Tessellation, Tiling or Surrounding a Point

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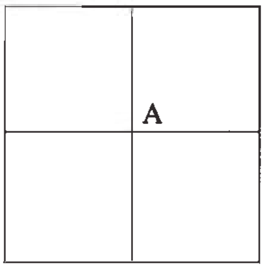
Tessellation (tiling) activities can be used effectively to present many of the 26 geometry concepts in the Alberta Education *Elementary Curriculum Guide.* Introductory activities should always emphasize the concrete mode, regardless of the grade level at which they are presented. Gradually a transition can be made to the pictorial and abstract modes. Although tiling can be done with a variety of two-dimensional geometric shapes, the activities that follow are based on seven regular polygonal regions whose perimeters are triangles, squares, pentagons, hexagons, octagons, decagons and dodecagons.

The illustrations in Figure 1 (see page 22) have a common edge length, 2.5 centimetres, and can be used as patterns to prepare black-line masters for duplication. It is recommended that a separate master be prepared for each of the seven shapes. Six dodecagons, 6 decagons, 12 octagons or 30 pentagons will fit on a regular sheet of paper. Because triangles, squares or hexagons can be drawn with common edges, a large number can be accommodated on a regular sheet of paper. When multiple copies are duplicated, it is recommended that heavy tag material with a different color for each kind of regular polygon be used. For demonstrations on an overhead projector, the masters can be used to prepare transparencies using a different color for each shape.

Figure 2



Tiling is usually considered a manipulative activity in which a surface is covered with two-dimensional geometric figures. It is also considered an activity in which a student surrounds a point. For example, 4 squares surround point A in Figure 2. Four square tiles can be manipulated to experience the concrete mode. The pictorial mode is used when students see that point A is surrounded in Figure 2, and the idea can be experienced abstractly by noting that the 360 degrees around point A are made up of four angles each containing 90 degrees.

Activity 1

1. Four squares surround a point. Take a number of triangles and see if you can surround a point. How many do you need? Can you keep on surrounding other points until you have covered a sheet of paper?
2. Try surrounding a point with some pentagons. Is it possible?
3. How many hexagons are needed to surround a point? Can you cover a sheet of paper with hexagons? How does the honeybee make use of hexagonal designs?

4. Can you surround a point using only octagons? Only decagons? Only dodecagons?

Activity 2

1. Show that 3 squares and 2 triangles surround a point. Can you cover a sheet of paper using just squares and triangles?
2. Can you surround a point using only hexagons and triangles? How many of each are needed?
3. Try using squares and octagons. How may of each are needed to surround a point?
4. Can you cover a sheet of paper using triangles and hexagons? Triangles and dodecagons?
5. You can surround a point using 2 pentagons and 1 decagon, but you cannot continue to use other copies of the same shape to cover a sheet of paper. Try it!
6. Try using decagons and triangles, decagons and squares, decagons and hexagons, decagons and octagons. Will any of these sets of shapes surround a point?

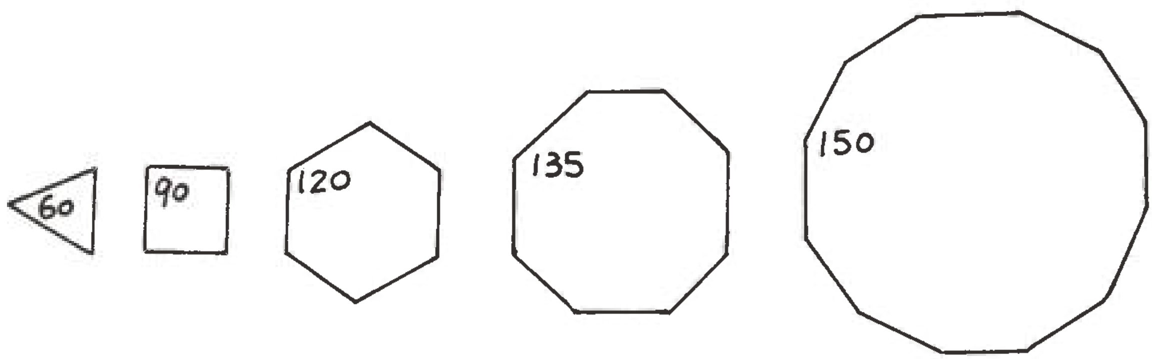
Activity 3

1. Show that a square, a hexagon and dodecagon surround a point. Can you cover a sheet of paper using many copies of these three shapes?
2. Try to cover a sheet of paper using only triangles, squares and dodecagons. How many of each do you need to surround a point?
3. Choose any three shapes and try to surround a point. Are there any other sets of three different kinds of shapes that will surround a point?

Activity 4

Figure 3 shows each of the five kinds of shapes. Use addition to make a list of the 11 ways in which a point can be surrounded. Three of them have been done for you.

Figure 3



1. 4 squares; 90 + 90 + 90 + 90 = 360
2. triangles; 60 + ____________ = 360
3. pentagons; ____________ = 360
4. 3 triangles and 2 squares; 60 + 60 + 60 + 90 + 90 = 360
5. ____________ and ____________ ; ____________ = 360
6. ____________ and ____________ ; ____________ = 360
7. ____________ and ____________ ; ____________ = 360
8. ____________ and ____________ ; ____________ = 360
9. 1 square and 1 hexagon and 1 dodecagon; 90 + 120 + 150 = 360
10. ____________ and ____________ and ____________ ; ____________ = 360
11. ____________ and ____________ and ____________ ; ____________ = 360

Activity 5

If you do not remember the number of degrees in each angle or a regular polygon (e.g., square), do the following:

1. Indicate a point which you assume to be the centre of the square,
2. Draw a line from this point to each of the vertices,
3. Calculate the number of degrees at each angle at the centre point (360 divided by 4 = 90),
4. Each angle of the square must be 90 (45 + 45).

1. How many degrees are in each angle at the centre of the triangle? 360 divided by 3 = ________. Therefore, how many degrees are in each vertex of the triangle?

2. How many degrees are in each angle at the centre of a pentagon? Therefore, what is the measure of each vertex of a pentagon?

3. Try this activity for a regular hexagon, octagon, decagon and dodecagon.

Activity 6

Complete the chart on page 23. Look for patterns in each of the four columns. Only 6 of the 10 regular polygons are illustrated. The ones not illustrated are named.

Activity 7

Use the procedures in each of the previous two activities to find the number of degrees in each vertex of a "centagon" (100 angles and edges).

References

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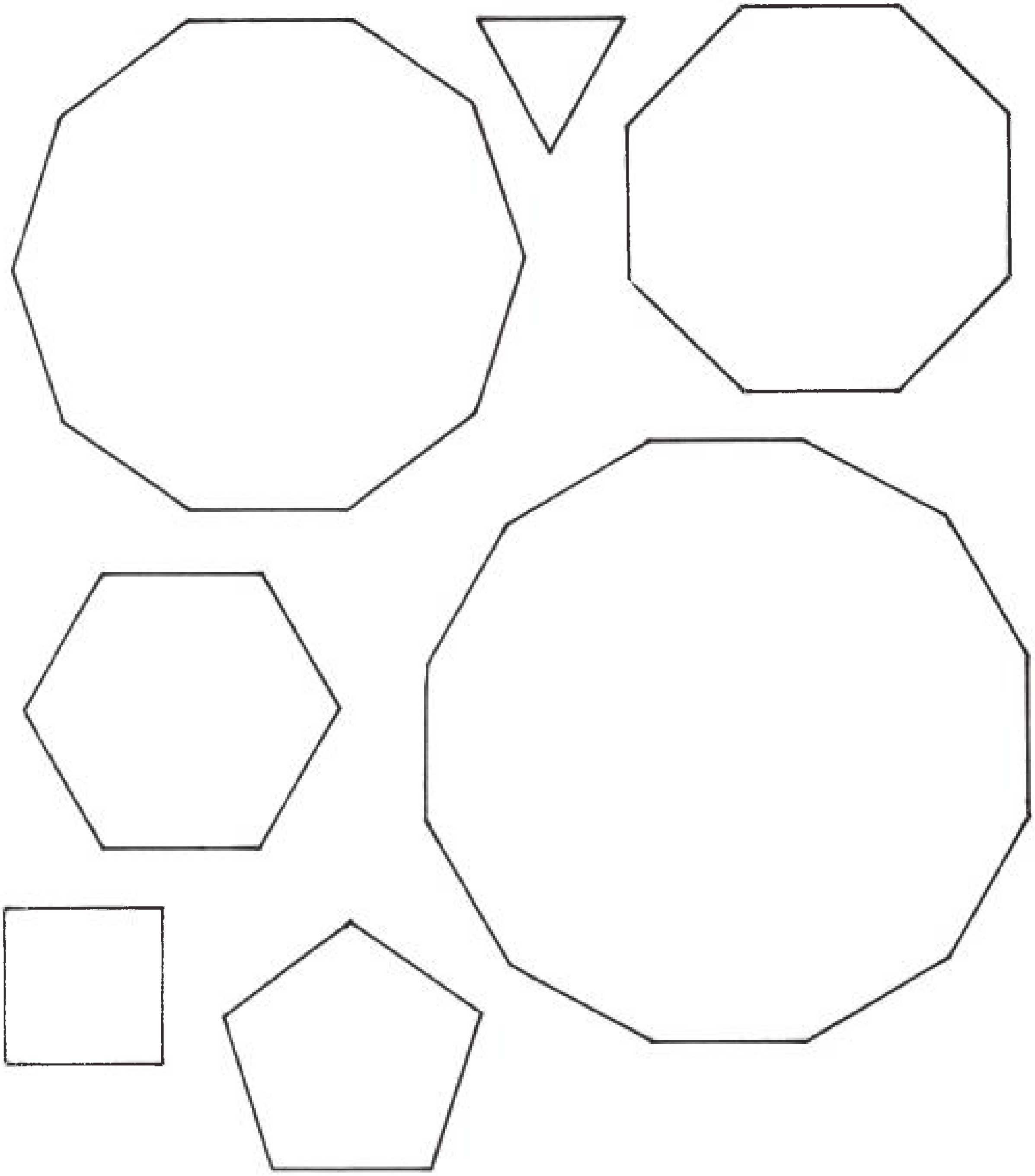
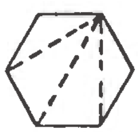
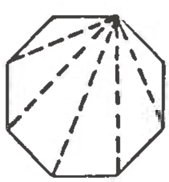
Figure 1



Activity 6

Regular polygon	Number of vertices	Number of triangles	Total number of degrees	Degrees in each vertex
	3	1	180	60
	4	2	360	90
	5			
		4	720	
septagon				
		6		
nonagon				
decagon				
"11-agon"