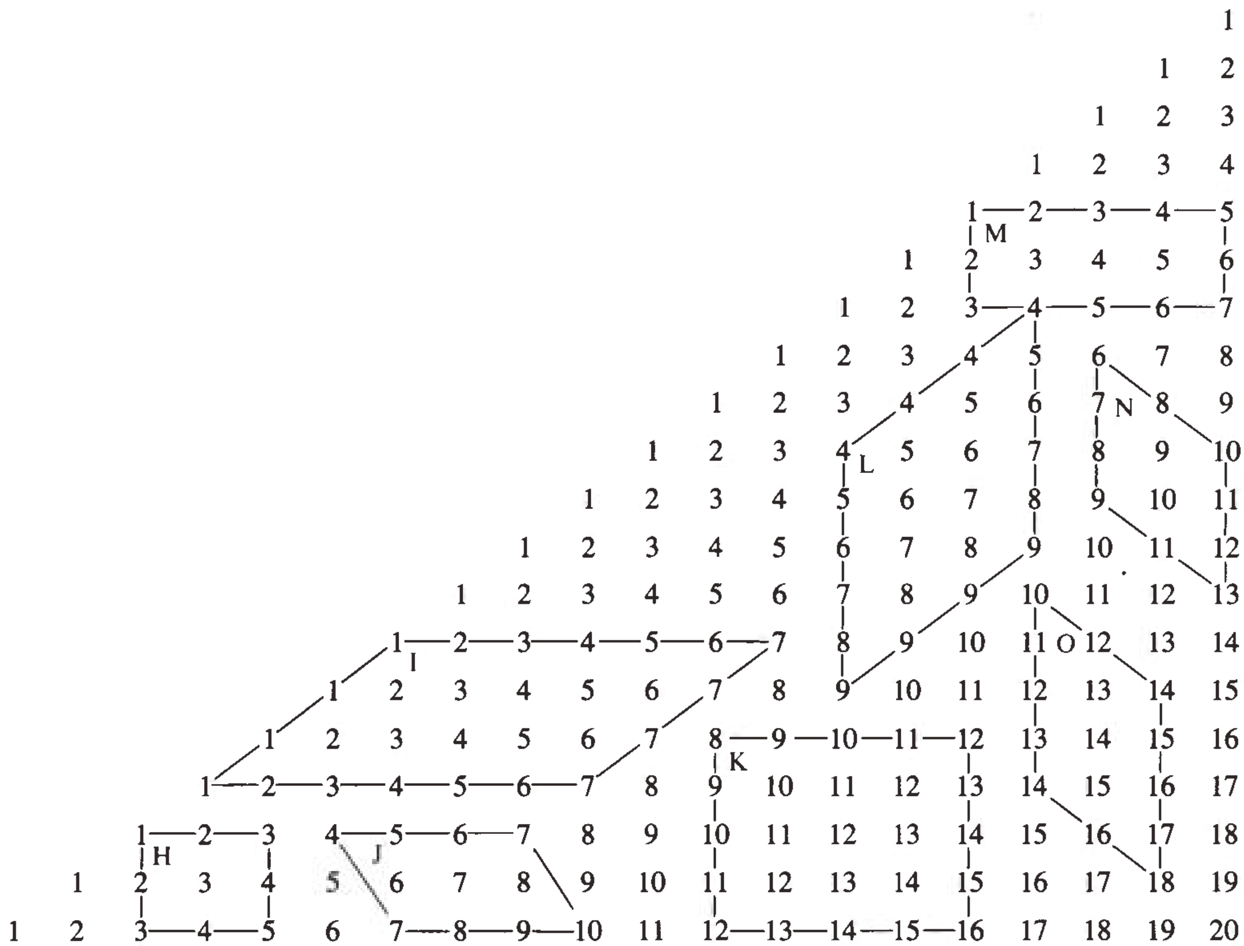
In each case, observe that $\frac{V}{I} = \frac{3}{N}$.

In other words, the ratio of the sum of the vertex numbers to the sum of the interior numbers equals the ratio of the number of vertex numbers (3) to the number of interior numbers. Have your students draw other isosceles right triangles to verify this.

Activity 2

Parallelograms have been drawn on Figure III. One pair of sides must lie on either the horizontal rows or the vertical columns of the number triangle.

Figure III



Follow the same steps as in Activity 1, except recall that a parallelogram has four vertices. Table 2 reports the results of our computations for parallelograms H through O.

Table 2

Parallelogram	V	I	N	$\frac{V}{I}$	$\frac{4}{N}$
H	12	3	1	$\frac{12}{3}$	$\frac{4}{1}$
I	16	40	10	$\frac{16}{40}$	$\frac{4}{10}$
J	28	21	3	$\frac{28}{21}$	$\frac{4}{3}$
K	48	108	9	$\frac{48}{108}$	$\frac{4}{9}$
L	26	52	8	$\frac{26}{52}$	$\frac{4}{8}$
M	16	12	3	$\frac{16}{12}$	$\frac{4}{3}$
N	38	19	2	$\frac{38}{19}$	$\frac{4}{2}$
O	56	42	3	$\frac{56}{42}$	$\frac{4}{3}$

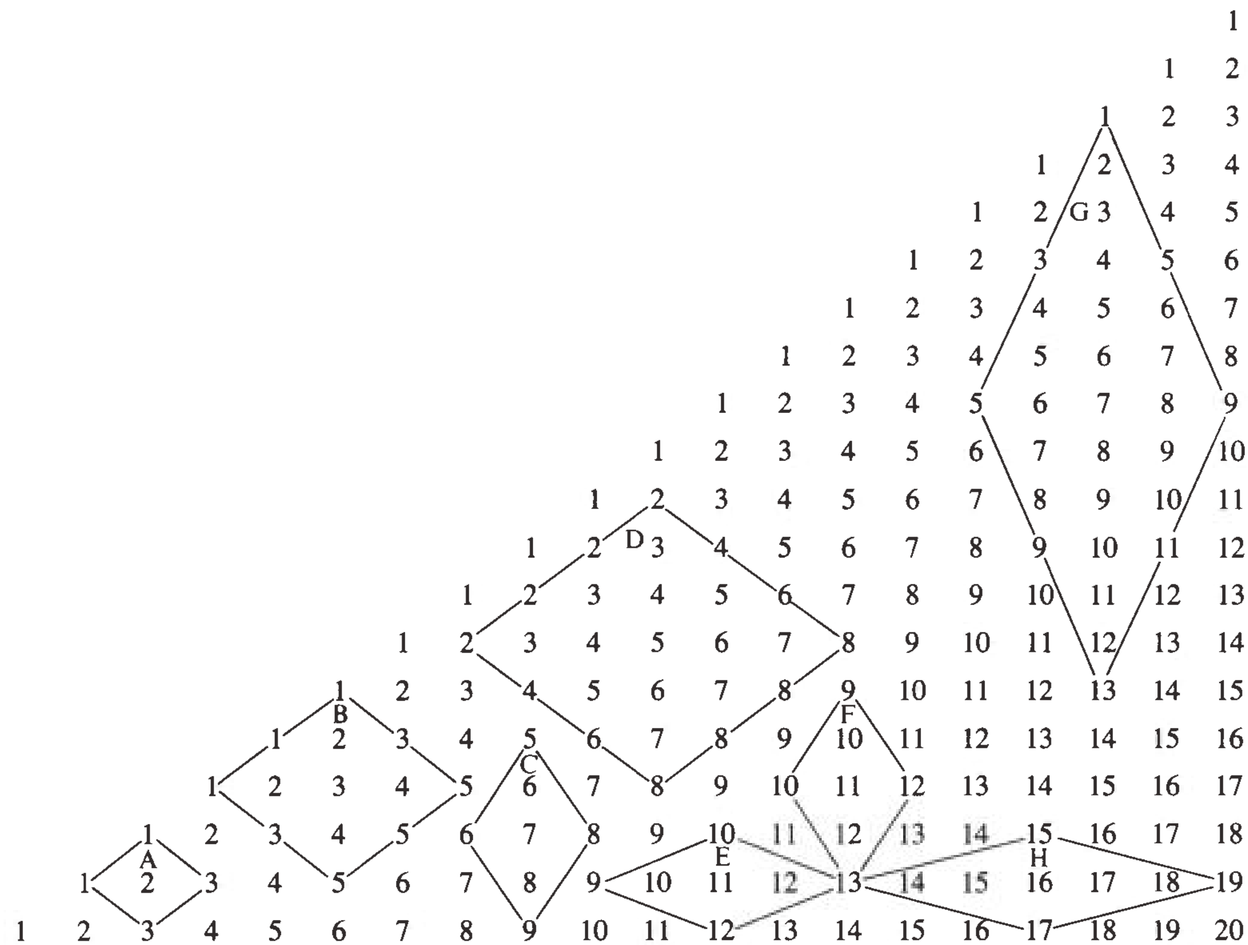
Observe that $\frac{V}{I} = \frac{4}{N}$,

that is, the ratio of the sum of the vertex numbers to the sum of the interior numbers equals the ratio of the number of vertex numbers (4) to the number of interior numbers. Have your students draw other parallelograms to verify this.

Activity 3

Diamonds have been drawn on Figure IV. The diagonals of the diamonds lie on the rows and columns of the number triangle.

Figure IV



Follow the steps of Activity 2. Table 3 reports the results of our computation for diamonds A through H.

Table 3

Diamond	V	I	N	$\frac{V}{I}$	$\frac{4}{N}$
A	8	2	1	$\frac{8}{2}$	$\frac{4}{1}$
B	12	15	5	$\frac{12}{15}$	$\frac{4}{5}$
C	28	21	3	$\frac{28}{21}$	$\frac{4}{3}$
D	20	65	13	$\frac{20}{65}$	$\frac{4}{13}$
E	44	33	3	$\frac{44}{33}$	$\frac{4}{3}$
F	44	33	3	$\frac{44}{33}$	$\frac{4}{3}$
G	28	147	21	$\frac{28}{147}$	$\frac{4}{21}$
H	64	80	5	$\frac{64}{80}$	$\frac{4}{5}$

Observe the pattern that Activity 2 holds. Challenges for the reader:

1. In Activity 1, the perpendicular sides of the isosceles right triangle were in rows and columns. If any of these conditions were changed, would the same patterns hold?

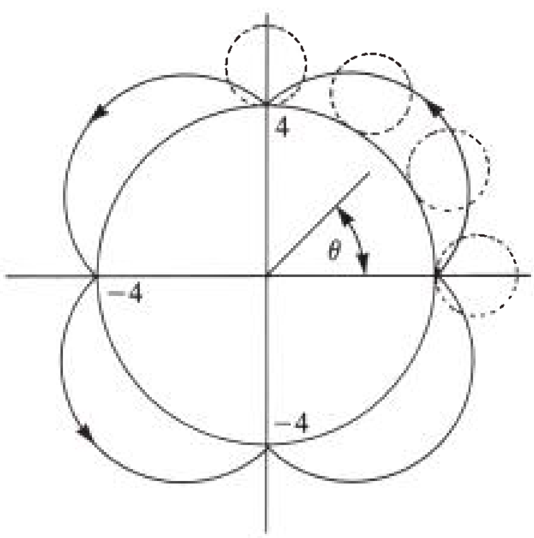
2. In Activity 2, would the same pattern hold if the sides of the parallelograms were not in rows or columns?

Applying a Fundamental Property of Integration

Sandra M. Pulver

Dr. Sandra Pulver is professor of mathematics, Pace University, New York, New York.

An interesting problem, in which students must ensure that the integrand is always positive between the limits of integration, is the following:



A circle of radius 1 is rolling around the circumference of a circle of radius 4. The path of a point on the circumference of the smaller circle is given by

$x = 5\cos\theta - \cos 5\theta$

$y = 5\sin\theta - \sin 5\theta$, tracing out an epicycloid.

Find the distance traveled by the point in one complete trip about the larger circle (that is, the arc length or perimeter of the epicycloid) (Larson and Hostetler 1982, 579).

Using the formula for arc length in parametric form:

If a curve is given by $x = f(\theta)$ and $y = g(\theta)$ (where f' and g' are continuous or the interval $[a,b]$), then the arc length of the curve over this interval is given by:

$$S = \int_{\theta = a}^{\theta = b} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dx}{d\theta}\right)^2}\, d\theta.$$

For example, we have

$$S = \int_{\theta = o}^{\theta = 2\pi} \sqrt{\left(\frac{d\,x}{d\,\theta}\right)^2 + \left(\frac{d\,x}{d\,\theta}\right)^2} \quad d\,\theta$$

$$S = \int \sqrt{(-5 \sin \theta + 5 \cos 5\,\theta)^2 + (5 \cos \theta - 5 \sin 5\,\theta)^2}\ d\,\theta$$

$$= 5 \cdot \int \sqrt{2 - 2 \sin \theta \sin 5\,\theta - 2 \cos \theta \cos 5\,\theta}\ d\,\theta$$

$$S = 5 \cdot \int_{0}^{2\pi} \sqrt{2-2 \cos 4\,\theta}\quad d\,\theta$$

$$= 5 \cdot \int \sqrt{4 \sin^2 2\,\theta}\quad d\,\theta$$

$$= 10 \cdot \int \sqrt{\sin^2 2\,\theta}\quad d\,\theta$$

$$= 10 \cdot \int \left| \sin 2\,\theta \right| \quad d\,\theta\,.$$

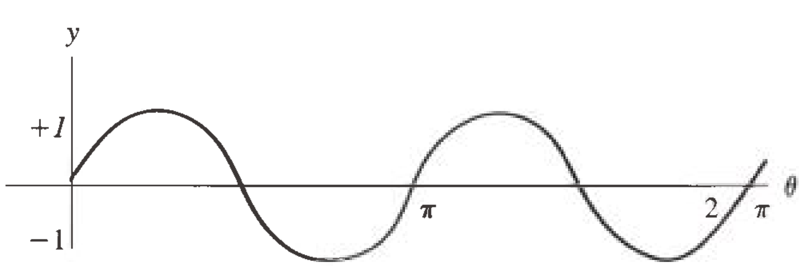
Without thinking, students now integrate between the limits of 0 and 2π, that is

$$S = \frac{10}{2} \int \sin 2\,\theta \cdot 2\,d\,\theta$$

$$= -5 \cdot \quad \cos 2\,\theta \Big]_{0}^{2\pi}$$

$$= -5 \cdot \quad (1-1) = 0, \text{ which is incorrect.}$$

Here, $\sin 2\,\theta$ must be positive on the interval of integration, that is, it must be broken up into parts. $\sin 2\,\theta$ has period $\frac{2\pi}{2} = \pi$, that is,



$\sin 2\,\theta$ is negative from $\frac{\pi}{2}$ to π, and again from $\frac{3\pi}{2}$ to $2\,\pi$.

Because of symmetry of this integral, we can change limits to $4\int_0^{\pi/2} \sin 2\theta \, d\theta$ to ensure that integral will always be positive on interval.

(Alternately, we could have used line $\pi \leq \theta \leq \frac{3\pi}{2}$ and multiplied by 4.)

Therefore, $S = (4)\ (10)\int_0^{\frac{\pi}{2}} \sqrt{\sin^2 2\theta}\ d\theta$

$$= 40\left(\frac{1}{2}\right)\int_0^{\frac{\pi}{2}} \sin 2\theta\ 2\ d\theta$$

$$= -20 \cos 2\theta \Big]_0^{\frac{\pi}{2}}$$

$$= -20\ [\cos \pi - \cos 0]$$

$$= -20\ [(-1) - \emptyset]$$

$$= 40$$

Reference

Larson, Roland, and Robert Hostetler. *Calculus with Analytic Geometry.* D.H. Heath and Company, Lexington, Massachusetts, 1982, 579.

Mathemagical Sub–terfuge

Stephen Forrester

Stephen Forrester is a substitute teacher in the Calgary public school system. He is currently editor of Substance, *a quarterly newsletter for substitute teachers. His first book* The Art of Street Magic, *published April 1989, contains a section on mathematical magic tricks.*

Tell me and I forget, teach me and I remember, involve me and I learn.

—Benjamin Franklin

Sub: (noun) Short for substitute. (verb) Act as substitute.

Substitute: (noun) Person acting or serving in place of another. (verb) Put instead of another, used instead of.

Subterfuge: (noun) Artifice, skilled handicraft or ingenuity adopted by a wise, prudent person or statesman.

Would you like to be able to walk into a class of kids you have never seen before and have them eating out of your hands? Substitute teachers, who are in a new situation during a limited time, can actually make kids warm up to them and bolster their enthusiasm. Many students who have become jaded with the regular procedures will welcome a new face, style and technique. The fact that the kids don't know you can be used to your advantage, and you will be judged with a clear slate.

Before I explain my tactics, let me discuss some qualities that kids like in a teacher. Teachers who genuinely take an interest in people and like to share something of themselves are liked by students.

I once had a teacher who would bring interesting things to class, like scuba diving gear, and show us how they worked. Sometimes he would get us into a discussion about cameras or calculators (novelties at the time) and end up teaching us some interesting points. I remember him saying, "Never take pictures of subjects with snow behind them because the light meter will give an incorrect reading due to the reflection of light from the snow."

Maybe what he taught during those moments wasn't part of the curriculum, but we became more interested and attentive. He became known as one you could "talk to." Students warmed up to him easily and were not afraid to ask questions in class.

What has all this to do with making an effect on kids as a substitute teacher? A substitute teacher must sometimes employ out-of-the-ordinary tactics to keep things running smoothly. As a substitute, I usually bring something fun to the classroom to "break the ice" at the start of the class, such as remote control answering machines or magic tricks or puzzles. Kids appreciate a sense of humor and like to discuss topics that are current or interesting. For example, ask them if they saw the movie about the deaf entitled "Children of a Lesser God," and offer to show the kids how to say hi in sign language.

I explain to the kids that if they finish all their homework in class, or if they at least make a good effort, I will show them something neat at the end of the class. This sounds like a bribe, but believe me, you are giving something of yourself and you can't have a relationship without an investment of yourself in some way.

After a while, you won't have to bring things in to class—they will bring things in to share with you! That is the real secret. Once the kids see that you can do something more than just teach, they will feel more relaxed and attentive around you, which makes the lessons smoother. I firmly believe that if kids like you as a person, they will be more willing to listen to what you have to say. If the things you share with the kids are interesting, and if these things help to build a positive relationship with the students, you will have an impact on them that they will never forget.

The teacher's enthusiasm is vital to the students' progress.

What are some interesting things that can turn on students? Here is an example of how a simple trick, dressed up like an apparent feat of mind reading, never fails to cause wonder.

Before class, find a large dictionary and look up the first word at the top left side of page 1,089. Let's assume that this word is poodle. Write this word on the bottom edge of the blackboard in small letters behind a chalk brush. After you have introduced yourself to the class, tell them that you have something interesting to show them before you start the lesson.

Ask someone nonchalantly to lend you a large book that you have not seen before. Explain that something like a dictionary would do fine, and point to the dictionary at the back that you have secretly looked at before. Ask someone to bring it forward. When you receive it, flip through it several times and explain that this is a fast way of memorizing all the words. Then slam it shut and put it down. Ask for a volunteer who is good at adding and subtracting to come up to the blackboard to help you do a little bit of math. Tell the other students that they can do this at their desks. Inform the students not to say out loud any of the numbers on the blackboard because you are not supposed to know what they are. Now stand with your back toward the blackboard so you can't see what your volunteer is writing.

Ask the student at the board to think of any three digit number (all the numbers must be different) and to write it on the board. Let us assume the number was 416. Ask him to find the reverse of the number. (The reverse of 416 is 614.) Ask the student to write this reversed number on top of the first number if it is larger than the original number; otherwise, write it below the first number. For example, 614 would be written on top of 416. If 835 was chosen, 538 would be written below 835. Instruct the students not to voice any of these numbers.

Ask the student at the board to subtract the smaller number from the larger number and to reverse the answer and add it to the answer again as follows:

$$\begin{array}{r} 614 \\ -\ 416 \\ \hline 198 \\ +\ 891 \\ \hline =1{,}089 \end{array} \qquad \text{or} \qquad \begin{array}{r} 835 \\ -\ 538 \\ \hline 297 \\ +\ 792 \\ \hline =1{,}089 \end{array}$$

No matter which three digit number the students pick, they will always end up with an answer of 1,089! Ask the students at their desks to watch the person at the board to make sure that an addition or subtraction error is not made.

When everyone has done this, tell the student at the board to pick up the dictionary and hold it. Ask another student to stand in front of the chalk brush that has the word written behind it. Explain that you made a prediction before class today. Ask the student with the dictionary to look at the final answer on the board (1,089). Tell the student to turn to that page in the dictionary and read off the first word at the top left hand side of the page. After they do this, ask the volunteer to read the word on the board. When the students hear that the words are the same, they will be amazed. Explain that this trick is really due to the magic of numbers and that any three-digit number will always give you an answer of 1,089 once it has gone through this process. Show that this is due to the properties of 9s as follows:

9	18	27	36	45. . . .
9=9	1+8=9	2+7=9	3+6=9	4+5=9. . . .

I also mention to them that if you pick any two-digit number with different digits and perform this process, the answer will always be 99:

$$\begin{array}{r} 73 \\ -\ 37 \\ \hline 36 \end{array} \qquad \begin{array}{r} 36 \\ +\ 63 \\ \hline 99 \end{array}$$

I conclude by telling them that the number 9 has almost magical properties since you can tell that a number is divisible by 9 if the sum of the digits is divisible by 9. For example, 7,236 is divisible by 9 since 7 + 2 + 3 + 6 = 18, which is divisible by 9. This is confirmed to be true since 7,236/9 = 804.

I thank them for watching me and tell them that they can try this trick at home on a brother or sister. The kids usually work well after this in the hopes of seeing another trick.

If the class I am teaching has a reputation for being rowdy, I often say, "I'd like to do something else at the end of the class, but that depends on you.'' If one student tries to cause trouble, remind him or her that one person can ruin the chances of the whole class seeing another trick. That class will hear about the trick from another class who saw it and feel they missed out. This places tremendous peer pressure on the ringleader not to ruin the fun for everyone else.

As an alternative, have the noisy student finish his/her homework out in the hall while you show the trick to the rest of the class who will eventually mention it to him/her later.

Of course one doesn't have to be a pro magician to have fun with the kids—any area of interest or skill the teacher has can be used to advantage. Also, one does not have to take up class time to do something interesting for the students. Usually one finds a willing audience in the hallways at lunch or after school.

The magic is believing in you.

—Doug Henning

An idea, by the well-known magician David Copperfield, called ''Project Magic'' uses magic as a therapeutic aid. Teaching physically or mentally handicapped children how to perform simple magic tricks lets them do something that everyone else can't do. As a result, they develop self-confidence and a good feeling about themselves. Additionally, learning a magical illusion can aid in increasing problem solving skills, visual skills and coordination. Here is an example of an amusing magic trick I call the linking paper clips.

Fold a dollar bill in thirds and place a paper clip around the first and second folds and another clip around the second and third folds (see Figure 1). As you are folding the bill, place a rubber band around the dollar and position it within one of the folds. If you pull the upper corners of the bill away from yourself in a quick motion, the paper clips become linked and will end up attached to the rubber band (see Figure 2).

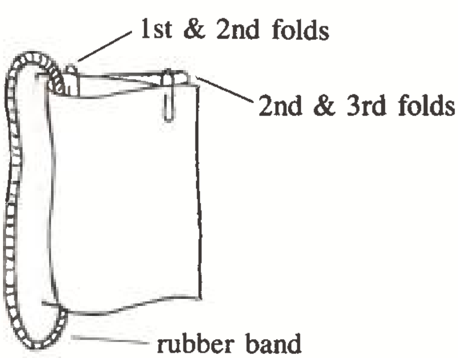


Figure 1

Abracadabra

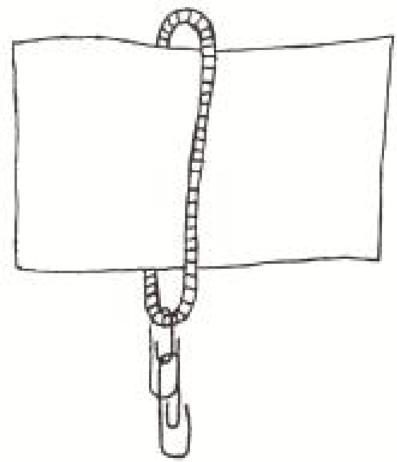


Figure 2

Here is another example of an interesting mathematical trick that seems to be a paradox and will challenge the smartest students to guess why this works. I narrate as follows: ''No one should underestimate the power of mathematics. Recently, a math professor related to me how he earns extra money. (1) He deposited $50 into a new bank account, then promptly withdrew $20 leaving a $30 balance. (2) The following day he withdrew $15 leaving a balance of $15. (3) Later that week, he withdrew $9 leaving a balance of $6. (4) He returned later that same day to withdraw the remaining $6 which left his account empty. The next week, however, he returned to the bank and withdrew another dollar. How?'' As you say this, write the following on the board:

Deposit of $50

	Withdrawal	Balance
(1)	$20	$30
(2)	$15	$15
(3)	$ 9	$ 6
(4)	$ 6	-0-
	$50	$51

Tell the kids, ''When you subtract the withdrawal column from the balance column, you see that the bank owes him $1. By using this special form of mathematics, he persuaded the manager that they still owed him $1. Challenge the smartest students in the class to explain how this is possible. If no one figures this out, explain that the balance column readings have nothing to do with the actual balance in the account. For example, if the man withdrew $10 five times, the balance column readings would add up to $100!

Happy are those who dream dreams and are ready to pay the price to make them come true.

Conclusion

- Remember that you are there to teach and not to babysit.
- If students are kept busy, they will not have time to get into trouble, so always have something extra on hand.
- Keep things consistent with the regular teacher by consulting a reliable student before class regarding the regular teacher's procedures.
- When subbing for a teacher who left unexpectedly for an emergency and has left no lesson plans, always be prepared with lessons, puzzles, work sheets and time-tested projects.

A teacher affects eternity; he can never tell where his influence stops.

—H. Adams

Student Problem Corner

John B. Percevault

Problem 1

To the teacher: Many problems of the type that follow can be developed by students. Although the problem appears simple, the successful problem solver must analyze as well as recall basic facts. Logic must be used. Students enjoy developing similar problems and vary conditions in the development of new problems.

The teacher may vary the description of the set of numerals used as replacements for the solution.

Review Exercise

□ + □ = 8	□ x □ = 35
□ ÷ □ = 0	□ − □ = 3
□ ? □ = 6	

Some suggested questions:

1. What equation(s) did you solve first? Why?
2. In the equation □ ÷ □ = 0, where did you place the zero? Why?
3. What was the last equation you solved?
4. What problem solving skills or strategies did you use?
5. Can you develop a similar problem?

Problem 2

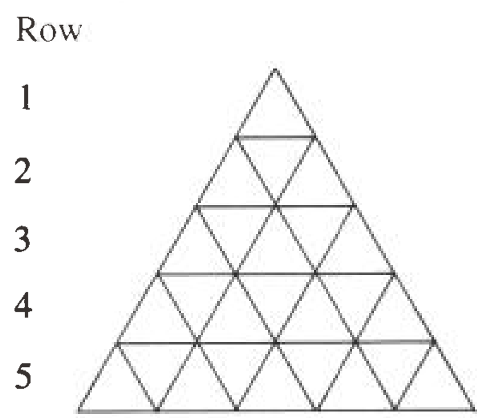
To the teacher: Many problem solving strategies that can be used in this problem include the following:

1. Counting
2. Classification (length of side of triangles)
3. Listing tables
   Example: Triangle of one unit length

Row	No. of triangles	Total
1	1	1
2	3	4
3	5	9

4. Using manipulatives—Model of triangle with sides of 1, 2 . . . units
5. General form

Counting Triangles



Sample Questions:

1. Can you find a triangle whose side length is 2? is 3?. . . .
2. How many triangles in row 1? row 2?. . . .
3. How many triangles (unit length) are in the first 2 rows?. . . .
4. Repeat the questions with reference to triangles whose side length is 2, 3. . . .
5. Many more questions could be defined.

Extensions

How many triangles with a unit length side would there be in the 8th row? 20th row? nth row? (Looks like the nth term of an arithmetic progression to me.)

What is the sum of all unit length triangles in the 8th row? nth row?

Develop extensions for number of and sum of triangles whose side length is 2.

Record (tabulate) results.

Research Clip

Teaching Mathematical Problem Solving to Primary School Children

Supplied by the National Council of Teachers of Mathematics

Teachers can improve the problem solving abilities of primary school children by providing sustained practice with problems that can be solved in a variety of ways and encouraging children to share their ideas rather than working for correct answers alone.

- Jennifer has exactly four different coins, with four different values: $.01, $.02, $.04 and $.08. How many different amounts can she make using one or more of her coins?

- Natalie has baked less than two dozen cookies. When she tried to divide them equally among 2, 3 or 4 of her friends, one cookie was always left over. How many cookies did she bake?

Problems such as these involve little reading and few procedural steps. They are solvable by informal means, such as counting, rather than by the use of abstract mathematics. They can be dramatized and are of interest to children. With regular exposure to such problems, primary school children can be guided to formulate and share their ideas without fear of being incorrect. As a consequence, they become more confident and proficient problem solvers, freer to take risks and tackle unfamiliar tasks. Classroom teachers make their greatest contribution when they provide a learning environment in which problem solving is viewed as an indispensable component of mathematics study and where it is understood that working for correct answers alone is not the most effective means of advancing young children's problem solving skills.

References

Bruni, J.V. "Problem Solving for the Primary Grades." *Arithmetic Teacher* 29 (February 1982): 10–15.

LeBlanc, J.F. "You Can Teach Problem Solving." *Arithmetic Teacher* 25 (November 1977): 16–20.

LeBlanc, J.F., L. Proudfit and I.J. Putt. "Teaching Problem Solving in the Elementary School." In *Problem Solving in School Mathematics*, 1980 Yearbook of the National Council of Teachers of Mathematics. Reston, Virginia: The Council, 1980.

Spencer, P.J., and F.K. Lester. "Second Graders Can Be Problem Solvers." *Arithmetic Teacher* 29 (September 1981): 15–17.

Suydam, M.N. "Update on Research on Problem Solving: Implications for Classroom Teaching." *Arithmetic Teacher* 29 (February 1982): 56–59.

MCATA Executive 1988/89

President
Louise Frame — Res. 251-5841
36, 2323 Oakmoor Drive SW — Bus. 278-3633
Calgary T2V 4T2

Past President
Robert Michie — Res. 246-8597
149 Wimbledon Crescent SW — Bus. 230-4743
Calgary T3C 3J2

Vice President
Marie Hauk — Res. 487-8841
315 Dechene Road — Bus. 492-4146
Edmonton T6M 1W3 — or 422-5480

Secretary
Mary-Jo Maas — Res. 553-4848
Box 484 — Bus. 553-4411
Fort MacLeod T0L 0Z0

Treasurer
Dick Kopan — Res. 271-5240
23 Lake Crimson Close SE — Bus. 271-8882
Calgary T2J 3K8

***delta-K* Editor**
John Percevault — Res. 328-1259
2510 22 Avenue S — Bus. 329-2725
Lethbridge T1K 1J5

***delta-K* Coeditor**
Linda Brandau — Res. 265-5395
2109, 1200 Sixth Street SW — Bus. 220-5635
Calgary T2R 1H3 — or 220-6288

Newsletter Editor
Art Jorgensen — Res. 723-5370
Box 2619
Edson T0E 0P0

1989 Conference Director
Dennis Burton — Res. 327-2222
3406 Sylvan Road — Bus. 328-9606
Lethbridge T1K 3J7

1990 Conference Director
George Ditto — Res. 282-6682
2713 17A Street NW — Bus. 294-6309
Calgary T2M 3S9

Department of Education Representative
Pat McLaughlin — Res. 281-4279
12216 Elbow Drive SW — Bus. 297-6353
Calgary T2W 1H2

Mathematics Representative
Alvin Baragar — Res. 469-5626
Department of Mathematics — Bus. 492-3398
University of Alberta
Edmonton T6G 2G1

PEC Liaison
Cliff R. Youngs — Res. 245-4415
2110 26 Avenue SW — Bus. 228-5363
Calgary T2T 1E6

ATA Staff Adviser
Bill M. Brooks — Bus. 265-2672
200, 540 12 Avenue SW — or 1-800-332-1280
Calgary T2R 0H4

Directors
Diane Congdon — Res. 526-7563
146 Fourth Street SW — Bus. 548-7516
Medicine Hat T1A 4E3

Bill Davidoff — Res. 627-4283
P.O. Box 574 — Bus. 627-4414
Pincher Creek T0K 1W0 — or 627-4415

George Ditto — Res. 282-6682
2713 17A Street NW — Bus. 294-6309
Calgary T2M 3S9

Nancy Hope — Res. 532-5915
8511 96 Street — Bus. 539-0333
Grande Prairie T8V 3C8

Wes Williamson — Res. 347-2332
106 Glendale Boulevard — Bus. 346-5010
Red Deer T4P 2P5

Director and NCTM Representative
Wendy Lukawesky — Res. 420-1466
1701, 9808 103 Street — Bus. 453-1576
Edmonton T5K 2G4

Regionals
South West
Dennis Burton — Res. 327-2222
3406 Sylvan Road — Bus. 328-9606
Lethbridge T1K 3J7

ISSN 0319-8367

Barnett House
11010 142 Street
Edmonton, Alberta
T5N 2R1