Actions and Assumptions: Their Relationship to Student Thinking

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Teaching involves the development of thought, the broadening of horizons, the awakening of interests, the piquing of curiosity and the arousal of intelligence. At least that is what we say. Learning involves the same things. At least that is what we hope. When teachers walk into classrooms they carry with them certain assumptions. Some of these assumptions are such an integral part of their character that they are not conscious of them. People who have an awareness of the forces that motivate them are likely to be receptive to positive changes within themselves. We, as students of teaching and learning, work to create an increased awareness of these subconscious forces within ourselves and to enact such changes.

By studying the work we did with four problem solving groups of students in Grades 3, 4, 5 and 6, we believe we have uncovered some of the underlying assumptions that drive our thinking processes and our behavior. Our actions, as captured in our speech patterns, and the results of our actions, as seen in some of the student records we obtained are examined here. We question some of our behaviors in the problem solving sessions and endeavor to judge their possible effects on the thinking processes of the students involved in the activity.

First, it might be worthwhile to discuss the nature of the problem chosen for this exercise (see Appendix for a description of the problem and materials). This study uncovers some of the assumptions that we hold regarding problem solving as an activity for students. We agreed that the activity chosen must engage students; that is, we wanted them to be intrigued and engrossed by it. The cooperation and interest of students is not gained automatically, and as such we consciously chose an activity that was fun and challenging. The problem was designed to offer some quick, easy solutions and some more difficult ones. This was based on the belief that problem solving does not work well in an atmosphere of frustration. Students had to believe that they had the capacity to solve the puzzle, so the activity was structured to give them some immediate and fairly certain success. The variety of strategies and solutions that a problem accepts is also important to children's perception of their ability to succeed. A true problem can be resolved in more than one way, and many different strategies may be involved in attaining any of the solutions. Because problems of this nature do not have "right" answers, children need not feel disabled in their problem solving if their patterns of thinking do not match that of other students. Finally, the problem also had to lend itself to some sort of a manipulative procedure. Solutions are easier to develop when the variables can be seen and handled. Additionally, the manipulative nature of the task was certainly important in its ability to engage the students.

Even though the topic for our math problem was open-ended and manipulative, it is arguable that the topic for discussion in any given class needs to be delineated; the boundaries of thought should be delimited. Alan Tom in his essay on the moral craft of teaching (1985, 149), states that "teaching [is] . . . the application of knowledge and skill to attain some practical end." He elaborates on this point by saying, "teachers . . . are inevitably involved in forming students in desirable ways.'' Thus we use our knowledge and our skill to help mold the thoughts of our charges. However, we need to fear the strength of that mold, the fidelity of the copy that we may create. Tom's argument is that each teacher has the moral responsibility to decide the desirable end for his students and the craft to enact those ends. We believe that while we as teachers carried out our moral responsibility, the nature of some of the assumptions we made during the problem solving activity interfered with the attainment of our objectives. Several salient points need to be addressed in this question: How restrictive do we need to be in order to perform our task effectively? What are the results, in terms of thinking, of these restrictions?

Pimm (1987, 32) describes a dilemma for the teacher, in terms of classroom communication:

> Teacher presence can interfere with developing pupil talk by overcontrolling it. The teacher may be too concerned with the form of what is being said, at the expense of the meaning which the pupil is trying to convey. On the other hand, if pupils are to become aware of the characteristics of disembodied speech, then considerable work needs to be done to encourage them to modify and expand their initial attempts. How to contend with this tension may be one of the central dilemmas of communication facing teachers.

This dilemma is a real one and extends well beyond the nature of the communication within our classrooms and into the actions that we use in our teaching. We want our students to develop independent thought, unimpeded by the restraints that we might place on it. We value original ideas and insights. Nonetheless, the discipline that can be applied to thought is sometimes as important as its freedom. The need exists for thought to take a form that meshes with the domain of thought extant in the classroom. The teacher's task is to strike a delicate balance between disciplined thinking and a free flow of ideas.

The necessity to create a framework of thought within a classroom and in an activity is apparent. If the teacher is to fulfill the role of "forming students in desirable ways" then there must be some kind of structure apparent within classroom activities. The absence of such direction would result in chaos and anarchy. These situations are rarely conducive to creating a considered outcome.

The following conversation with a Grade 4 class, which indicates the nature of delimitations that we placed on children's thinking, illustrates the necessity of structure. Our action was not capricious; the planned activity demanded measurement and excluded estimation.

Teacher: We can measure 3, and we can measure 7, what other numbers. . . . We were just going to try to figure out what other numbers we can measure with these jars. Anyone got any ideas of what we can figure out with these jars?

Students: We could get 8 . . . 2.

T: Do you have any plans for how we could do that?

S: Well for this, for the 3 maybe we could put this much down.

T: Oh, I see. Now would that be measuring or estimating?

S: Estimating.

Later, at the end of the instructions

T: Remember there's no estimating in this game. You can only. . . .

S: Measure.

While estimation is a useful skill and one that is used often in mathematics, it was not a skill that was appropriate to the solution of the problem posed to the children. It is notable that all groups embraced it as a first solution and thus it was necessary, in all groups, to delineate the problem. This is not an unrealistic practice. Many problems have easier, but perhaps less accurate or less legitimate solutions.

But what happens to the nature of the thinking that a student pursues when this kind of delineation has taken place? It is not unrealistic to believe that an insistence on the rules might lead to a less adventurous, less individual approach to problem solving than one unimpeded by such conventions. Although the following situation involving a Grade 4 class illustrates confusion, and not necessarily a sense of restriction, it is reasonable to infer that the perplexity of the student stems from a fear of transgressing the rules.

Student 1: How about 10–7?

Student 2: No, but it has to be a subtract. I mean a plus? Does it have to be a plus?

Student 1: No, it doesn't (said simultaneously with Teacher 1)

Teacher 1: It can be either. You can use adding and subtracting, but you're not allowed to pour into that one.

S 1: Ha ha, ha ha ha!

T 2: You can only use the 3 and the 7 jars.

S 2: What is that one?

T 2: That one's just a jar to store in.

The process of delineating the parameters of the problem, although perhaps a necessary one, had quite effectively "frozen" the student's thinking processes. He was unwilling to assert whether he should add or subtract, and was unable to proceed toward a solution.

In another situation, a student who was less affected by our procedural instructions developed a unique resolution to a problematic process within the activity, one that destroyed some of our assumptions. Zoe, a Grade 6 student, used the smallest vial, which we had used to fill the jars, to empty the first centimetre or so of water from her storage jar prior to pouring the rest of its contents into the measuring jar. In doing so, she avoided spilling the water and was able to achieve more accurate measurements. What is interesting here is that we had not considered using the vial this way and had described it in a prescriptive sense. We had indicated that these vials were to be used to accurately fill the 3 and 7 jars. Had we questioned our assumptions more carefully and simply indicated that the vials were not to be used for measurement, we would have invited the students to find uses for them. Perhaps others would have discovered Zoe's method also.

The two examples quoted above show an interesting counterpoint in terms of teacher instruction and of students' reaction to that instruction. In teaching, some delimitation is necessary in order to discipline thought and to foster desired outcomes. However, teachers need to examine their thinking and to be sensitive to how they may be limiting children's thinking. We needed to eliminate student estimations; we didn't need to delineate the uses of all of the other tools available to them. We could have given up some control and still attained our goals, but we were too preoccupied with the form of the problem solving and not concerned enough with its meaning.

One assumption that we had made prior to starting the problem solving exercise was that we as teachers were to be unobtrusive within the process. The situation was analogous to one described by Pimm (1987, 37) in which a teacher used a poster picture of a great stellated dodecahedron to initiate student talk and to "remove herself from centre stage." The teacher in Pimm's account was not as successful in her initial attempt as she would have liked. Her tendency to pose direct questions weakened her attempt to be unobtrusive. We were fortunate in that we had four attempts at the same lesson. Although we remained prescriptive in some aspects even in the last lesson, we withdrew from directing the process. Thus, the Grade 5 lesson on the first day started off as follows:

T: What we want you to do is figure out how to get every volume from 1 to 10 using these two jars. Now how many volumes does this jar hold?

S: Three.

T: Three. Can you figure how to get 3 volumes?

S: Fill it up.

S: Fill it up to the black line.

T: Fill it up! How many does this one hold?

S: Seven.

T: So how do you get 7?

S: Fill it up to the black line.

T: The question is, How do you get 1 and 2 and 3 and 4 and 5 and 6 and 7 and 8 and 9 and 10? That's your job. Some are pretty easy. Some are real brain teasers. Which are easy?

S: Three, 7 and 10.

Whereas, at the beginning of the last lesson, the Grade 6 lesson we hear the following conversation:

T: Not using anything else, just using water and jars, any ideas on what we might be doing?

S: Measuring.

T: You're right. You're right. We're measuring.

S: We're getting the volume of the water.

T: There are numbers on these. This one's a 3. The little one's a 3, and the big one's a 7.

S: That means that there are 7,700 ml.

T: Well, actually it doesn't mean that. It just means that this one contains 7 units of water if it's filled to the black line, and this one contains 3 units of water if it's filled up to the black line.

S: So if that's filled up to the black line, it'll go to 3 on here?

T: Right.

S: So what we're going to do is put like, okay, so if you put. . . .

S: Seven plus 3 equals 10.

S: Seven and 3, it'll get up to 10. It should equal 10.

T: Okay. 7 + 3 = 10. Okay and that's it—so you have to get all the numbers from 1 to 10. Think you can do that?

There is a notable difference in the approach taken in the two classes. Although the students solve the problem of what to do in both classes, the amount of direction involved in the second class has decreased, and the number of declarative statements has increased. Of course there are other variables to consider, such as the ability levels of the two groups, but even the initial statements demonstrate the teacher's attempt to move away from "centre stage."

Some of the intrusive suggestions that were used in earlier sessions became less abundant in later sessions. In early sessions, students were given models of what to write. In later sessions, we gave only descriptors, such as "write a recipe for each number," or "write an explanation as though a Grade 3 student was going to read it." We reflected upon our teaching methods and tried to make the directions a little less stringent. We felt that this was a positive step in growth for us.

Our ability to withdraw from the process and to allow the students more freedom was short-lived in the Grade 6 class. The group worked in a very task-oriented and efficient manner. They solved the problem within 20 minutes; it had not been previously completed in less than an hour! What happened as a result was very interesting. We did not trust the students to be able to develop a problem to challenge themselves, nor did the classroom teacher or our professor. Even though, together we developed some very creative ideas in terms of extending the problem, not one of us proposed that the students be responsible for creating an idea themselves. Thus, on the advice of the professor, they started working to measure numbers greater than 10 and looking for patterns in the solutions. On the advice of the classroom teacher, they tried to look at the elegance, or simplicity of their solutions ("pretend that every pour costs $10"). We tried to discover why we lacked faith in the students. We felt that part of the problem stemmed from the fact that we were dealing with children whom we did not know extremely well. We also felt that because we had been invited into the classroom, we had a certain responsibility to maintain control. In areas where we felt in control we were willing to subdue our role in the problem solving situation. Where we were less certain of our ultimate control, we became concerned and restricted the nature of the activity.

The attempt to strike a balance between independent thinking and a productive classroom situation became a dilemma in these problem solving groups. At times, the dilemma was exacerbated by the nature of our talk and the conception that we had of our role. We often worked to solve small problems to free students up for the main problem, and in doing so channeled student thinking processes and perhaps affected the nature of the solutions. Certainly the solutions to auxiliary problems and possibly those in the principal problem were less varied than they may have otherwise been. On the other hand, there was some improvement within the area where we were comfortable with our role.

An examination of the data alerted us to some of the underlying assumptions that we had made about writing mathematical ideas in the classroom. Although we have a good grounding in the theory of writing and expression in mathematics, and we hold the belief that mathematics should be presented as a subject where reflection is important, the writing tasks that we asked of the children were product and not process oriented. Terms such as "recipe" do not evoke an image of reflection and did not solicit reflection from the students.

To a large extent, we focused on the manipulation aspect of the problem rather than on recording the problem solving process, even though we value it. Thus the question of recording became an issue in all of the problem solving groups. The students were happy to work at solving the problem, but did not seem to embrace the written aspect of the activity. Certainly their written accounts do not provide a very insightful record of their thinking or of their solutions. Most of the records follow a pattern and indicate a minimum of effort.

One volume measurement that required some application and ingenuity was the measurement of two units of water. The following accounts of this difficult procedure indicate the amount of description given by each group in their written records:

Grade 3: (Group 1) I took 7 jars and took [out] 4 3-jars.
(Group 2) Fill 7 jar (twice). Take away 3 and 3 and 3. We got 2.

Grade 4: (Group 1) 9–7 = 2. Pour 3 3s into 10, get 9, subtract 7.
(Group 2) We made 2 by taking 6 away from 7, which made 1. We poured the 1 into the storage jar, then I did the same thing over again and poured that into the storage jar and made 2.
(Group 3) We made 14 and subtracted 4 3s.

Grade 5: (Group 1) Like 1 to do again I will have 2.
(Group 2) I took a 7 then I took away 2 3s and store the 1. Then did the same method again.
(Group 3) 1 + 1

Grade 6: (Group 1) We did #1 again.
(Group 2) We filled the #3 jar 3 times. Then took 1 #7 jar from the 9.

We feel that these records are brief and do not reflect the difficulty of the task. Why were the students so reluctant to write? An important assumption shows up here. What we requested of the students, in terms of writing, would not in any real sense serve as an indication of their thinking processes. What we asked for was a "recipe," a "description of the procedure for a younger child," a simplistic model of the successes they experienced as they worked through the problem. We neither asked for, nor enabled the students to give us a reflective account of their thinking processes during the problem solving situation; we devalued misguided efforts by not requesting a description or account of them.

Pimm (1987, 47) offers a couple of ideas that may explain the brevity of the writing done during the activity. It could be linked to what Pimm refers to as the "status quo," or pupils' views of what mathematics is. It might also relate to the nature of communication the pupils perceived as taking place. With regard to the former, Pimm states

> The status quo can be hard to alter, even by a teacher who has decided to try to introduce a more discursive atmosphere. Pupils' views and expectations of what should go on in mathematics lessons are often quite rigid.

The students with whom we were dealing were probably not used to writing during mathematics lessons. This would partially explain their reticence to engage in the task of writing in the first place. Given that their view of mathematics may preclude any written explanations, it is unrealistic to expect students' descriptions to contain precise detail. The students' writing may also have been affected by their reasons for writing. Pimm (1987, 38) suggests that there are two types of speech: message-oriented and listener-oriented.

> In message-oriented speech, the speaker is goal directed and wishes to express a particular message, to "change the listener's state of knowledge"—it matters that the listener understands correctly. With listener-oriented speech the primary aim is the "establishment and maintenance of good social relations with the listener."

Although Pimm refers to speech, a parallel may be drawn with writing. Students may have been attempting to maintain good relations with the "teachers." We had made a specific request that they describe their solutions, something that they likely viewed as extraneous to mathematics, and rather than educating us as to the real nature of mathematics (get the answer), they simply cooperated and wrote something, thus avoiding any "unpleasantness or hurt feelings."

Other explanations are also possible. Time is almost always viewed as an evil in North American schools. Despite the fact that there was no real need to hurry in any of the classes, the unspoken message from us, as well as the expectations bound in with the idea of "status quo" lead to a feeling of a need to hurry. Stigler (1988, 27) proposes the variable of pace as a fundamental distinction that exists between American and Japanese classrooms. The author posits a relationship between the idea of pace and the amount of discussion and talk that occurs in the classrooms of these two nations. Japanese teachers consistently devote more time to talk and deal with fewer problems in greater depth. As members of our goal-oriented society, the need to complete the job was likely a strong motivating factor for the students and for us. The students thus would be induced to spend little time recording or thinking about their solutions and more time "doing" them. This explanation in no way precludes the previous ones; in fact, it intertwines with the ideas of types of communication, "status quo" and with our assumptions about writing.

We learned a lot about ourselves, about our thoughts as teachers and about some of the assumptions we make when we deal with children. We realized that even after the considerable thought that we gave to choosing a problem conducive to creative thinking, we "directed" the children's thinking. We would like to think that all the theories of learning that we have been studying for the past year are an integral part of our being now and that we can invoke them spontaneously. We were dismayed, despite a perceptible improvement over the course of the four lessons, to realize that we could still be so directive in the classroom. The reflection involved in the creation of this paper helped us to realize some of the assumptions that we had made. Once the underlying assumptions that drive our teaching are uncovered, we will be able to release them and to adopt a new understanding of the teaching process.

Appendix

Procedure

Students were given three jars and a pail of water. Two of the jars were used for measuring tasks; the third served a storage and verification function only: water could be poured into it, but none could be removed except to start over. The volume was marked with black lines. Volumes were measured by placing the jar on the table at eye level. This procedure maximized the accuracy and reinforced correct measuring techniques. The two measuring jars contained 400 ml and 170 ml, respectively. This approximates a 7/7 : 3/7 relationship. One unit = 57 ml. 400/7 = 57.143. 170/3 = 56.666. Using the jars, students were asked to measure out all possible volumes of water between 1 unit and 10 units.

Units	Possible Solutions
1	7-3-3
2	2.33 small poured into large–remainder in small = 2
3	3
4	7–3, 3 + 1, 2 + 2
5	3 + 2, 3 + 1 + 1
6	3 + 3
7	7
8	3 + 3 + 2, 7 + 1
9	3 x 3
10	7 + 3

The lesson consisted of a presentation of the problem, an explanation and a demonstration of the two measuring jars. The teachers remained seated at the table during the explanation and during the outset of the problem solving when the students were most likely to feel intimidated and incapable. Subsequent to the initial presentation, the teachers restricted their activities to monitoring and observing. They assisted "stuck" groups or individuals and assessed results. When students obtained a result they were asked to show their resultant volume and to describe their solution.

Students were allowed to decide who they wished to work with. They were limited only by the amount of equipment available (four sets of jars).

We worked on this project as coparticipants. In other words, we team taught sessions and collected two separate sets of field notes. A transcript was compiled from audiotaped sessions.

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