Estimating with Decimals

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The decimal number system provides mathematical models for a large number of the practical problems that students are likely to encounter. Yet research studies show that students perform poorly on tasks that require decimal computation, and that most often the students apply a memorized computational rule in a meaningless way (Bell, Swan and Taylor 1981; Hiebert and Wearne 1984).

The following question was asked of Math 15 students on a recent test concerning decimal knowledge.

Estimate the answer, then place the decimal point in the given answer.

1. 2.42 x 3.610 = 08736200
2. 3.20 ÷ .08 = 0040000
3. .42 x .23 = 00096600
4. 30 ÷ .6 = 00500000
5. 4.5 x 51.62 = 023229000

The test was administered to 23 students. Only a very small number of students gave the correct answer.

Table 1 includes the question, the number of students who responded with the correct answer and the number of students who gave the most common response. We can see by Table 1 that the majority of students did not give the correct answer. If we examine the most common response, it seems to suggest that a certain procedure was used, a procedure used when multiplying two decimals. The students seem to have counted from the right the number of places equal to the sum of the number of digits to the right of the decimal point in the numbers of the problem and inserted the decimal point there. Isn't this the "little trick" we tell our students when we teach multiplication of decimals?

Owens and Haggerty (1987) observed the processes of children as they form concepts and attach meaning to multiplication of decimals. Children are often taught to count the places after the decimal point in order to place the decimal in the product. Such algorithmic strategies are often used without understanding, and their use can lead to difficulties. In Table 1 we see that the procedure of counting places was used with both the multiplication and division problems. The majority of students did not discriminate between multiplication and division.

In order to better understand the procedures and processes used by the students, a sample of the students who wrote the test was interviewed. The interview questions dealt with problems that were on

Table 1. Number of students with correct response and most common response(N = 23).

Question	Correct Response	N	Most Common Response	N
1. 2.42 x 3.610 = 08736200	8.7362	2	87.362	15
2. 3.2 ÷ .08 = 0040000	40.0	4	4.0	10
3. .42 x .23 = 00096600	0.0966	1	9.66	15
4. 30 ÷ .6 = 00500000	50.0	2	50,000.0	12
5. 4.5 x 51.2 = 02322900	232.29	2	23,229.0	14

the test. The students were given a clean copy of the question and asked to place the decimal point in the answer. After the student placed the decimal point where he or she thought appropriate, the interviewer asked the student, "How do you know that it goes there?" The students who gave the most common response answered that they had "counted places." The answer for the division problem was the same; they counted places. These students were very concerned about the procedure used.

To get a clearer picture of why procedures are so important to students, we should look at some of the research done in the field of decimal number knowledge. Research on the instruction of decimal numbers is fairly recent. The work of Hiebert and Wearne has been the most comprehensive attempt made to delineate the cognitive aspects of decimal number knowledge. Hiebert and Wearne (1986, 199) argue that "mathematical competence is characterized by connections between conceptual and procedural knowledge . . . that mathematical incompetence often is due to an absence of connections between conceptual and procedural knowledge."

What is conceptual and procedural knowledge? Conceptual knowledge is knowledge of those facts and properties of mathematics that are recognized as being related in some way. When a fact or property becomes part of a larger network through the recognition or construction of a relationship between the fact and a network that is already in place, then we say that that fact becomes part of conceptual knowledge.

Procedural knowledge is limited to knowledge of how written mathematical symbols behave according to syntactic rules. Procedural knowledge of symbols does not include knowing what the symbol "means," that is knowing that the symbol represents an external referent. Procedural knowledge also includes the set of rules or algorithms that are used to manipulate the symbols and solve mathematical problems. For example, counting places and not knowing why.

These Math 15 students used their procedural knowledge. They manipulated the symbols according to a rule they knew in order to solve the problem. However, in this case, applying a known procedure did not produce the correct response. What went wrong?

Hiebert and Wearne (1984) indicate that there are three levels, points, or "sites" in the process of computing with decimal numbers that demarcate the primary sources of students' difficulty. At Site 1 many students do not know what the symbols mean. They fail to connect decimal symbols with meaningful referents. At Site 2 many students do not know why the computation procedure works. Based on individual interviews and analysis of written errors (Hiebert 1985), most students' computation activity consists of recalling and applying memorized rules for which they connect absolutely no rationale. At Site 3, many students are not aware that answers should be reasonable. To be able to check whether an answer to a decimal computation problem is reasonable, one must connect at least an intuitive idea of the arithmetic operation with appropriate meanings for symbols. Hiebert and Wearne (1987) interpret the difficulties that students exhibit at each of the three sites in the computation process as a consequence of a divorce between procedural and conceptual knowledge.

It appears that the majority of students who wrote this test failed to connect the decimal symbols with meaningful referents (Site 1), recalled and applied a memorized rule for which they seem to have no rationale (Site 2), and were unaware that the answers given were unreasonable (Site 3). The interviews also showed that the students' only concern was the procedure. After placing the decimal point, the students did not check to see whether the answer was reasonable or not.

A short excerpt from one of the interviews follows.

Interviewer: Are you sure the decimal point goes there?

Student: Yes.

I: How do you know that it goes there?

S: All you have to do when you multiply decimals is count places.

I: Is the answer correct?

S: It has to be if you follow the rule.

Upon further questioning and working with the rounding off of numbers the student was able to see that her answer was incorrect. She then realized that "extra" zeros had been added to the "real" answer. So why was the student not able to estimate the correct answer the first time?

Kieren (1987), in his reflections on fraction number research, comments that several of the studies reflected that traditional instruction makes an early and probably unwarranted emphasis on symbolic manipulation and computation with common or decimal fractions. If this is true, then children are probably forced to treat these symbols as concrete objects and hence build knowledge based inappropriately on patterns in the symbols (e.g., count the number of decimal places). What seems clear is that a fraction curriculum in which symbols are not tied to meaningful object actions has inhibiting effects.

Lichtenberg and Lichtenberg (1982, 143) report "the typical approach to decimals does not allow enough time for developing meaning, whereas inordinate amounts of time are devoted to the computational procedures." The emphasis here is on teaching computational skills and how to manipulate the symbols to arrive at a correct answer. As students move through school they memorize an abundance of task-specific rules for manipulating symbols (Hiebert 1984). The problem is that few links are constructed between the understandings they have and the symbols and rules they are taught. Many students have not acquired adequate meanings for the symbols they use; they do not understand the procedures they apply to manipulate the symbols, and they fail to test the reasonableness of the outcomes. Hiebert sees the critical instructional problem not as one of teaching additional information, but rather as one of helping students see connections between pieces of information that they already possess.

Questions about how students learn mathematics, and how they should be taught, turn on speculations about which type of knowledge (conceptual or procedural) is more important or what might be an appropriate balance between them.

Many learning problems in mathematics can be attributed to the absence of connections between the memorized, mechanically applied rules and conceptual understandings (Hiebert and Wearne 1984). How can these connections or links be attained? The critical instructional problem may be one of helping students connect pieces of information that they already possess.

Post, Behr and Lesh (1982) feel that students' difficulty in learning decimal knowledge is due in part to the fact that school programs tend to emphasize procedural skills and computational aspects rather than the development of important foundational understandings.

Readings related to conceptual and procedural knowledge show that the two are often two distinct sets of knowledge and that procedural understanding may be easier for teachers to teach and easier for students to understand than conceptual knowledge. As a consequence, decimal number knowledge may be taught mostly from a procedural point of view. Therefore, there is need to examine what procedures the students know, how they are using them and whether it is possible to arrive at conceptual understanding.

Suggestions for Teachers

Reys (1986) has found that computational estimation skills can be taught and do improve with instruction. Computational estimation refers to obtaining a reasonable approximation rather than an exact answer to a problem without having to depend on pencil and paper algorithms or calculators. Due to the increasingly technological world in which our students live, it would be wise to teach them estimation strategies.

In order to teach estimation strategies, we must establish what a reasonable approximation is. A student with good estimation skills should be able to decide whether his answer seems reasonable. Questions such as the following can be answered without computing an exact answer.

1. What is the length of this room?
2. How much pizza and pop would we have to order for lunch for this group?
3. If milk is ¢.89 a litre and bread is $1.09, can I buy both with $2.00?

Each of the above problems can be solved by the use of different estimation techniques that cater to the particular numbers and operations of each problem. Thus, different estimation problems will lead to the students using a variety of estimation strategies.

Usually when we speak of estimation we include the strategy of rounding. However, this connection is not always clear to the student. Therefore, rounding exercises should be done in conjunction with estimating exercises.

The strategy of rounding can be used in association with

1. estimating the sum of numbers
   27.546–0.3926 is about ______________

2. finding the approximate product

   .5091 x 380

   19
   190
   1,900
   19,000

3. choosing a reasonable quotient
   28.76 ÷ .4 equals about
   a) .07 b) 7 c) 70 d) 700

4. working with large numbers
   6,000,000 x 2.114 is about ______________ .

Seymour (1981) has published two books on developing estimation skills that contain worksheets for duplication. The activities in these books deal with reasoning, computation, measurement, pricing, counting and estimation techniques, worldly knowledge and problem solving. The activities were designed to help students in Grades 6 and 7 (Book A) and Grades 8 and 9 (Book B) develop their estimating abilities and learn to use approximate numbers.

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