Mathematics for a Magic Theme

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Yvonne Pothier, a professor at Mount St. Vincent University, Halifax, Nova Scotia and Daiyo Sawada, a professor at the University of Alberta, Edmonton, are involved in a collaborative mathematics teaching project at Mount Carmel School in Edmonton, Alberta. Gail Brooks is one of the teachers participating in the project.

Parents and relatives were invited to celebrate Education Week 1989. They came to see class presentations dealing with the magic theme and to view students' creative work.

I, Yvonne Pothier, had been teaching the mathematics problem solving strand to the Grade 4 class as part of a collaborative teaching project. Because 3 by 3 magic squares had recently stimulated a great deal of interest during this unit, the idea of a magic square presentation seemed to be "just the thing." Gail Brooks, the teacher I was working with, agreed and so the short skit presented here was written and eventually performed by an enthusiastic group of students. We offer it to anyone who is interested.

Lesson 1

During the problem solving lessons devoted to magic squares, students were given a large rectangular piece of newsprint paper (50 cm x 30 cm) and were directed to "make the largest square they could by folding the paper." This task proved to be challenging to many students, and the successful ones took on the role of "teacher" for their classmates. When everyone had constructed a large square by making two folds, students were then told to fold their square to make 9 small squares of equal size within the large square. Most students were able to make 4 and then 16 small squares, but some found it impossible to get 9 squares. Again, with the help of classmates, everyone produced the required 3 x 3 square. Some students' 9 squares were scarcely recognizable because so many folds had been made during the process of experimentation; therefore, students were told to draw lines highlighting the "good" folds.

Lesson 2

In lesson two, the students were given a sheet of paper on which a 3 x 3 square was printed. They were reminded of the lessons in which they folded the large square to get 9 small squares, and then the figure on the paper was examined to identify rows, columns and diagonals. Students were then given 9 small squares cut from colored, gummed paper with the numerals from one to nine written on the squares, and asked to arrange them in a 3 x 3 square. The next step was to change the square into a magic square. A magic square was defined, and the students set about moving the colored squares in an attempt to place them in such way that each row, column and diagonal had a sum of 15. Most of the students discovered the solution by the time class ended, and they shared solutions and glued the colored squares onto the sheet of paper to form a magic square (see Figure 1).

Figure 1
A 3 x 3 Magic Square

1	2	3
4	5	6
7	8	9

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8	1	6
3	5	7
4	9	2

The collaborative project has been made possible, in part, by a research grant from Mount St. Vincent University.

Lesson 3

During lesson three, students discussed their solutions, and we asked them what helped them find their solution and which number they tried to "figure out" first. One solution was written on the blackboard, and students discussed why five was the centre number. They also noted the constant difference between the two numbers on either side of the five and directly above and below it. Finally, students noted that the corner numbers were all even. Students discussed the possibility of constructing other magic squares and suggested using different sequences of nine numbers or doubling the first nine numbers. Within minutes, excited cries, such as "Doubling works!" and "Look! I've made one with a sum of 45" were heard.

In addition to presenting a skit, students agreed to make a magic square booklet in which each student constructed a 3 x 3 magic square and explained the number sequence selected. Some number sequences are presented in Figure 2.

Figure 2

Children's Sequences for 3 x 3 Magic Square

a. 2, 4, 6, 8, 10, 12, 14, 16, 18

b. 3, 6, 9, 12, 15, 18, 21, 24, 27

c. 7, 9, 11, 13, 15, 17, 19, 21, 23

d. 11, 12, 13, 14, 15, 16, 17, 18, 19

e. 31, 32, 33, 34, 35, 36, 37, 38, 39

f. 10, 20, 30, 40, 50, 60, 70, 80, 90

g. 8, 16, 24, 32, 40, 48, 56, 64, 72

h. 20, 40, 60, 80, 100, 120, 140, 160, 180

Lesson 4

A final lesson was devoted to "looking for sets of four numbers with a sum of 34 in a 4 x 4 magic square. Some of the students' solutions are presented in Figure 3.

The evening of the school celebration arrived, and the Grade 4s put on a great performance.

Figure 3

A Sample of Four Number Sets With a Sum of 34 in a 4 x 4 Magic Square

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

a. 3, 2, 10, 11 e. 11, 8, 7, 12
b. 5, 8, 7, 14 f. 5, 9, 8, 12
c. 3, 10, 13, 8 g. 2, 13, 4, 15
d. 2, 10, 7, 15 h. 16, 3, 14, 1

It's Mathematics Time—It's Mathemagic Time

A Skit About Magic Squares

Characters: Nine children holding number cards, two narrators, four pupils and one magician. To accommodate all pupils in a larger class, another group of nine children could demonstrate the second magic square, or narrators, pupils and magicians could be added.

Stage Set-Up: Nine children are on stage in a 3 x 3 square formation. Each holds a set of four cards: one card is blank, two have numbers from one to nine on them, and one card is a multiple of 2. The narrators stand on either side of the stage; the pupils and the magician stand at the side of the square.

Magician: With a sweep of the wand, the magician announces, "It's math-a-MAGIC time!"

Narrator 1: Our class wishes to demonstrate some mathematics magic. You see before you a square shape. It's called a 3 x 3 square because there are nine small squares within it.

Narrator 2: We see three rows, three columns and two diagonals. Here's one and here's another. (Magician uses wand to point to each row, column and diagonal in turn.)

N1: To make a magic square, we first number each small square using the numbers one to nine, like this . . . 1, 2, 3, 4, 5, 6, 7, 8, 9! (In sequence, children show cards with numerals on them.)

N2: Now for some magic! We want to move the numbers so that each row, column and diagonal has a sum of 15. (Magician indicates rows, columns and diagonals.)

N1: Let's begin. Which three numbers add up to 15? (Magician moves wand across each row in turn while the children show their third card.)
8 + 1 + 6 = 15
3 + 5 + 7 = 15
4 + 9 + 2 = 15

S1: What about the columns?

N2: Alright, you check them.

S1: 8 + 3 + 4 = 15
1 + 5 + 9 = 15
6 + 7 + 2 = 15

N2: Great! But there's one more thing to check.

S2: The diagonals!

N2: Alright, you do this.

S2: 8 + 5 + 2 = 15
6 + 5 + 4 = 15 Wow!

N1: Bravo! We have made a magic square! The rows add up to 15; the columns add up to 15, and the diagonals add up to 15.

S1: I wonder what would happen if we did something to the numbers.

S2: What do you mean?

S1: Well, we could double each number.

N2: A good idea! Let's try it. We double each number in row 1 (magician points, children change their number cards), double the numbers in row 2 and in row 3.

N1: Now let's check.

S1: 16 + 2 + 12 = 30
6 + 10 + 14 = 30
8 + 18 + 4 = 30 It works so far.

S2: Now the columns. 16 + 2 + 12 = 30
2 + 10 + 18 = 30, 12 + 14 + 4 = 30
Great!

S1: Don't forget the diagonals!

S2: 16 + 10 + 4 = 30, 12 + 10 + 8 = 30

N2: Voilà! Another magic square!

S1: I wonder if we would get another magic square if we multiplied each number by three, by four or by any other number?

S2: I wonder what would happen if we took the numbers in one magic square and added them to the corresponding numbers in another magic square? Would that make a new magic square?

S1: I wonder if we could make a magic square using other numbers like the numbers from 11 to 19, or from 21 to 29 or any other sequence?

S2: I wonder if it's possible to make a 4 x 4 magic square?

All on stage: That's some magic for YOU (pointing to the audience) to try!

Curtain.