

# The Ups and Downs of Elevator Probability

*Sandra Pulver*

*Sandra Pulver is a professor of mathematics at Pace University, New York, N. Y.*

An elevator paradox exists. Whenever I am in a building waiting for the elevator to go up, the elevators are always on their way down. Are the elevators manufactured on the roof and sent down to the basement to be stored? And when I am on an upper floor waiting to go down, the elevators are usually on their way up. Are the elevators being constructed in the basement and carried off the roof by helicopters?

Actually, the intuitive feelings here are correct, and the probabilities are not difficult to compute.

Suppose that an elevator is traveling up and down at constant speed and in continuous cycles in a 20-storey building. If I am on the 4th floor waiting to go up, I will have 3 floors below me and 16 above.

Therefore, the probability is 16/19 that the elevator is above me and will be on its way down when it stops. However, if I am on the 17th floor waiting to go down, I will have 3 floors above me and 16 floors below me. Therefore, the probability is 16/19 that the elevator is on some floor below me and will be moving up when it stops!

The solution for two or more elevators is complicated by conditional probability, but Donald E. Knuth, a computer scientist at Stanford University, has made a discovery. As the number of elevators approaches infinity, the probability that the first elevator, going up or down, will stop on any floor, except the top or bottom floors, approaches exactly 1:2.

## References

Knuth, Donald E. *Scientific American*. February 1973, 138-39.