Conceptual Understanding: Promoting Talk in the Mathematics Classroom

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As a visitor from British Columbia, allow me to share with you a few of the changes in our province, recent and contemplated, that have had or may have an impact on teaching and learning mathematics. In 1987, a new *Mathematics Curriculum Guide* was published. The guidelines of this document suggested that conceptual understanding and problem solving should be the focus of mathematics teaching and learning. Recent draft documents published by the Ministry of Education, based on recommendations made by the Sullivan Commission, suggest that at some levels, mathematics could be taught as part of an integrated curriculum via themes or interesting projects. At present, the Ministry of Education has solicited papers on the topic of making thinking the major focus of teaching. A recent publication by the National Council of Teachers of Mathematics, entitled *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) suggests a framework for teaching mathematics and articulates five general goals. According to this document, all students should

1. learn to value mathematics,
2. become confident in their ability to do mathematics,
3. become mathematical problem solvers,
4. learn to communicate mathematically,
5. learn to reason mathematically.

These events and these publications indicate that changes in mathematics are in progress. The question that might be posed is, should all of these changes take place?

The reasons for saying yes to most of them are manifold. Let's look at a few possible reasons. Our views and our knowledge about how students learn mathematics and how they transfer knowledge have changed since our parents trained to become teachers. We all come from at least a two-calculator home. Mathematical expectations for new employees in industry have changed and are changing. A summary of these expectations by Pollak is listed in the *Curriculum Standards and Evaluation for School Mathematics,* and this list includes

1. the ability to set up problems,
2. knowledge of a variety of techniques,
3. understanding underlying mathematical features,
4. the ability to work with others,
5. the ability to see the applicability of mathematical ideas
6. preparation for open problem situations
7. belief in the utility and value of mathematics.

The key to attaining the expectations and the five general goals, especially the goal of becoming a problem solver, is the ability to understand the mathematics that is learned or the possession of conceptual understanding (Greenwood and Anderson 1983) or relational as opposed to instrumental, understanding (Skemp 1987). What types of classroom settings and what types of activities are conducive to the acquisition of conceptual understanding?

Greenwood and Anderson (1983), as part of their discussion of the operational and the conceptual domain, present a workable structure that suggests some

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very effective instructional approaches. They suggest an environment that "promotes, indeed provokes, communication by the student." They also suggest that teaching techniques "require a demonstration of conceptual understanding before expecting computational proficiency."

Post (1988) proposes that "the stereotypical model of students working alone at their desks needs to be expanded to include group discussions, project work and students working cooperatively rather than competitively. Students must talk about mathematical ideas and concepts."

Cobb (1985) agrees with Post and states that

> the most needed curricular innovation is . . . to encourage children to talk with each other and with the teacher about mathematics—their mathematics. Such interactions might sustain the belief that it is acceptable to think about mathematics and that mathematics involves understanding and the gaining of insights rather than finding ways to give the impression that one is behaving "appropriately."

An emphasis on student talk implies that teachers will have to be good listeners. Easley and Zwoyer coined the phrase "teaching by listening," and they conclude that

> If you can both listen to children and accept their answers, not as things to be judged right or wrong but as pieces of information which may reveal what the child is thinking, you will have taken a giant step toward becoming a master teacher rather than merely a disseminator of information. (1975)

Being a good listener means resisting what Kilpatrick calls "teacher lust" (de Groot 1988), or resisting the urge to control what students are doing and thinking. Kilpatrick states that mathematics teachers seem especially likely to be afflicted with teacher lust; after having asked a student to explain something, they often jump in, with scarcely a pause, to provide a clearer explanation themselves.

Are we as parents tempted to succumb to the same affliction as the mathematics teachers described by Kilpatrick? At times, are we tempted to share our wisdom with our children quickly, rather than to sit back and provide the opportunity to teach by listening and questioning?

After arriving home, a colleague was faced by his five-year-old son who, after looking at his digital watch announced, "Dad, it's 4:41. Twenty-one minutes to Scoobie Doo." After a lengthly explanation explanation by the father, which involved pointing out that from 41 to 50 is nine minutes and from 50 to 60 is ten minutes, so you'll have to wait nineteen minutes, the boy responded with, "But Dad, there are two minutes of news first!"

Results from various international studies have shown Japanese superiority in mathematics. I don't think we should ever contemplate copying anther system, but if some of the major reasons for this outcome are known, then perhaps a few of the effective teaching strategies could be kept in mind when we teach mathematics to our students. After observing between one and four mathematics lessons in 16 schools, the Illinois Council of Teacher of Mathematics delegation to Japan (1989) describes a typical mathematics lesson.

1. Students rise and bow	
2. Review previous day's problems or introduce problem solving topic	5 minutes
3. Understanding the problem	5 minutes
4. Problem solving by students, working in pairs or small groups (cooperative learning)	20-25 minutes
5. Comparing and discussing (students put proposed solutions on small or large blackboards)	10 minutes
6. Summing up by teacher	5 minutes
7. Exercises (2 to 4 problems only)	
8. Soft gong sounds to indicate the end of class. Students rise and bow.	

Stigler (1988), in his article about Japanese and American schools, states that the pace of instruction in Japanese classrooms is relaxed, and Japanese teachers constantly stop to discuss and explain. Both of these reports about mathematics teaching indicate the importance of talking in the classroom.

Class discussions are also important at higher levels of education, and my sessions with teachers-to-be are frequently interrupted with periods where students are given the opportunity to discuss points, conjectures or ideas in a cooperative setting. However, there is no truth to the rumor that I am trying to implement the first step of a typical Japanese mathematics lesson!

Allowing students the opportunity or more opportunities to talk is easily arranged, and teachers can use existing materials. Ask yourself what types of questions you could pose using existing computational exercises. What types of questions or instructions could contribute to the conceptual understanding of the ideas on that page of exercises? What types of questions and discussions could you use to increase time spent on developmental tasks and reduce the time students work on their own when little or no new mathematics is learned? What kind of questions could you pose that might lead students to gain new insights into some of the ideas presented?

Teachers in preservice and inservice sessions usually express the hope that by the time the training is completed or the meeting is over, they will have so many ideas and strategies at their disposal that the last thing that would ever come to mind is asking their students to complete more than three or four similar tasks on their own. I hope these teachers will have begun to think about using their ingenuity and creativity to generate ideas and activities for their students that will increase the time spent on developmental activities.

Types of tasks and settings would best be illustrated with excerpts from actual classroom settings. Brief excerpts from videotaped lessons were used during the actual presentation; simulated settings were also used. However, a little imagination will help you think of discussions that groups of two or three students might have after being asked the following questions:

1. Which of the items do you think is easiest (most difficult)?
2. Which of the items has an answer less than (greater than ______)?
3. Which items are similar or which two items do you think are similar?
4. Which items are different?
5. Which items are in some way the same yet will have answers that are different?
6. Which items are different but will yield answers that are similar?
7. The answer is ______ . Which items do not match this answer? Which item(s) could this answer belong to?
8. The estimate is ______ . Which item(s) could this estimate be for? Which items could/should be excluded?
9. Which items would have an "even" number (fractional number) for an answer?
10. Arrange the first five items in order of their estimated answers.

One major advantage of a cooperative setting is that students will have to discuss solutions and reach an agreement before responses are solicited and then compared. As different groups report, it will become obvious that there are different ways of thinking about a question or request and that different ways of looking at a set of items exist. Delaying a report by a group and having the members of the class guess the possible strategies behind the response, is likely to show that there may be different ways of thinking about the same results. Some unique or very creative thinking will surface in any classroom setting. These types of outcomes are valuable because they provide students with the opportunity to think and to think about their thinking. Students will also find out how others think.

Discussion opportunities can also be integrated with assignments or homework. As you picture a set of computational exercises, consider the following questions: What type of thinking skills/strategies are involved in responding to the given instructions or requests that follow? How might you adapt these questions or requests to make them suitable for the grade level you teach? Which tasks or combinations of tasks would you use with your students?

1. You do not have to do all of the items. Just solve those items that have an answer greater than (less than ______ ; greater than ______ and less than ______ .

2. Find the answers to two items that you think are "very easy" and for two items that you think are "not so easy." Give reasons for your choices. How would you explain how to find the answers for those "not so easy" items to someone in a lower grade? Write out your explanations. If possible, use diagrams.

3. Find the answers to two items you think are "very different."

4. Choose two items, and make up word problems that you could find the answers for. Make up a word problem for one item that you could not find the answer for. Why couldn't you find the answer? What would you need to know that would enable you to find the answer?

5. Design a multiple choice question for two items. As part of your choices, include two incorrect answers for each question. Provide your reasons for these choices. Why might someone, who "does not know as much as you do," select these choices?

6. After you have found the answers to four items, think of some mistakes younger students might make if they were asked to solve these tasks. What might they do wrong, and what would you teach them so they wouldn't make these mistakes?

7. Choose five items. Beside each item, list people (professions) who you think use these skills. Make up word problems for each example.

Let's turn our attention to the students and to our goal of having them acquire conceptual understanding of the mathematics they learn. This means that they should be able to talk about what they have learned in their own words. Students should be able to demonstrate their understanding with the assistance of a manipulative. They should also be able to pause and give meaning to their work at any time (Skemp 1987).

If we recognize our terminology, our phrases or our "chants" while listening to our students, we should become concerned. Davis (1986) speaks of "disaster studies" when he reports of the students' inability to talk about mathematics in their own words. He speculates that one reason for this is that teaching mathematics is treated like learning how to sing German lieder without possessing any real command of German as a language.

Some of us may remember the mathematics teacher turned singer/entertainer, Tom Lehrer. He took an algorithmic chant, the procedure for subtraction with regrouping, put it to piano music and wrote a song about the new math. Nightclub patrons loved it; it was mathematics made easy!

If every student is to reach the five general goals suggested by the NCTM standards, classrooms must allow for student discussion, foster self-confidence, lower or eliminate test anxiety, encourage students to take risks and give students an opportunity to think.

I hope the suggestions and ideas presented here will stimulate you to think about such classroom settings.

References

Cobb, P. "A Reaction to Early Number Papers." *Journal for Research in Mathematics Education* 16, no. 2 (March 1985): 141-45.

Davis, R. *Learning Mathematics—The Cognitive Science Approach to Mathematics Education*. Norwood, N.J.: Ablex Publishing Corporation, 1986.

de Groot, I. "Did You Know That . . . ?" *Vector* 29, no. 2 (Winter 1988): 7-8.

Easley, J., and R. Zwoyer. "Teaching by Listening—Toward a New Day in Math Classes." *Contemporary Education* 47 (1975): 19-25.

Greenwood, J., and R. Anderson. "Some Thoughts on Teaching and Learning Mathematics." *Arithmetic Teacher* 31, no. 3 (November 1983): 42-49.

ICTM Japan Mathematics Delegation. *Mathematics Teaching in Japanese Elementary and Secondary Schools*. Carbondale, Ill.: Southern Illinois University, 1989.

Liedtke, W. "Let's Talk About Talking Mathematics." *Arithmetic Teacher* 35, no. 2 (April 1988): 2.

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: National Council of Teachers of Mathematics, 1989.

Post, T. *Teaching Mathematics in Grades 1-8 (Research-Based Methods)*. Toronto, Ont.: Allyn and Bacon, 1988.

Skemp, R. *The Psychology of Learning Mathematics*. Hillsdale, N.J.: Lawrence Erlbaum Associates, 1987.

Stigler, J. "The Use of Verbal Explanations in Japanese and American Classrooms" (Research into Practice). *Arithmetic Teacher* 36, no. 2 (October 1988): 27-29.