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delta-k

JOURNAL OF THE
MATHEMATICS COUNCIL
OF THE ALBERTA
TEACHERS' ASSOCIATION

Volume 28, Number 1 April 1990

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Contextualizing Mathematics Through Manipulatives and Estimation

Before discussing this issue's theme, I want to take this opportunity to thank John Percevault, editor of *delta-K* for many years. The time and energy John devoted to this journal are much appreciated. He has also been invaluable to me in the "changing of the guard."

Any changes in editorship result in changes to the journal. I hope you will inform me of any changes you would like to see, and provide feedback on what you like and do not like about *delta-K*. Alberta educators must communicate their questions and concerns about mathematics education. A letters department or a dialogue department can be created if you indicate that you would like one. My role as editor is to create dialogue and to provoke and encourage responses from you; therefore, I will be articulating some issues and/or questions related to the articles in each journal. Although these will be my issues and concerns, this journal is for you; your thoughts and questions are welcome.

This issue of *delta-K* is organized around the theme of contextualizing mathematics, of placing problems and exercises in a setting meaningful to students. Often we assume that because mathematics deals with abstractness, we as teachers need to be abstract. But we do our students a disservice by being too abstract and by teaching them basic facts, such as 4 + 6, without connecting these facts to the real world or to manipulatives. When we move beyond Grade 1 or 2, we seem to feel that contextualizing mathematics is not as necessary as it is for younger students. So students in Grades 4 to 6 and especially in Grades 7 to 13 are rarely offered alternate, contextual ways of making sense of abstract mathematical symbols and concepts.

To cope with the abstractness, students in the classroom often memorize mathematical rules and procedures, and attempt to use them by manipulating symbols. This memorization approach to learning and doing mathematics stems partially from viewing the discipline as decontextualized. Often students deal solely with symbols.

Students who memorize rules and procedures with little understanding of these procedures or why they use them concern me. Through my work with elementary teachers and with students studying to be elementary teachers, this concern has recurred. But this concern is not only mine; it is expressed by mathematicians and mathematics educators at all levels.

The relationship between students memorizing rules and procedures and a decontextualized view of mathematics is complex. One way of exploring the relationship is to observe children working through mathematics problems and to speak with them about what they are doing. In one of the methods courses I teach at the University of Calgary, my students enter elementary classrooms and conduct interviews with schoolchildren of various ages. Students asked the children how the mathematics they are learning was used outside the classroom. Most often, the response was that mathematics was used to do their homework or to help their younger brothers and sisters do their homework. What startled my students was that the children did not connect a problem, like 43–17, with any real world context. It had become totally abstract and decontextualized.

Even for many of the schoolchildren who have used manipulatives to learn the algorithms of addition, subtraction, multiplication and division, there still exists a "memorize what to do" attitude toward mathematics.

When my methods students asked the children about the connection between the manipulatives and the paper and pencil algorithms, they saw little relationship between the two. The children can operate using the manipulatives, but often it seems as though they have memorized how to use them, just like they memorize how to do the paper and pencil procedures.

If children are so attached to memorization as a way of learning mathematics, we need to question how we are using manipulatives in the classroom. Manipulatives are an excellent way to contextualize mathematical concepts because children can see and touch the materials, but we must not assume that the materials themselves will assure student understanding of the concepts. We have assumed that merely using base-10 blocks, for example, will automatically help children understand place value. This may or may not be the case. We need to explore what children do and do not understand about the materials and how they are related to the concepts we are teaching. In essence, we need to examine the use and misuse of manipulatives more closely.

Similarly, estimation skills can be learned in a rote, mechanical manner with little understanding of their purpose. Often we ask children to estimate the answer to problems like 998 + 665 or 1.25 x .56. Again, by giving only the numerical symbols, we have removed any relevant context, and we may have unconsciously promoted the memorization of estimation rules.

This issue of *delta-K* deals with manipulatives and estimation. Because both of these topics are of current interest in mathematics education, and seem to get enormous coverage in journals such as *The Arithmetic Teacher* and other publications of the National Council of Teachers of Mathematics (NCTM), I want to take an alternate approach in this issue. I wish to emphasize the importance of using manipulatives as a way of contextualizing mathematics for students at all grade levels and to emphasize how estimation skills can also be contextualized.

The article by A. Craig Loewen deals with some theoretical issues related to using manipulatives in the classroom. Loewen offers some excellent contextual ideas for illustrating algebraic concepts, particularly for concepts that students have difficulty visualizing.

The second article by Bernard Yvon and Anne Fortin offers a concrete idea for motivating young children to work with place value ideas in a meaningful way. In her article, Karen Ibbotson offers some practical ideas for using manipulatives to teach the basic ideas of addition, subtraction, multiplication and division of whole numbers.

The article by K. Allen Neufeld on tessellations offers some excellent activities for contextualizing geometric concepts. If you are interested in tessellation in the arts, I recommend the work of M.C. Escher.

The article by Jeanette Parow-Jarman and Dave Whiteside describes a problem solving activity they conducted with elementary schoolchildren and explores the relationship between their own behaviors and assumptions, and how they affected the children's thinking. As well as using manipulatives to concretize and contextualize the problem, the children were given a writing activity to describe how they solved the problem.

The last two articles deal with the topic of estimation. The article by Yvette d'Entremont offers some background on research done with students' concepts of decimal estimation and discusses the idea of procedural knowledge and how students who do not connect decimals to a meaningful context learn to memorize and manipulate symbols. She also offers some real world suggestions for contextualizing the estimation of decimals. The article by Katherine Willson offers some excellent suggestions for bringing estimation ideas into a meaningful context for students in Grades 4 to 7.

These articles obviously offer but a small sample of ideas and issues surrounding the topics of manipulatives and estimation. I hope they will stimulate you to think about the topics and to enter into discussions about them and about the theme "contextualizing mathematics."

Linda I. Brandau

Implementing Manipulatives in Mathematics Teaching

A. Craig Loewen

A. Craig Loewen is an assistant professor in the Department of Education at the University of Lethbridge, Lethbridge, Alberta.

Manipulative materials have emerged in mathematics instruction as more than just a means to add variety to lessons; they are an essential element for effective mathematics instruction. However, for manipulatives to be used successfully in the classroom, a great deal of thought must precede their implementation. What role do manipulatives play in mathematics instruction? What factors influence the effective implementation of manipulatives in the instructional process? This paper presents alternative answers to these questions.

A Definition of Manipulatives

The purpose of manipulatives is to make mathematics more concrete. Manipulatives enable students to play with, experience and develop for themselves mathematical principles, relationships and ideas. For manipulatives to have any place in the mathematics classroom, they must embody or physically represent specific mathematical concepts (Wiebe 1983). Consider two examples and one counterexample.

A concept that many elementary mathematics students struggle with is why the remainder after division can never exceed the divisor. This concept may be illustrated when teaching division using a balance beam (Knifong and Burton 1985). To model the equation $7 \div 2$, the student would place 1 weight on the balance a distance of 7 units to the left of the fulcrum (see Diagram 1). Because the divisor is 2, weights are hung 2 units from the fulcrum on the right side until the beam is balanced. The situation quickly arises that when 3 weights are hung on the right side, the beam tips to the left, but when another weight is added, the beam tips to the right. Where then should the final weight be hung in order to balance the beam? Through experimentation, it is obvious that hanging the weight any further to the right (e.g., a value greater than the divisor) is counterproductive, and thus a position closer to the fulcrum (e.g., a value less than the divisor) must be selected. In this case, the weight must be hung 1 unit to the right of the fulcrum to completely balance the beam. The remainder must always be less than the divisor; this mathematical concept is actually embodied within the manipulative materials.

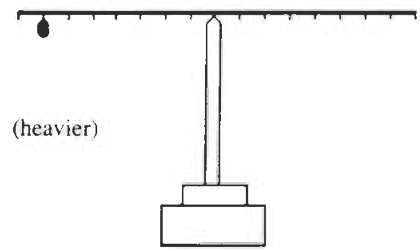
Poker chips may be manipulated to model the subtraction of negative integers. Assume that a blue poker chip represents $+1$ and a red poker chip represents -1. Thus the subtraction of -4 from 3 in the equation $3 - {}^{-}4 = ?$ may be modeled as follows. Set out 3 blue poker chips $(+3)$ and then remove 4 red ones (-4). It is obvious that no red chips may be removed because there are only blue chips available. However, note that a blue and a red chip together total zero (e.g., $-1 + 1 = 0$). Thus, any number of pairs may be added without changing the value of the expression. Pairs are added until there are enough red chips such that 4 may be removed (add 4 pairs). Now, when 4 red chips are removed, 7 blue chips remains. This process illustrates that $3 - {}^{-}4 = 7$. This model makes it clear why the difference is greater than the minuend when subtracting a negative subtrahend.

As a counterexample, consider the common exercise in which students pair numbered cards with corresponding word cards (see Diagram 2). This

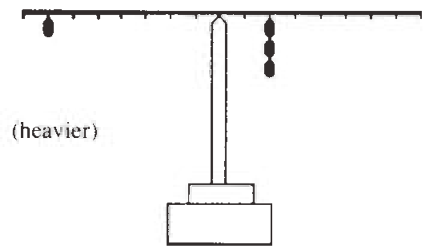
Diagram 1

Using a balance beam to illustrate division. Solve $7 \div 2 = ?$

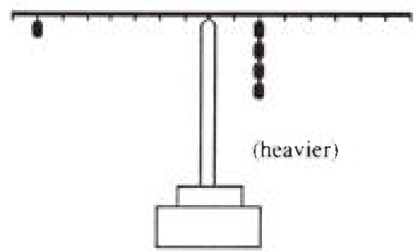
A. Begin with one weight seven units to the left of the fulcrum.



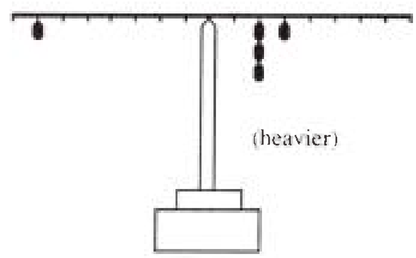
B. Add weights 2 units to the right of the fulcrum to balance the beam. The left side is still too heavy.



C. The right side is now too heavy, so the whole number quotient must be 3. Experiment with one weight to find the remainder.



D. The right side is still heavier, so the remainder must be less than 2, which is the divisor.



E. The beam is now balanced. The remainder must always be less than the divisor.

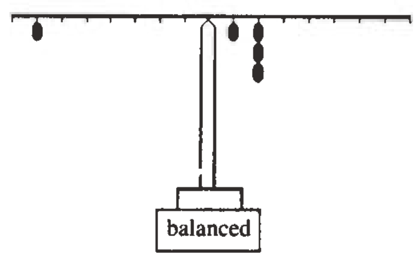
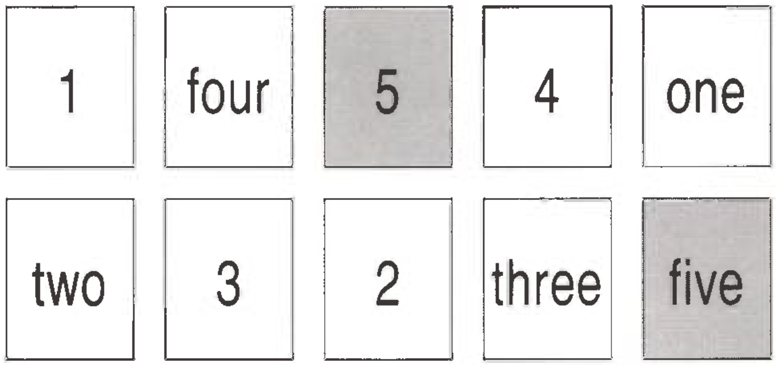


Diagram 2
Matching numbered cards with name cards.



The shaded pair is a match and may be removed. This activity is sometimes played as a game which begins with all cards face down. Two cards, one at a time, are turned over by a player. If a match is found then those cards are removed from the game. If no match is found, the cards are turned back face down and the other player takes a turn. The winning player is the one having made the greatest number of matches once all the cards have been used.

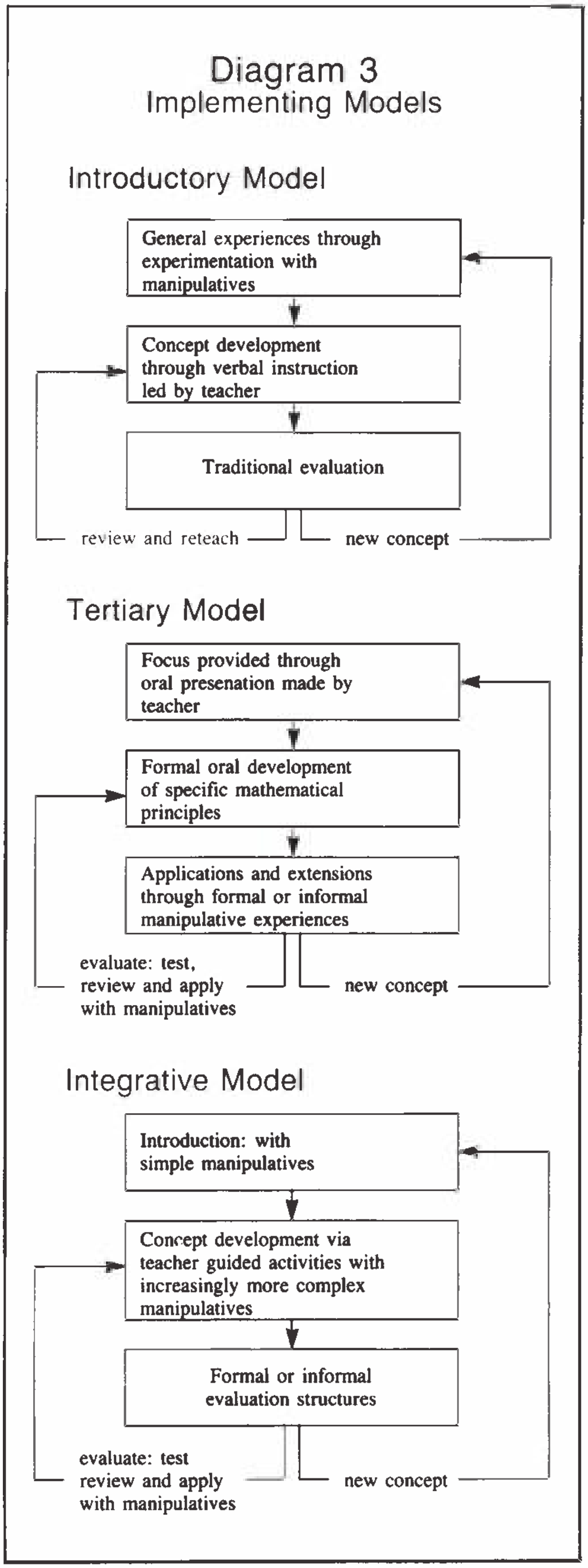
activity, and others like it, may be called manipulative only in that students are given some object (cards) that they may touch and move. The cards do not embody any mathematical concept however, and this exercise only serves to help students develop correspondence between names and symbols (a vocabulary exercise). For the student, no greater understanding of "oneness," "twoness," or "fiveness" is developed simply by matching symbols to words; memory skills are drilled.

Three Implementation Models

What role do manipulatives play in mathematics instruction? Where do manipulatives fit into the typical instructional sequence: introduce, develop, review and evaluate? The following three general models for implementing manipulatives offer some alternative answers to these questions. These models may be applied to either individual lessons or to complete units.

The first implementation model is called the introductory model (see Diagram 3) because the manipulatives are used only in the beginning stages of instruction. The purpose of the manipulative in this model is to introduce the mathematics concept to be learned and to provide a body of concrete experiences that can be drawn upon or synthesized during later formal instruction. The manipulative also fulfills the purpose of increasing student interest and motivation. In some cases the manipulative also provides a sense of relevance to the later formal instruction delivered by the teacher. The learning sequence flows from the concrete to the abstract. In this model, knowledge is organized from the general to the specific; general concrete experiences are provided first and followed by highly structured formal sessions in which specific concepts are revealed through an oral exposition delivered by the teacher. The major assumptions of this model are that students require a context for effective formal instruction, and that general concrete experiences facilitate the learning of specific abstract concepts.

The second implementation model is called the tertiary model (see Diagram 3) because the manipulatives are not introduced until the latter stages of instruction. Early instruction is teacher-controlled, but later experimentation is less closely monitored.



In the initial stages of this model, the teacher provides formal focused instruction on specific abstract concepts; the focus is not on understanding but on the awareness of principles. These specific principles are later linked to create more general knowledge through informal experimentation with manipulatives; knowledge is organized from the specific to the general, while experiences are organized from the abstract to the concrete. The manipulative serves a synthesis role and functions as a context in which learned concepts may be applied. In this model, the manipulative may also serve as a means for the teacher to evaluate student progress and understanding, as well as a means to undertake review of specified concepts. The second model is built upon the assumption that students require basic skills and knowledge before they can fully benefit (e.g., draw conclusions and formalize mental structures) from the experiences and environment manipulatives provide.

The final model is the integrative model (see Diagram 3). In this model, manipulatives are used continually throughout instruction; knowledge and skills are introduced, developed, reviewed and evaluated through concrete experiences with physical representations of mathematical concepts. By using manipulatives at all points during instruction it is hoped that high motivation and interest levels will be maintained throughout the entire instructional cycle. Using a manipulative for all phases of instruction eliminates the need to introduce more than one set of materials. The major assumptions of this model are that students learn better and retain longer what is learned in a single familiar context, and that all phases of the instructional cycle may be delivered easily using manipulatives.

No one model is correct or better than another. Instead, the teacher should use the model that best suits the material to be taught, the needs of the students and his or her own instructional style. The teacher may wish to consider the mathematics skills and motivation levels of the students, the students' learning styles, the synthesis and generalization skills of the students, the ease with which students master and apply learned concepts, as well as the complexity of the mathematics concepts to be taught. Each model possesses its own advantages, disadvantages and assumptions. The teacher must select the model in which the disadvantages are minimized, the assumptions appear realistic and the advantages are exploited. When these conditions exist, the purpose of the manipulative is maximized and effective implementation is achieved and measured by improved student learning.

Some Factors Influencing Effective Implementation

Manipulatives may be evaluated according to a variety of criterion. Hynes (1986) has suggested that manipulatives may be evaluated according to both their pedagogical and physical attributes. With respect to pedagogical attributes, manipulatives must provide a clear representation of a mathematical concept, be appropriate for the student level, interest the students, be versatile, contribute to the building of a mathematical concept, assist in developing vocabulary, improve spatial visualization, promote problem solving, provide a sense of proof and promote creativity. With respect to physical attributes, the manipulative must be durable, simple, attractive, manageable, cost-effective and reasonable in terms of the quantity required. Not all manipulatives exemplify all of these attributes, but generally, the better the manipulative, the more conditions it will satisfy.

The attributes that Hynes describes are valuable when discussing the relative differences between manipulatives, but the true value of manipulatives lies in how effectively they may be employed in teaching and learning situations. The most versatile, motivating and attractive manipulative will not be effective unless properly employed. Therefore, the manner in which the activity is conducted may be just as important as the materials themselves.

The first implementational consideration is the degree to which the student has control over concept development. If given time to simply experiment and play with the objects, will students develop the desired concept on their own? To what extent must the students' interaction with the manipulatives be guided by the teacher? To allow students to discover and develop concepts independently is often too time consuming, and there is no guarantee that the concept will ever be clearly or correctly formalized; however, concepts that are developed independently are more likely to be retained and treasured. The teacher must decide which form of manipulative is preferable based upon such considerations as students' past experiences with discovery learning, students' learning styles, the time available for the development of a concept, the motivation level of the students and the ease with which the concept may be summarized from the play experience.

The second implementational consideration pertains to the degree to which the student may control the manipulative. Is it desirable that each student has his or her own set of manipulatives, or is it sufficient that the teacher manipulate one set for the benefit of all? When the teacher manipulates the materials for the students, then the visual experience is substituted for the tactile. When working with individuals or small groups, the discovery learning approach is possible, and this approach necessitates a tactile experience. When working with a large group, a visual learning experience is more practical. In essence, the teacher defines his or her own role as that of consultant or group leader. The choice of role dictates the degree to which the teacher intervenes in and controls the concept development process.

The third implementational consideration is the degree to which the mathematical concept embodied within the manipulative is obvious to the students. When the concept is obvious, then the materials are appropriate for developing mathematical relationships or facts. When the concept is less obvious, then the manipulative serves as a data-keeping tool. When the manipulative is used as a data-keeping tool, the student is relieved of data-keeping functions and may concentrate more fully on process skills. The following examples clarify the data-keeping and concept development natures of manipulatives.

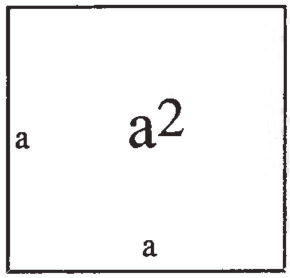
The first example is illustrated in Diagram 4. In this example, paper is folded and cut (Bober and Percevault 1987) in such a way as to illustrate why $a^2 - b^2 = (a-b)(a+b)$. Once the activity is completed, an inherent sense of proof or obviousness makes it difficult tc contest that the relationship is true.

The second example is illustrated in Diagram 5. In this example, the students model the process of solving simple algebraic equations through the manipulation of ordinary objects such as paper cups and circles cut from colored construction paper. In this exercise, the process of solving equations is emphasized, and the materials serve the purpose of keeping track of the various symbols and quantities found on each side of an algebraic equation. Certain key relationships, such as $-x + x = 0$ and $-1 + 1 = 0$, are not made more obvious through this exercise.

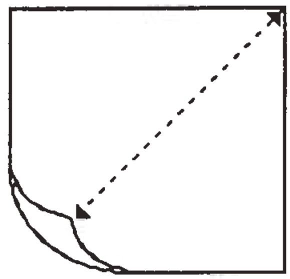
Experience with these manipulatives simply provides the students with an alternate way of conceptualizing and remembering a process; it does not necessarily impart a greater understanding as to why the inherent relationships within the process are true.

Diagram 4

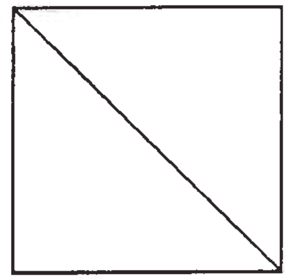
Illustrating the difference of squares: $a^2 - b^2 = (a-b)(a+b)$



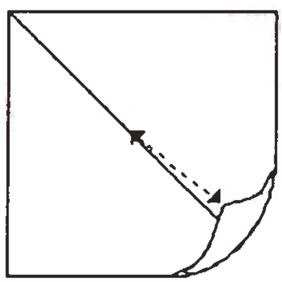
1. Begin with a square piece of paper.



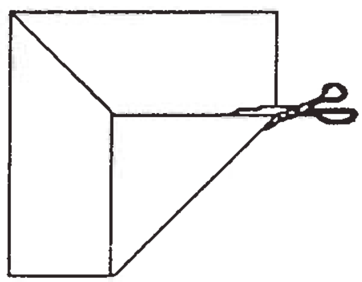
2. Fold diagonally.



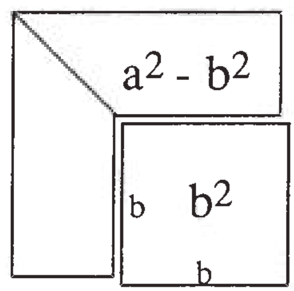
3. Crease.



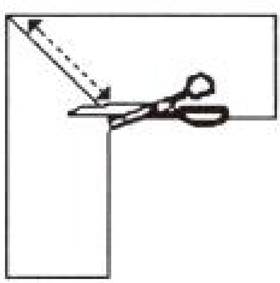
4. Fold one corner on the diagonal crease to any point part way along the crease.



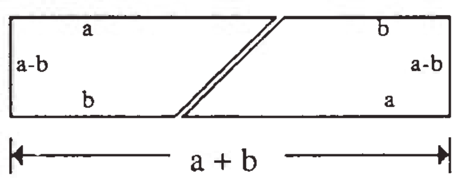
5. Cut along the paper's edge.



6. Remove b^2.



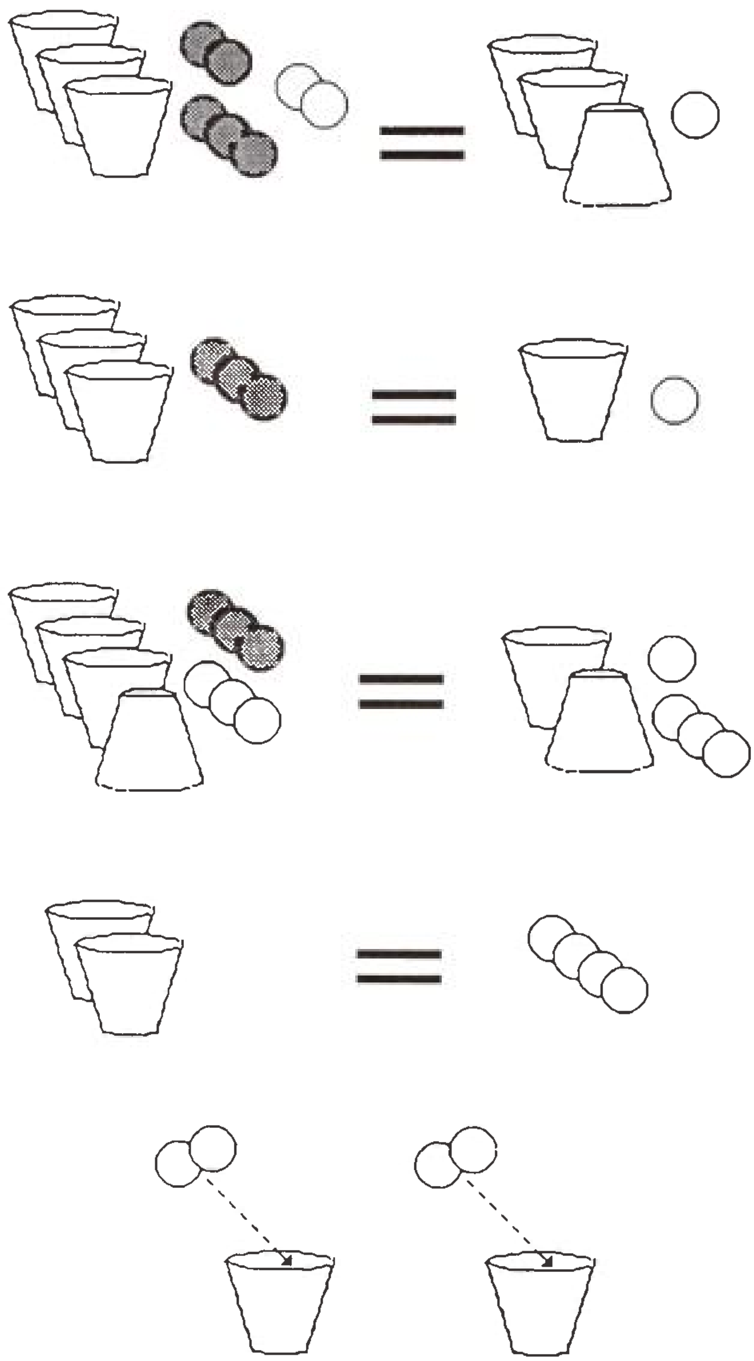
7. Cut along the remaining diagonal fold.



8. Rearrange.

Diagram 5

Modeling the process of solving algebraic equations with paper cups and colored paper chips.



$3x - 5 + 2 = 2x - x + 1$
Remove opposites to simplify.

$3x - 3 = x + 1$
Add opposites to get like expressions on the same side of the equation.

$3x - x + 3 - 3 = x - x + 1 + 3$
Remove opposites to simplify.

$2x = 4$

Distribute.
$x = 2$

The manipulatives are useful in learning the process because it is easier to remember how to distribute paper chips equally to paper cups than it is to correctly divide both sides of an equation by a constant value. The teacher must decide under what circumstances it is preferable to select manipulatives that facilitate a deeper or more complete understanding of a concept as opposed to manipulatives that simply promote an alternate conceptualization of a process. Both alternatives have value depending upon the students with whom the teacher is working and the objectives that are to be met.

The fourth implementational consideration is the number of concepts a manipulative supports within an instructional unit. If many concepts within an instructional unit are to be taught using manipulatives, then it is desirable to use similar materials for each topic within the unit. Using similar materials helps students link and relate these topics, relieves the need to constantly introduce and familiarize students with new materials, and provides a sense of continuity and coherence to the unit. However, a manipulative can be effective even if it supports only one concept, especially when used to review a concept or provide a brief extension to a previously developed concept. The manipulative must fit the instructional purposes and processes that the teacher has designed.

The fifth implementational consideration is the degree of familiarity students need with the materials in order to use them properly. How much time must be spent introducing the materials to the students and developing necessary vocabulary? If students are not properly familiarized with the materials, they will spend less time focusing on mathematical principles and more time trying to remember the manipulative procedure. Furthermore, if students are not familiar with the materials, they will not possess the vocabulary or language necessary to ask questions of themselves and others or to summarize their new knowledge. Certain materials require a longer introduction time, and generally, materials that require more introduction are less desirable. In order to justify a longer introduction time, the teacher must consider how well the manipulative embodies the mathematical concept, the number of concepts that may be taught using the materials, the required degree and extent of teacher-student interaction and whether students will work with their own sets of materials.

Well-constructed manipulative materials do not guarantee effective instruction. Even good manipulative materials will only be as effective as the process through which they are employed, and this process requires careful thought and reflection by those who understand the mathematics curriculum as well as children's thinking processes, capabilities, needs and interests.

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Bring a Big, Bright Smile to Math with Plinko!

Bernard Yvon and Anne Fortin

Bernard Yvon is a professor of education and child development at the University of Maine in Orono, Maine. Anne Fortin is an elementary teacher at Dedham School in East Holden, Maine.

Do your students really understand the meaning of "one's place," "ten's place," and "hundred's place"? Could they explain in their own words the relationships between each digit in a number? Unfortunately, many students cannot. They know the mechanics needed to finish the worksheet or assignment, but they lack a true understanding of the task.

It was surprising to learn how little Grade 2 students understood about place value. They knew the terms, but when asked what each term meant in relation to the others, blank stares and a few wild guesses were received. It was then I realized that my students needed a solid understanding of "known" math concepts, because these concepts are the foundation for much of the mathematics students will encounter later on. They need these concepts so that they can function intelligently in an uncertain future world.

How can this understanding be fostered in students? One way is to provide concrete experiences that allow children to manipulate objects as they learn, and to have fun at the same time. This idea was the motivation for creating "Plinko," a place-value teaching aid guaranteed to bring a smile to students' faces and the gleam of comprehension to their eyes.

How to Play "Plinko"

In order to interest my students in learning about place value, I decided to create an activity that students could do by themselves. A student stands behind the panel, and drops three disks, labeled "ones," "tens" or "hundreds" anywhere along the top row of pegs. The disks must be pressed flat against the panel and dropped one at a time. The disk works its way down the panel and drops into a numbered bin at the bottom (see photograph). The student then calls out the number of the bin in which the disk lands. Other students record that number in the correct place-value position. For example, if the disk labeled "tens" lands in the bin marked "8," students should place an 8 in the "tens" column. The numbers may be written on paper or recorded by having students physically manipulate objects such as blocks, beads or popsicle sticks (see Sketch 1). This will help them get used to finding the "hundreds place," and if manipulatives are used, they will help students see these place values as well.

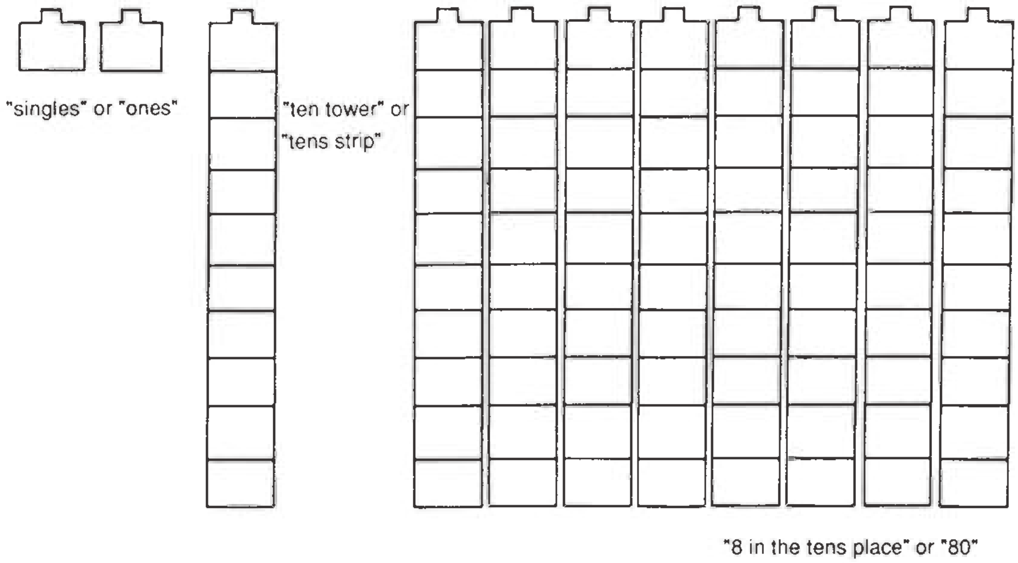
The activity continues until all the disks have been played, called out and recorded. The disks are left in the bins until all three have been played. The students then orally calculate the number and check the disks to see if they have recorded the numbers correctly. If so, then another student may come to the panel and begin the process all over again. A running score may be kept for individuals and/or teams to incorporate the element of competition.

Some children, however, may inadvertently place a number in the incorrect column, or they may misunderstand the concept of place value. If a student sees no connection or relationship between the value positions, such as 10 "ones" equals 1 "ten," or even that the hundred's column is always to the left of the "tens" column, explanations should be made immediately. These corrections may be verbal explanations by yourself or by another student.

Sketch #1: Manipulatives: blocks, beads, popsicle sticks

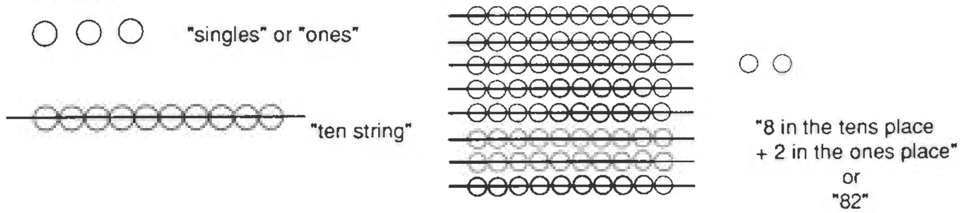
BLOCKS

Example:



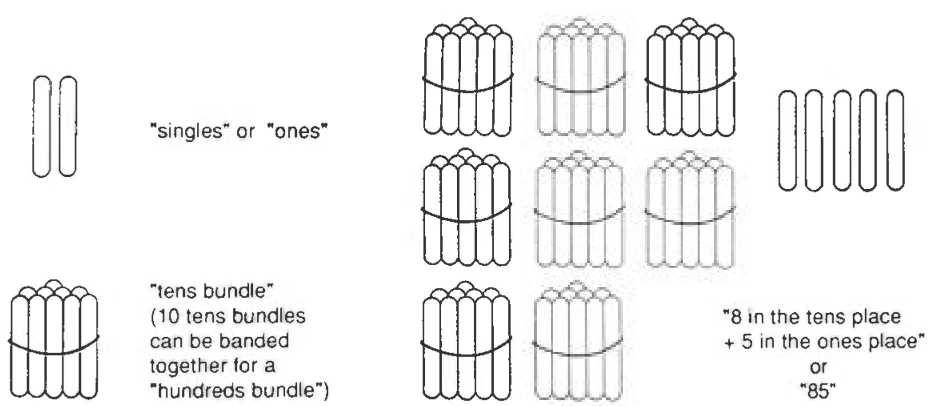
BEADS

Example:

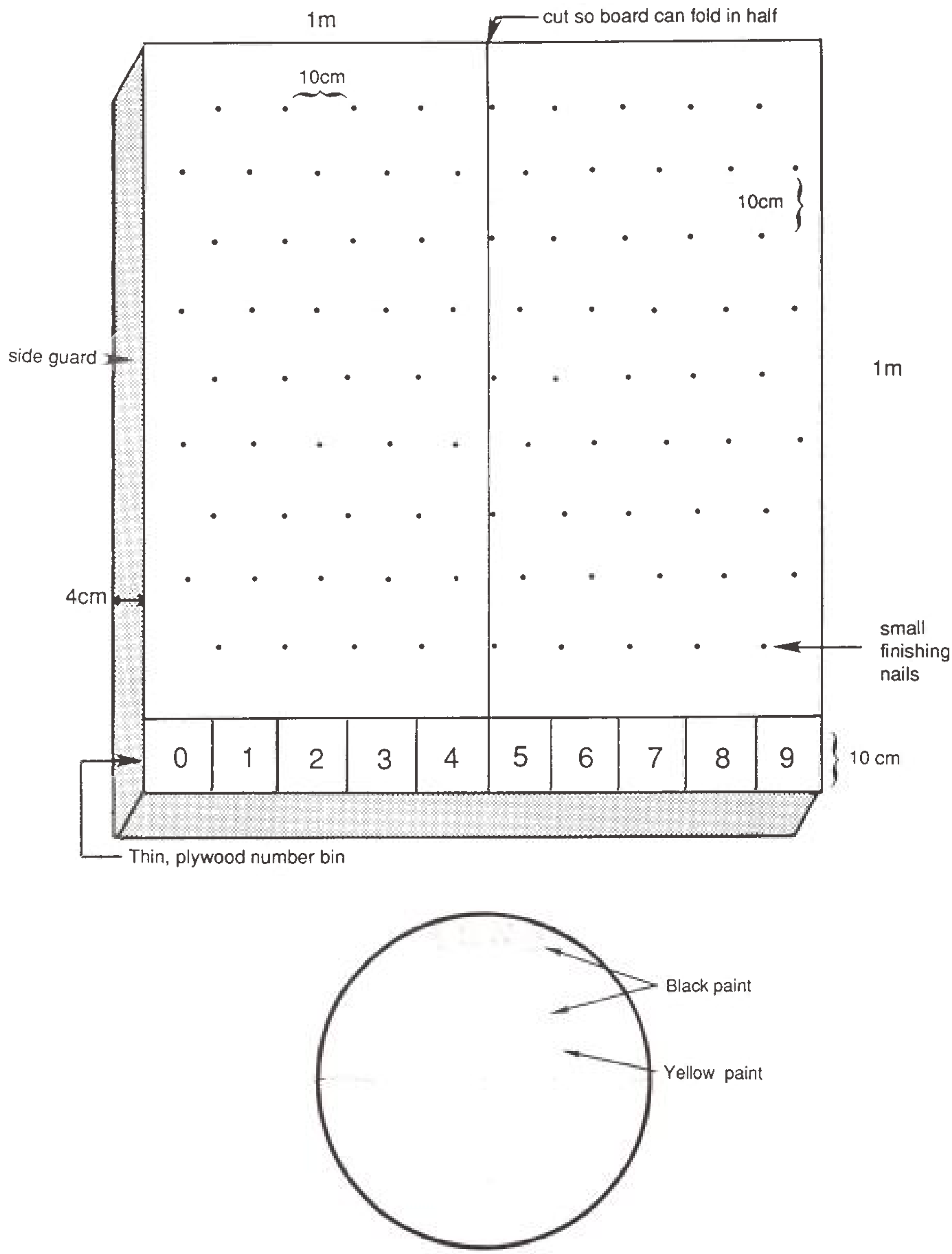


POPSICLE STICKS

Example:



Sketch #2: Gameboard Measurements



Manipulatives can also be used to further explain the concept and to reinforce place value in concrete terms. For example, the child could physically stack 10 individual blocks to see that they equal one "ten" block. Therefore, if some students do not have the correct number after checking with the disks, discuss what might have gone wrong and correct it at that time.

To expand on this activity, the concept of rounding numbers could be incorporated. Once the resulting number is agreed upon, students could round that number to the nearest 10, 100 or even 1,000, depending on their grasp of the concept.

Bingo is also a great way of using the numbers produced in "Plinko." Students can look for the "Plinko" number on their individual bingo cards. If the number is on their card, they place a marker on the number, just as in a bingo game. This "game" provides not only another method of practising place value, but it also adds a little extra fun to learning.

Materials

You will need
1. yellow and black paint;
2. approximately 90 small finishing nails;
3. a large plywood square, about 1 metre square, 2 to 3 cm thick;
4. thin plywood for the bins;
5. three hinges for centre cut; and
6. flat, circular wooden disks.

Benefits

Aside from place values, "Plinko" can also be beneficial in teaching addition, subtraction, multiplication, division, fractions, decimals and probability. It enables a wide range of students to make use of and enjoy the aid at several different levels. For addition and subtraction, students could ignore the labels on the disks and simply add or subtract the single-digit numbers as the disk lands on them. They can check the sums and differences for patterns of odd or even numbers, prime numbers and so on, or students could simply identify the number as an even, odd or prime and add and subtract it. As their abilities increase, they can work up to adding and subtracting the two- and three-digit numbers created by the original game procedure. Negative numbers could be incorporated simply by subtracting the larger number from the smaller or by adding disks with negative numbers marked on them.

Front view of game board

The same type of activity could be performed for developing multiplication and division skills. Rather than adding or subtracting the resulting numbers, simply multiply or divide them. Decimals and fractions can be produced and studied by using the same procedure.

The concepts of probability and statistics could be taught by using "Plinko" to record and/or predict the frequency with which numbers will occur when the disks are dropped.

Therefore, whether you are a kindergarten teacher or a Grade 6 teacher, this aid can be useful to you and to your students whatever your particular math curriculum may entail.

Students have enjoyed "Plinko" very much. They often ask if they can work with "Plinko" instead of having recess, and they often say math is fun. When I hear these comments, I realize just how helpful "Plinko" is in reaching children affectively and cognitively. It also helps students answer questions with more confidence and accuracy.

"Plinko" has strengthened not only my students' understanding of the foundations of math, but it has also given them something to smile about. A big, bright smile in a classroom really can make a difference. Don't you and your students deserve this refreshing change?

Hands Off the Textbook: Hands On the Manipulatives

Karen Ibbotson

Karen Ibbotson is an elementary teacher at Sherwood Community School in Calgary, Alberta.

Manipulatives are very important in Division II, but for some reason, they often magically disappear around the time students move from Division I to Division II. Teaching math with manipulatives in Division II is an exciting teaching opportunity. Though manipulatives may be time-consuming to prepare, the students are more attentive, eager to learn and achieve mastery of math concepts more quickly than when the traditional lecture/textbook approach is used. Manipulatives make math exciting and give the students opportunities to learn through discovery.

Materials

Materials come in many varieties. You may use ready-made base-10 blocks. These are durable and well proportioned. Making base-10 materials is also possible. Even when ready-made materials are used, it is often helpful to make materials with the students. The process of making materials enhances the learning process. Beansticks are popular and fun to make. They are the most durable of the student-made materials. You may also wish to use popsicle sticks and elastics, beads on pipe cleaners or beans in medicine cups. Other materials that might come in handy are plastic muffin trays; dice made from wooden cubes; "place mats" marked "ones," "tens" and "hundreds"; "place mats" marked work area and storage area; and blackboard models of the base-10 materials you are using.

Rules and Routines

It is important to establish routines when working with manipulatives because students often get excited and noisy. Have them use the same routine in preparing and cleaning up materials. Individual sets of base-10 materials make this process easier. When introducing a new manipulative, give the students an opportunity to explore for a few minutes. They will be better able to concentrate on the task if their natural desire to explore has been satisfied. Have students define a work area and a storage area to help them organize their materials.

Place Value

Have students make a large pile of "ones." Thirty-four should do. Walk around and question them about what they are making by asking them questions. Are you sure that's 34? Can you prove to me that your pile has 34? After a few similar activities, ask them if they could think of a way to organize their units to help them prove that they have the correct number of units. Students will quickly begin to group their units. Discuss the advantages of one type of grouping over another. They will discover that counting by "tens" is easiest.

At this point, students are ready to make base-10 materials. After they've been given the instructions, students should be able to work on their own. Walk around the classroom and ask questions. How many "tens" do you have? How much does that make all together? How many is this number? (Separate some "tens" and "ones.")

After the materials are prepared, the students will enjoy a game of "Race to 100." Students play the game in pairs. Between them, they have one die, 40 "ones" and 22 "tens." They take turns rolling the die and collecting the number of units indicated on the face of the die. When a student has enough "ones" to make 10, he or she must trade them in for a 10-stick. If a student neglects to trade, the other student may take 1 unit from his or her partner. The first student to reach 100 is the winner. For a more challenging game, have the students race backward from 100. Once the students have grasped the concept of "ones" and "tens," they are ready to do a similar activity with "hundreds."

Operations

The concepts of addition, subtraction and some parts of multiplication will be taught using a very similar method. Most of the concepts will be taught using the base-10 materials and the place mats. Other games and activities should accompany these teaching methods so that the concepts are reinforced.

Addition problems that do not require regrouping should be demonstrated first. Most students in Division II have the ability to do this operation, but many do not understand the actual concept behind it. The following activities will help your students gain an understanding of activities that they used to do by rote.

Have students place a number such as 23 on their "ones," "tens" and "hundreds" place mat. Place the 2 10-sticks in the "tens" area and the 3 ones in the "ones" area. Then have the students place another number, such as 36, on their place mat and follow the same procedure. To determine the answer to 23 + 36 students need only count the number of "ones" and "tens" on their place mat. As the students work, the teacher records what is happening on the blackboard. Later, students can record the numbers for themselves.

Another activity that students enjoy involves using teacher-made cards of different colors. Decide what color will represent "ones," "tens," "hundreds" and so on. For example, pink might represent "ones"; green might represent "tens"; and blue might represent "hundreds." Number cards of each color from 1 to 9. Keeping colors separate, shuffle each pile, and place it at the top of the appropriate section on the place mat. Have the students flip one card in each pile and then make the number indicated on the cards with base-10 materials. One student may write the numbers as the other student makes it. Students then flip another card in each pile and make that number. The student then adds the two numbers together.

Subtraction

Teaching subtraction is similar to teaching addition. Once the students need to regroup numbers they will see how important it is for them to begin by counting the "ones" first. They will realize that when there are not enough units to subtract they have to trade 1 10-stick in for 10 "ones." Using the term "trade" instead of "borrow" is preferable because students understand the notion of trading a dollar for 10 dimes. Trading means that when you give something you get something of equal value in return. Borrowing implies that it must be returned, and this does not happen in subtraction.

Multiplication

Multiplication may also be taught with the place mats. An alternative method is to use plastic muffin trays or something else that keeps units separate. Students begin by putting equal numbers of units in each cup. Then, they count the total number of units. This activity helps students understand the concept of "repeated addition" and reinforces basic mathematic facts.

For the multiplication algorithm, students place 14 units in each of 3 muffin cups. Ask students how they should begin. They will know that $4 \times 3 = 12$, and will therefore know how many "ones" there are. Students then exchange 10 "ones" for 1 10-stick and place it NEXT TO their muffin tray. Next, they multiply the 10-sticks, 1×3, and ADD the extra 10-stick. Again, the teacher records the students' work. This process works for any number no matter how large.

Division

The technique for division is very similar to the technique used for multiplication. Students make a number on their work area. Then they divide or "deal" these into the number of muffin cups that will be used. Begin with a number such as 55. Tell the students that they are to divide it by 4, and that they should begin with 10-sticks. Students will put 1 10-stick in each of 4 muffin cups. They will have 1 10-stick left over. Ask students what they should do with this 10-stick. They know that they should trade it in for 10 "ones." Then they can add this to the 5 "ones," and divide them among the 4 muffin cups.

Have students turn their notebooks sideways when working on division problems. The lines will help keep the problem lined up. Once students begin working with double and triple digit divisors, have them write the rounded off number in brackets underneath the actual divisor.

Conclusion

There are many other mathematical concepts that can be taught using manipulatives. For instance, decimals could be taught using the same activities and concepts. Students who have worked with manipulatives in learning place value and operations will have no trouble learning decimals.

Be brave and experiment. Students enjoy all hands-on activities. You will be surprised how successful you feel when students are smiling and happy during math class. Look at your students, and you will know that it is time for hands off the textbooks, hands on the manipulatives.

National Council of Teachers of Mathematics

Proclamation

Whereas, mathematical literacy is essential for citizens to function effectively in society; and,

Whereas, mathematics is used every day—both in the home and in the workplace; and,

Whereas, the language and processes of mathematics are basic to all other disciplines; and,

Whereas, our expanding technologically based society demands increased awareness and competence in mathematics; and,

Whereas, school curricula in mathematics provide the foundation for meeting the above needs;

Now, therefore, I, Shirley M. Frye, President of the National Council of Teachers of Mathematics, do hereby proclaim the month of April 1990 as

Mathematics Education Month

To be observed in schools and communities in recognizing the increased importance of mathematics in our lives.

In witness thereof, I have hereunto set my hand and caused the corporate seal of the National Council of Teachers of Mathematics to be affixed on this 1st day of September 1989.

Shirley M. Frye

President

Tessellation, Tiling or Surrounding a Point

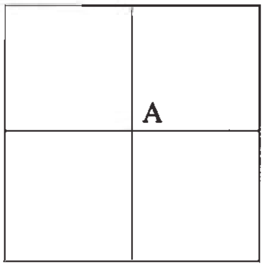
K. Allen Neufeld

K. Allen Neufeld teaches undergraduate and graduate courses in mathematics education and has been elementary practicum coordinator at the University of Alberta since 1982. He was president of MCATA from 1975 to 1977 and has edited monographs on metrication and calculators.

Tessellation (tiling) activities can be used effectively to present many of the 26 geometry concepts in the Alberta Education *Elementary Curriculum Guide*. Introductory activities should always emphasize the concrete mode, regardless of the grade level at which they are presented. Gradually a transition can be made to the pictorial and abstract modes. Although tiling can be done with a variety of two-dimensional geometric shapes, the activities that follow are based on seven regular polygonal regions whose perimeters are triangles, squares, pentagons, hexagons, octagons, decagons and dodecagons.

The illustrations in Figure 1 (see page 22) have a common edge length, 2.5 centimetres, and can be used as patterns to prepare black-line masters for duplication. It is recommended that a separate master be prepared for each of the seven shapes. Six dodecagons, 6 decagons, 12 octagons or 30 pentagons will fit on a regular sheet of paper. Because triangles, squares or hexagons can be drawn with common edges, a large number can be accommodated on a regular sheet of paper. When multiple copies are duplicated, it is recommended that heavy tag material with a different color for each kind of regular polygon be used. For demonstrations on an overhead projector, the masters can be used to prepare transparencies using a different color for each shape.

Figure 2



Tiling is usually considered a manipulative activity in which a surface is covered with two-dimensional geometric figures. It is also considered an activity in which a student surrounds a point. For example, 4 squares surround point A in Figure 2. Four square tiles can be manipulated to experience the concrete mode. The pictorial mode is used when students see that point A is surrounded in Figure 2, and the idea can be experienced abstractly by noting that the 360 degrees around point A are made up of four angles each containing 90 degrees.

Activity 1

1. Four squares surround a point. Take a number of triangles and see if you can surround a point. How many do you need? Can you keep on surrounding other points until you have covered a sheet of paper?
2. Try surrounding a point with some pentagons. Is it possible?
3. How many hexagons are needed to surround a point? Can you cover a sheet of paper with hexagons? How does the honeybee make use of hexagonal designs?

4. Can you surround a point using only octagons? Only decagons? Only dodecagons?

Activity 2

1. Show that 3 squares and 2 triangles surround a point. Can you cover a sheet of paper using just squares and triangles?
2. Can you surround a point using only hexagons and triangles? How many of each are needed?
3. Try using squares and octagons. How may of each are needed to surround a point?
4. Can you cover a sheet of paper using triangles and hexagons? Triangles and dodecagons?
5. You can surround a point using 2 pentagons and 1 decagon, but you cannot continue to use other copies of the same shape to cover a sheet of paper. Try it!
6. Try using decagons and triangles, decagons and squares, decagons and hexagons, decagons and octagons. Will any of these sets of shapes surround a point?

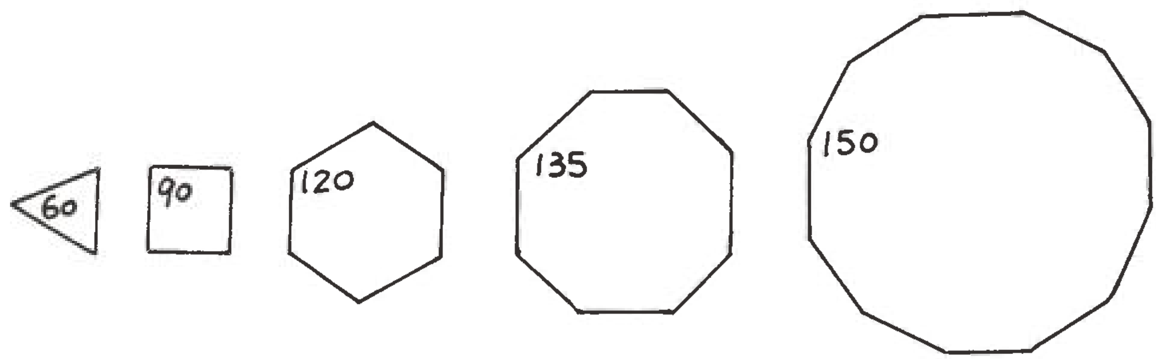
Activity 3

1. Show that a square, a hexagon and dodecagon surround a point. Can you cover a sheet of paper using many copies of these three shapes?
2. Try to cover a sheet of paper using only triangles, squares and dodecagons. How many of each do you need to surround a point?
3. Choose any three shapes and try to surround a point. Are there any other sets of three different kinds of shapes that will surround a point?

Activity 4

Figure 3 shows each of the five kinds of shapes. Use addition to make a list of the 11 ways in which a point can be surrounded. Three of them have been done for you.

Figure 3



1. 4 squares; 90 + 90 + 90 + 90 = 360
2. triangles; 60 + ____________ = 360
3. pentagons; ____________ = 360
4. 3 triangles and 2 squares; 60 + 60 + 60 + 90 + 90 = 360
5. ____________ and ____________ ; ____________ = 360
6. ____________ and ____________ ; ____________ = 360
7. ____________ and ____________ ; ____________ = 360
8. ____________ and ____________ ; ____________ = 360
9. 1 square and 1 hexagon and 1 dodecagon; 90 + 120 + 150 = 360
10. ____________ and ____________ and ____________ ; ____________ = 360
11. ____________ and ____________ and ____________ ; ____________ = 360

Activity 5

If you do not remember the number of degrees in each angle or a regular polygon (e.g., square), do the following:

1. Indicate a point which you assume to be the centre of the square,
2. Draw a line from this point to each of the vertices,
3. Calculate the number of degrees at each angle at the centre point (360 divided by 4 = 90),
4. Each angle of the square must be 90 (45 + 45).

1. How many degrees are in each angle at the centre of the triangle? 360 divided by 3 = ______. Therefore, how many degrees are in each vertex of the triangle?

2. How many degrees are in each angle at the centre of a pentagon? Therefore, what is the measure of each vertex of a pentagon?

3. Try this activity for a regular hexagon, octagon, decagon and dodecagon.

Activity 6

Complete the chart on page 23. Look for patterns in each of the four columns. Only 6 of the 10 regular polygons are illustrated. The ones not illustrated are named.

Activity 7

Use the procedures in each of the previous two activities to find the number of degrees in each vertex of a "centagon" (100 angles and edges).

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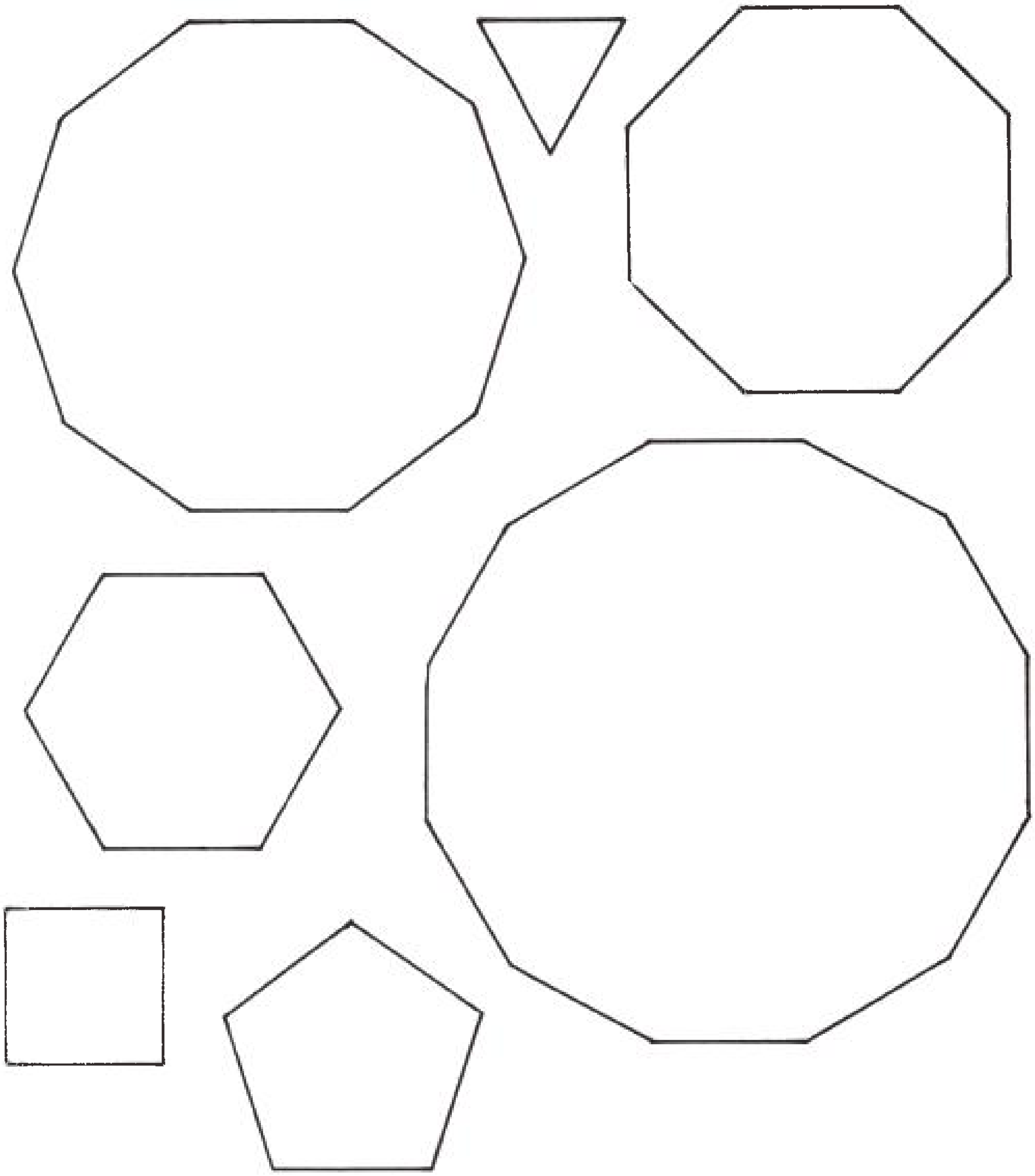
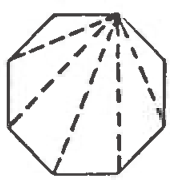
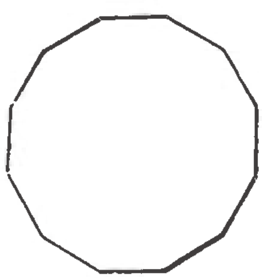
Figure 1



Activity 6

Regular polygon	Number of vertices	Number of triangles	Total number of degrees	Degrees in each vertex
	3	1	180	60
	4	2	360	90
	5			
		4	720	
septagon				
		6		
nonagon				
decagon				
"11-agon"				
				

Actions and Assumptions: Their Relationship to Student Thinking

Jeanette Parow-Jarman and Dave Whiteside

Jeanette Parow-Jarman recently completed her bachelor's degree in elementary mathematics education at the University of Calgary. Dave Whiteside is a Grade 3 teacher in Calgary, Alberta. He has also taught high school biology.

Teaching involves the development of thought, the broadening of horizons, the awakening of interests, the piquing of curiosity and the arousal of intelligence. At least that is what we say. Learning involves the same things. At least that is what we hope. When teachers walk into classrooms they carry with them certain assumptions. Some of these assumptions are such an integral part of their character that they are not conscious of them. People who have an awareness of the forces that motivate them are likely to be receptive to positive changes within themselves. We, as students of teaching and learning, work to create an increased awareness of these subconscious forces within ourselves and to enact such changes.

By studying the work we did with four problem solving groups of students in Grades 3, 4, 5 and 6, we believe we have uncovered some of the underlying assumptions that drive our thinking processes and our behavior. Our actions, as captured in our speech patterns, and the results of our actions, as seen in some of the student records we obtained are examined here. We question some of our behaviors in the problem solving sessions and endeavor to judge their possible effects on the thinking processes of the students involved in the activity.

First, it might be worthwhile to discuss the nature of the problem chosen for this exercise (see Appendix for a description of the problem and materials). This study uncovers some of the assumptions that we hold regarding problem solving as an activity for students. We agreed that the activity chosen must engage students; that is, we wanted them to be intrigued and engrossed by it. The cooperation and interest of students is not gained automatically, and as such we consciously chose an activity that was fun and challenging. The problem was designed to offer some quick, easy solutions and some more difficult ones. This was based on the belief that problem solving does not work well in an atmosphere of frustration. Students had to believe that they had the capacity to solve the puzzle, so the activity was structured to give them some immediate and fairly certain success. The variety of strategies and solutions that a problem accepts is also important to children's perception of their ability to succeed. A true problem can be resolved in more than one way, and many different strategies may be involved in attaining any of the solutions. Because problems of this nature do not have "right" answers, children need not feel disabled in their problem solving if their patterns of thinking do not match that of other students. Finally, the problem also had to lend itself to some sort of a manipulative procedure. Solutions are easier to develop when the variables can be seen and handled. Additionally, the manipulative nature of the task was certainly important in its ability to engage the students.

Even though the topic for our math problem was open-ended and manipulative, it is arguable that the topic for discussion in any given class needs to be delineated; the boundaries of thought should be delimited. Alan Tom in his essay on the moral craft of teaching (1985, 149), states that "teaching [is] . . . the application of knowledge and skill to attain some practical end." He elaborates on this point by saying, "teachers . . . are inevitably involved in forming students in desirable ways." Thus we use our knowledge and our skill to help mold the thoughts of our charges. However, we need to fear the strength of that mold, the fidelity of the copy that we may create. Tom's argument is that each teacher has the moral responsibility to decide the desirable end for his students and the craft to enact those ends. We believe that while we as teachers carried out our moral responsibility, the nature of some of the assumptions we made during the problem solving activity interfered with the attainment of our objectives. Several salient points need to be addressed in this question: How restrictive do we need to be in order to perform our task effectively? What are the results, in terms of thinking, of these restrictions?

Pimm (1987, 32) describes a dilemma for the teacher, in terms of classroom communication:

> Teacher presence can interfere with developing pupil talk by overcontrolling it. The teacher may be too concerned with the form of what is being said, at the expense of the meaning which the pupil is trying to convey. On the other hand, if pupils are to become aware of the characteristics of disembodied speech, then considerable work needs to be done to encourage them to modify and expand their initial attempts. How to contend with this tension may be one of the central dilemmas of communication facing teachers.

This dilemma is a real one and extends well beyond the nature of the communication within our classrooms and into the actions that we use in our teaching. We want our students to develop independent thought, unimpeded by the restraints that we might place on it. We value original ideas and insights. Nonetheless, the discipline that can be applied to thought is sometimes as important as its freedom. The need exists for thought to take a form that meshes with the domain of thought extant in the classroom. The teacher's task is to strike a delicate balance between disciplined thinking and a free flow of ideas.

The necessity to create a framework of thought within a classroom and in an activity is apparent. If the teacher is to fulfill the role of "forming students in desirable ways" then there must be some kind of structure apparent within classroom activities. The absence of such direction would result in chaos and anarchy. These situations are rarely conducive to creating a considered outcome.

The following conversation with a Grade 4 class, which indicates the nature of delimitations that we placed on children's thinking, illustrates the necessity of structure. Our action was not capricious; the planned activity demanded measurement and excluded estimation.

Teacher: We can measure 3, and we can measure 7, what other numbers. . . . We were just going to try to figure out what other numbers we can measure with these jars. Anyone got any ideas of what we can figure out with these jars?

Students: We could get 8 . . . 2.

T: Do you have any plans for how we could do that?

S: Well for this, for the 3 maybe we could put this much down.

T: Oh, I see. Now would that be measuring or estimating?

S: Estimating.

Later, at the end of the instructions

T: Remember there's no estimating in this game. You can only. . . .

S: Measure.

While estimation is a useful skill and one that is used often in mathematics, it was not a skill that was appropriate to the solution of the problem posed to the children. It is notable that all groups embraced it as a first solution and thus it was necessary, in all groups, to delineate the problem. This is not an unrealistic practice. Many problems have easier, but perhaps less accurate or less legitimate solutions.

But what happens to the nature of the thinking that a student pursues when this kind of delineation has taken place? It is not unrealistic to believe that an insistence on the rules might lead to a less adventurous, less individual approach to problem solving than one unimpeded by such conventions. Although the following situation involving a Grade 4 class illustrates confusion, and not necessarily a sense of restriction, it is reasonable to infer that the perplexity of the student stems from a fear of transgressing the rules.

Student 1: How about 10–7?

Student 2: No, but it has to be a subtract. I mean a plus? Does it have to be a plus?

Student 1: No, it doesn't (said simultaneously with Teacher 1)

Teacher 1: It can be either. You can use adding and subtracting, but you're not allowed to pour into that one.

S 1: Ha ha, ha ha ha!

T 2: You can only use the 3 and the 7 jars.

S 2: What is that one?

T 2: That one's just a jar to store in.

The process of delineating the parameters of the problem, although perhaps a necessary one, had quite effectively "frozen" the student's thinking processes. He was unwilling to assert whether he should add or subtract, and was unable to proceed toward a solution.

In another situation, a student who was less affected by our procedural instructions developed a unique resolution to a problematic process within the activity, one that destroyed some of our assumptions. Zoe, a Grade 6 student, used the smallest vial, which we had used to fill the jars, to empty the first centimetre or so of water from her storage jar prior to pouring the rest of its contents into the measuring jar. In doing so, she avoided spilling the water and was able to achieve more accurate measurements. What is interesting here is that we had not considered using the vial this way and had described it in a prescriptive sense. We had indicated that these vials were to be used to accurately fill the 3 and 7 jars. Had we questioned our assumptions more carefully and simply indicated that the vials were not to be used for measurement, we would have invited the students to find uses for them. Perhaps others would have discovered Zoe's method also.

The two examples quoted above show an interesting counterpoint in terms of teacher instruction and of students' reaction to that instruction. In teaching, some delimitation is necessary in order to discipline thought and to foster desired outcomes. However, teachers need to examine their thinking and to be sensitive to how they may be limiting children's thinking. We needed to eliminate student estimations; we didn't need to delineate the uses of all of the other tools available to them. We could have given up some control and still attained our goals, but we were too preoccupied with the form of the problem solving and not concerned enough with its meaning.

One assumption that we had made prior to starting the problem solving exercise was that we as teachers were to be unobtrusive within the process. The situation was analogous to one described by Pimm (1987, 37) in which a teacher used a poster picture of a great stellated dodecahedron to initiate student talk and to "remove herself from centre stage." The teacher in Pimm's account was not as successful in her initial attempt as she would have liked. Her tendency to pose direct questions weakened her attempt to be unobtrusive. We were fortunate in that we had four attempts at the same lesson. Although we remained prescriptive in some aspects even in the last lesson, we withdrew from directing the process. Thus, the Grade 5 lesson on the first day started off as follows:

T: What we want you to do is figure out how to get every volume from 1 to 10 using these two jars. Now how many volumes does this jar hold?

S: Three.

T: Three. Can you figure how to get 3 volumes?

S: Fill it up.

S: Fill it up to the black line.

T: Fill it up! How many does this one hold?

S: Seven.

T: So how do you get 7?

S: Fill it up to the black line.

T: The question is, How do you get 1 and 2 and 3 and 4 and 5 and 6 and 7 and 8 and 9 and 10? That's your job. Some are pretty easy. Some are real brain teasers. Which are easy?

S: Three, 7 and 10.

Whereas, at the beginning of the last lesson, the Grade 6 lesson we hear the following conversation:

T: Not using anything else, just using water and jars, any ideas on what we might be doing?

S: Measuring.

T: You're right. You're right. We're measuring.

S: We're getting the volume of the water.

T: There are numbers on these. This one's a 3. The little one's a 3, and the big one's a 7.

S: That means that there are 7,700 ml.

T: Well, actually it doesn't mean that. It just means that this one contains 7 units of water if it's filled to the black line, and this one contains 3 units of water if it's filled up to the black line.

S: So if that's filled up to the black line, it'll go to 3 on here?

T: Right.

S: So what we're going to do is put like, okay, so if you put. . . .

S: Seven plus 3 equals 10.

S: Seven and 3, it'll get up to 10. It should equal 10.

T: Okay. 7 + 3 = 10. Okay and that's it—so you have to get all the numbers from 1 to 10. Think you can do that?

There is a notable difference in the approach taken in the two classes. Although the students solve the problem of what to do in both classes, the amount of direction involved in the second class has decreased, and the number of declarative statements has increased. Of course there are other variables to consider, such as the ability levels of the two groups, but even the initial statements demonstrate the teacher's attempt to move away from "centre stage."

Some of the intrusive suggestions that were used in earlier sessions became less abundant in later sessions. In early sessions, students were given models of what to write. In later sessions, we gave only descriptors, such as "write a recipe for each number," or "write an explanation as though a Grade 3 student was going to read it." We reflected upon our teaching methods and tried to make the directions a little less stringent. We felt that this was a positive step in growth for us.

Our ability to withdraw from the process and to allow the students more freedom was short-lived in the Grade 6 class. The group worked in a very task-oriented and efficient manner. They solved the problem within 20 minutes; it had not been previously completed in less than an hour! What happened as a result was very interesting. We did not trust the students to be able to develop a problem to challenge themselves, nor did the classroom teacher or our professor. Even though, together we developed some very creative ideas in terms of extending the problem, not one of us proposed that the students be responsible for creating an idea themselves. Thus, on the advice of the professor, they started working to measure numbers greater than 10 and looking for patterns in the solutions. On the advice of the classroom teacher, they tried to look at the elegance, or simplicity of their solutions ("pretend that every pour costs $10"). We tried to discover why we lacked faith in the students. We felt that part of the problem stemmed from the fact that we were dealing with children whom we did not know extremely well. We also felt that because we had been invited into the classroom, we had a certain responsibility to maintain control. In areas where we felt in control we were willing to subdue our role in the problem solving situation. Where we were less certain of our ultimate control, we became concerned and restricted the nature of the activity.

The attempt to strike a balance between independent thinking and a productive classroom situation became a dilemma in these problem solving groups. At times, the dilemma was exacerbated by the nature of our talk and the conception that we had of our role. We often worked to solve small problems to free students up for the main problem, and in doing so channeled student thinking processes and perhaps affected the nature of the solutions. Certainly the solutions to auxiliary problems and possibly those in the principal problem were less varied than they may have otherwise been. On the other hand, there was some improvement within the area where we were comfortable with our role.

An examination of the data alerted us to some of the underlying assumptions that we had made about writing mathematical ideas in the classroom. Although we have a good grounding in the theory of writing and expression in mathematics, and we hold the belief that mathematics should be presented as a subject where reflection is important, the writing tasks that we asked of the children were product and not process oriented. Terms such as "recipe" do not evoke an image of reflection and did not solicit reflection from the students.

To a large extent, we focused on the manipulation aspect of the problem rather than on recording the problem solving process, even though we value it. Thus the question of recording became an issue in all of the problem solving groups. The students were happy to work at solving the problem, but did not seem to embrace the written aspect of the activity. Certainly their written accounts do not provide a very insightful record of their thinking or of their solutions. Most of the records follow a pattern and indicate a minimum of effort.

One volume measurement that required some application and ingenuity was the measurement of two units of water. The following accounts of this difficult procedure indicate the amount of description given by each group in their written records:

Grade 3: (Group 1) I took 7 jars and took [out] 4 3-jars.
(Group 2) Fill 7 jar (twice). Take away 3 and 3 and 3. We got 2.

Grade 4: (Group 1) 9–7 = 2. Pour 3 3s into 10, get 9, subtract 7.
(Group 2) We made 2 by taking 6 away from 7, which made 1. We poured the 1 into the storage jar, then I did the same thing over again and poured that into the storage jar and made 2.
(Group 3) We made 14 and subtracted 4 3s.

Grade 5: (Group 1) Like 1 to do again I will have 2.
(Group 2) I took a 7 then I took away 2 3s and store the 1. Then did the same method again.
(Group 3) 1 + 1

Grade 6: (Group 1) We did #1 again.
(Group 2) We filled the #3 jar 3 times. Then took 1 #7 jar from the 9.

We feel that these records are brief and do not reflect the difficulty of the task. Why were the students so reluctant to write? An important assumption shows up here. What we requested of the students, in terms of writing, would not in any real sense serve as an indication of their thinking processes. What we asked for was a "recipe," a "description of the procedure for a younger child," a simplistic model of the successes they experienced as they worked through the problem. We neither asked for, nor enabled the students to give us a reflective account of their thinking processes during the problem solving situation; we devalued misguided efforts by not requesting a description or account of them.

Pimm (1987, 47) offers a couple of ideas that may explain the brevity of the writing done during the activity. It could be linked to what Pimm refers to as the "status quo," or pupils' views of what mathematics is. It might also relate to the nature of communication the pupils perceived as taking place. With regard to the former, Pimm states

> The status quo can be hard to alter, even by a teacher who has decided to try to introduce a more discursive atmosphere. Pupils' views and expectations of what should go on in mathematics lessons are often quite rigid.

The students with whom we were dealing were probably not used to writing during mathematics lessons. This would partially explain their reticence to engage in the task of writing in the first place. Given that their view of mathematics may preclude any written explanations, it is unrealistic to expect students' descriptions to contain precise detail. The students' writing may also have been affected by their reasons for writing. Pimm (1987, 38) suggests that there are two types of speech: message-oriented and listener-oriented.

> In message-oriented speech, the speaker is goal directed and wishes to express a particular message, to "change the listener's state of knowledge"—it matters that the listener understands correctly. With listener-oriented speech the primary aim is the "establishment and maintenance of good social relations with the listener."

Although Pimm refers to speech, a parallel may be drawn with writing. Students may have been attempting to maintain good relations with the "teachers." We had made a specific request that they describe their solutions, something that they likely viewed as extraneous to mathematics, and rather than educating us as to the real nature of mathematics (get the answer), they simply cooperated and wrote something, thus avoiding any "unpleasantness or hurt feelings."

Other explanations are also possible. Time is almost always viewed as an evil in North American schools. Despite the fact that there was no real need to hurry in any of the classes, the unspoken message from us, as well as the expectations bound in with the idea of "status quo" lead to a feeling of a need to hurry. Stigler (1988, 27) proposes the variable of pace as a fundamental distinction that exists between American and Japanese classrooms. The author posits a relationship between the idea of pace and the amount of discussion and talk that occurs in the classrooms of these two nations. Japanese teachers consistently devote more time to talk and deal with fewer problems in greater depth. As members of our goal-oriented society, the need to complete the job was likely a strong motivating factor for the students and for us. The students thus would be induced to spend little time recording or thinking about their solutions and more time "doing" them. This explanation in no way precludes the previous ones; in fact, it intertwines with the ideas of types of communication, "status quo" and with our assumptions about writing.

We learned a lot about ourselves, about our thoughts as teachers and about some of the assumptions we make when we deal with children. We realized that even after the considerable thought that we gave to choosing a problem conducive to creative thinking, we "directed" the children's thinking. We would like to think that all the theories of learning that we have been studying for the past year are an integral part of our being now and that we can invoke them spontaneously. We were dismayed, despite a perceptible improvement over the course of the four lessons, to realize that we could still be so directive in the classroom. The reflection involved in the creation of this paper helped us to realize some of the assumptions that we had made. Once the underlying assumptions that drive our teaching are uncovered, we will be able to release them and to adopt a new understanding of the teaching process.

Appendix

Procedure

Students were given three jars and a pail of water. Two of the jars were used for measuring tasks; the third served a storage and verification function only: water could be poured into it, but none could be removed except to start over. The volume was marked with black lines. Volumes were measured by placing the jar on the table at eye level. This procedure maximized the accuracy and reinforced correct measuring techniques. The two measuring jars contained 400 ml and 170 ml, respectively. This approximates a 7/7 : 3/7 relationship. One unit = 57 ml. 400/7 = 57.143. 170/3 = 56.666. Using the jars, students were asked to measure out all possible volumes of water between 1 unit and 10 units.

Units	Possible Solutions
1	7-3-3
2	2.33 small poured into large–remainder in small = 2
3	3
4	7–3, 3 + 1, 2 + 2
5	3 + 2, 3 + 1 + 1
6	3 + 3
7	7
8	3 + 3 + 2, 7 + 1
9	3 x 3
10	7 + 3

The lesson consisted of a presentation of the problem, an explanation and a demonstration of the two measuring jars. The teachers remained seated at the table during the explanation and during the outset of the problem solving when the students were most likely to feel intimidated and incapable. Subsequent to the initial presentation, the teachers restricted their activities to monitoring and observing. They assisted "stuck" groups or individuals and assessed results. When students obtained a result they were asked to show their resultant volume and to describe their solution.

Students were allowed to decide who they wished to work with. They were limited only by the amount of equipment available (four sets of jars).

We worked on this project as coparticipants. In other words, we team taught sessions and collected two separate sets of field notes. A transcript was compiled from audiotaped sessions.

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Estimating with Decimals

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The decimal number system provides mathematical models for a large number of the practical problems that students are likely to encounter. Yet research studies show that students perform poorly on tasks that require decimal computation, and that most often the students apply a memorized computational rule in a meaningless way (Bell, Swan and Taylor 1981; Hiebert and Wearne 1984).

The following question was asked of Math 15 students on a recent test concerning decimal knowledge.

Estimate the answer, then place the decimal point in the given answer.

1. 2.42 x 3.610 = 08736200
2. 3.20 ÷ .08 = 0040000
3. .42 x .23 = 00096600
4. 30 ÷ .6 = 00500000
5. 4.5 x 51.62 = 023229000

The test was administered to 23 students. Only a very small number of students gave the correct answer.

Table 1 includes the question, the number of students who responded with the correct answer and the number of students who gave the most common response. We can see by Table 1 that the majority of students did not give the correct answer. If we examine the most common response, it seems to suggest that a certain procedure was used, a procedure used when multiplying two decimals. The students seem to have counted from the right the number of places equal to the sum of the number of digits to the right of the decimal point in the numbers of the problem and inserted the decimal point there. Isn't this the "little trick" we tell our students when we teach multiplication of decimals?

Owens and Haggerty (1987) observed the processes of children as they form concepts and attach meaning to multiplication of decimals. Children are often taught to count the places after the decimal point in order to place the decimal in the product. Such algorithmic strategies are often used without understanding, and their use can lead to difficulties. In Table 1 we see that the procedure of counting places was used with both the multiplication and division problems. The majority of students did not discriminate between multiplication and division.

In order to better understand the procedures and processes used by the students, a sample of the students who wrote the test was interviewed. The interview questions dealt with problems that were on

Table 1. Number of students with correct response and most common response(N = 23).

Question	Correct Response	N	Most Common Response	N
1. 2.42 x 3.610 = 08736200	8.7362	2	87.362	15
2. 3.2 ÷ .08 = 0040000	40.0	4	4.0	10
3. .42 x .23 = 00096600	0.0966	1	9.66	15
4. 30 ÷ .6 = 00500000	50.0	2	50,000.0	12
5. 4.5 x 51.2 = 02322900	232.29	2	23,229.0	14

the test. The students were given a clean copy of the question and asked to place the decimal point in the answer. After the student placed the decimal point where he or she thought appropriate, the interviewer asked the student, "How do you know that it goes there?" The students who gave the most common response answered that they had "counted places." The answer for the division problem was the same; they counted places. These students were very concerned about the procedure used.

To get a clearer picture of why procedures are so important to students, we should look at some of the research done in the field of decimal number knowledge. Research on the instruction of decimal numbers is fairly recent. The work of Hiebert and Wearne has been the most comprehensive attempt made to delineate the cognitive aspects of decimal number knowledge. Hiebert and Wearne (1986, 199) argue that "mathematical competence is characterized by connections between conceptual and procedural knowledge . . . that mathematical incompetence often is due to an absence of connections between conceptual and procedural knowledge."

What is conceptual and procedural knowledge? Conceptual knowledge is knowledge of those facts and properties of mathematics that are recognized as being related in some way. When a fact or property becomes part of a larger network through the recognition or construction of a relationship between the fact and a network that is already in place, then we say that that fact becomes part of conceptual knowledge.

Procedural knowledge is limited to knowledge of how written mathematical symbols behave according to syntactic rules. Procedural knowledge of symbols does not include knowing what the symbol "means," that is knowing that the symbol represents an external referent. Procedural knowledge also includes the set of rules or algorithms that are used to manipulate the symbols and solve mathematical problems. For example, counting places and not knowing why.

These Math 15 students used their procedural knowledge. They manipulated the symbols according to a rule they knew in order to solve the problem. However, in this case, applying a known procedure did not produce the correct response. What went wrong?

Hiebert and Wearne (1984) indicate that there are three levels, points, or "sites" in the process of computing with decimal numbers that demarcate the primary sources of students' difficulty. At Site 1 many students do not know what the symbols mean. They fail to connect decimal symbols with meaningful referents. At Site 2 many students do not know why the computation procedure works. Based on individual interviews and analysis of written errors (Hiebert 1985), most students' computation activity consists of recalling and applying memorized rules for which they connect absolutely no rationale. At Site 3, many students are not aware that answers should be reasonable. To be able to check whether an answer to a decimal computation problem is reasonable, one must connect at least an intuitive idea of the arithmetic operation with appropriate meanings for symbols. Hiebert and Wearne (1987) interpret the difficulties that students exhibit at each of the three sites in the computation process as a consequence of a divorce between procedural and conceptual knowledge.

It appears that the majority of students who wrote this test failed to connect the decimal symbols with meaningful referents (Site 1), recalled and applied a memorized rule for which they seem to have no rationale (Site 2), and were unaware that the answers given were unreasonable (Site 3). The interviews also showed that the students' only concern was the procedure. After placing the decimal point, the students did not check to see whether the answer was reasonable or not.

A short excerpt from one of the interviews follows.

Interviewer: Are you sure the decimal point goes there?

Student: Yes.

I: How do you know that it goes there?

S: All you have to do when you multiply decimals is count places.

I: Is the answer correct?

S: It has to be if you follow the rule.

Upon further questioning and working with the rounding off of numbers the student was able to see that her answer was incorrect. She then realized that "extra" zeros had been added to the "real" answer. So why was the student not able to estimate the correct answer the first time?

Kieren (1987), in his reflections on fraction number research, comments that several of the studies reflected that traditional instruction makes an early and probably unwarranted emphasis on symbolic manipulation and computation with common or decimal fractions. If this is true, then children are probably forced to treat these symbols as concrete objects and hence build knowledge based inappropriately on patterns in the symbols (e.g., count the number of decimal places). What seems clear is that a fraction curriculum in which symbols are not tied to meaningful object actions has inhibiting effects.

Lichtenberg and Lichtenberg (1982, 143) report "the typical approach to decimals does not allow enough time for developing meaning, whereas inordinate amounts of time are devoted to the computational procedures." The emphasis here is on teaching computational skills and how to manipulate the symbols to arrive at a correct answer. As students move through school they memorize an abundance of task-specific rules for manipulating symbols (Hiebert 1984). The problem is that few links are constructed between the understandings they have and the symbols and rules they are taught. Many students have not acquired adequate meanings for the symbols they use; they do not understand the procedures they apply to manipulate the symbols, and they fail to test the reasonableness of the outcomes. Hiebert sees the critical instructional problem not as one of teaching additional information, but rather as one of helping students see connections between pieces of information that they already possess.

Questions about how students learn mathematics, and how they should be taught, turn on speculations about which type of knowledge (conceptual or procedural) is more important or what might be an appropriate balance between them.

Many learning problems in mathematics can be attributed to the absence of connections between the memorized, mechanically applied rules and conceptual understandings (Hiebert and Wearne 1984). How can these connections or links be attained? The critical instructional problem may be one of helping students connect pieces of information that they already possess.

Post, Behr and Lesh (1982) feel that students' difficulty in learning decimal knowledge is due in part to the fact that school programs tend to emphasize procedural skills and computational aspects rather than the development of important foundational understandings.

Readings related to conceptual and procedural knowledge show that the two are often two distinct sets of knowledge and that procedural understanding may be easier for teachers to teach and easier for students to understand than conceptual knowledge. As a consequence, decimal number knowledge may be taught mostly from a procedural point of view. Therefore, there is need to examine what procedures the students know, how they are using them and whether it is possible to arrive at conceptual understanding.

Suggestions for Teachers

Reys (1986) has found that computational estimation skills can be taught and do improve with instruction. Computational estimation refers to obtaining a reasonable approximation rather than an exact answer to a problem without having to depend on pencil and paper algorithms or calculators. Due to the increasingly technological world in which our students live, it would be wise to teach them estimation strategies.

In order to teach estimation strategies, we must establish what a reasonable approximation is. A student with good estimation skills should be able to decide whether his answer seems reasonable. Questions such as the following can be answered without computing an exact answer.

1. What is the length of this room?
2. How much pizza and pop would we have to order for lunch for this group?
3. If milk is ¢.89 a litre and bread is $1.09, can I buy both with $2.00?

Each of the above problems can be solved by the use of different estimation techniques that cater to the particular numbers and operations of each problem. Thus, different estimation problems will lead to the students using a variety of estimation strategies.

Usually when we speak of estimation we include the strategy of rounding. However, this connection is not always clear to the student. Therefore, rounding exercises should be done in conjunction with estimating exercises.

The strategy of rounding can be used in association with

1. estimating the sum of numbers
   27.546–0.3926 is about ________________
2. finding the approximate product

   .5091 x 380

   19
   190
   1,900
   19,000

3. choosing a reasonable quotient
   28.76 ÷ .4 equals about
   a) .07 b) 7 c) 70 d) 700

4. working with large numbers
   6,000,000 x 2.114 is about ____________ .

Seymour (1981) has published two books on developing estimation skills that contain worksheets for duplication. The activities in these books deal with reasoning, computation, measurement, pricing, counting and estimation techniques, worldly knowledge and problem solving. The activities were designed to help students in Grades 6 and 7 (Book A) and Grades 8 and 9 (Book B) develop their estimating abilities and learn to use approximate numbers.

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Introducing Estimation

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Current elementary mathematics textbooks, such as *MathQuest* and *Journeys in Math,* have recognized the importance of estimation skills. Not only have the number of estimation activities increased in this series of textbooks, but estimation has been integrated throughout as well. However, many students exposed to these textual series still feel uncomfortable or seem reluctant to estimate.

This reluctance may be due to the fact that since Grade 1 they have been conditioned to believe that mathematics produces a single exact answer, or they may have been taught estimation skills without being taught the reasons for using estimation. Not only is it important to establish that estimation is a highly useful tool, it is also important to create a climate conducive to developing an estimation "mind-set."

Before beginning to teach specific estimation strategies, students should be provided with a rationale for estimation. The following suggestions or sequences of brief activities have been used successfully to introduce estimation to students in Grades 4 to 7. They are intended to provide a rationale as well as help establish an estimation mind-set and should be included in initial work with estimation in the intermediate grades.

Define Estimation

Begin by asking each student to write a definition for "an estimate." The majority of students will define an estimate as "a guess." A few students will record more specific answers, such as a good guess, a smart guess, an educated guess or an approximation. Have students give examples to illustrate what they mean by a "good guess" as opposed to "a guess."

Tell students they are not competing against other students for correct answers when they estimate. Tell them they are competing against themselves to become better estimators. Only through practising estimation skills will they become more accurate with their estimates.

Brainstorm for Possible Uses of Estimation

Have students record as many situations as possible to illustrate where they or their family members have used estimates. After sharing their information with the class, ask students to interview parents and family members for the next class in order to determine additional uses of estimation. The information that students acquire from the family survey usually creates a lively class discussion and an awareness of the frequency and diversity of estimation in our daily lives.

Examples of Estimation Recorded by Students

1. How much time is left? (Asked during recess.)
2. How long will it take to get home after school?
3. How long will my homework take?
4. How much tissue paper do I need for the art project?
5. How much money can I save before Christmas?
6. Will I have enough money to buy three books at the book fair?
7. Will I have enough money for Hot Dog Day?

Examples of Estimation from Family Interviews

1. Adding ingredients when cooking
2. Determining how many kilometres the car will run for each litre of gas
3. Buying material for sewing
4. Estimating the time it takes to drive somewhere
5. Tipping in a restaurant
6. Estimating the amount of money needed at a grocery store
7. Buying fertilizer for the lawn

Identify Situations Where an Exact Number is Required

Questions such as the following establish that there are times when an exact answer is essential, and an estimate would make little sense.

1. What is your address?
2. How old are you?
3. What is your phone number?
4. What time is it?
5. How many sisters and brothers do you have?
6. What year were you born?
7. How many library books are overdue?
8. How much will dinner cost?

Have students identify whether an exact answer or an estimate is required for each question, and ask them to justify their answer. Students may ask more questions about when an exact answer is required.

Identify Situations Where Estimates Are More Appropriate

In the situations that follow, estimates are not only acceptable, exact answers would be unrealistic or inappropriate.

1. What is the population of Canada?
2. What is the population of the world?
3. At what altitude are we flying?
4. How many hamburgers have McDonald's restaurants sold worldwide?

Ask students to explain why an estimate is more appropriate than an exact answer for each of the questions. Students may ask additional questions about when an exact answer is unrealistic.

Introduce Newspaper Headlines

Have students collect newspaper headlines that illustrate estimates as well as headlines that illustrate exact answers. Students find this activity highly motivational, and it usually evokes lively class discussions. Creating class posters from the headlines that represent estimates and exact answers provides a good visual reminder.

Examples of Newspaper Headlines

1. Civic pride fined $50
2. 57 Dead in Plane Crash
3. 25,000 Homeless After Quake
4. 68 Carted Off
5. $65 Welfare Cut Restored
6. 30% Off All Cameras
7. Jackpot Lotto 649 $4.4 Million
8. 59,000 Watch Grey Cup
9. Unemployment Rate at 10%
10. $143,000 for Script at Auction

Emphasize the Language of Estimation

Students should become familiar with the language of estimation. The following are examples of common phrases that refer to estimation:

1. About 35 and a half
2. In between 7 and 8, but closer to 7
3. Just about 80
4. Approximately 500 km
5. Close to size 10
6. A little more than 16, a little less than 35
7. Around $100

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146 Fourth Street SW — Bus. 548-7516
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Publicity Director
Bill Davidoff — Res. 627-4283
P.O. Box 574 — Bus. 627-4414
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Professional Development Director
Jim Neilsen — Res. 238-3919
264 Cedardale Place SW — Bus. 276-5521
Calgary T2W 5J2

Directors at Large
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6 Greenhill Street — Bus. 475-1737
St. Albert T8N 2B4

Wes Williamson — Res. 347-2332
106 Glendale Boulevard — Bus. 346-4755
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