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Who Are We?

It's the end of another school year, and it's time to reflect on our role as mathematics educators. It's time to consider what we are doing, why we are doing it and who we are.

We manifest who we are every time a student approaches us to ask a question or to seek help. "I don't understand." "Where do I start?" "I'm confused." "Can I do the problem this way?"

Our responses to these requests, implicitly and explicitly, tell our students what we believe is important about learning. Do we make statements like "Go read the problem again, or Let me show you how to do that"? If so, we may be giving the message that either we do not know how to help that student or that we believe "helping" is doing the problem for the student.

Telling students to reread a problem is often frustrating for students who do not know how to start a problem. Often they have reread it many times and wonder why rereading it again would necessarily help them. By "showing" students how to do a problem, we are not being helpful because now the students are not doing the problem, we are.

When it comes time for them to do problems on their own, students often say, "I understood the problem when I watched you do it, but I can't do it without your help." To solve problems, students need our help in different ways. We need to respond to students by giving hints, those ideas that truly lead students in helpful directions, but at the same time, not to solve problems for them. We need to model how we do problems, not in an artificial manner after we have solved a problem, but during the problem solving process so that students see the kinds of strategies we use, and also so they see that the process is not as neat and orderly as textbooks would have them believe.

If we react to their questions in this manner, we convey a different sense of what learning mathematics is all about. We convey a sense of support and at the same time the belief that students are responsible for their own learning. We convey a sense of authenticity. We let students know that mathematics is not simply a mechanical procedure; it is a way of knowing when and where to use procedures. We tell students that mathematics is not always procedural; it is a method of knowing what information is relevant and what is not, and this helps them decide what kinds of mathematical questions to ask.

Our responses indicate what we believe to be educationally important as does our curricula. Students in my university classes often wonder how they can use textbooks or mandated curricula creatively. They assume that because they are being told what to teach or what materials to use, they are also being told *how* to teach. I tell them that the same curricula will "look" different in classroom practice because of the variety of methods and approaches that can be used.

If a drill and practice approach is being emphasized, students are given the message that this is what constitutes mathematics. A recent episode of television's "60 Minutes" devoted a portion of the program to the work of John Saxon, a self-proclaimed expert in mathematics education from the United States. Mr. Saxon has written a series of self-published mathematics textbooks based on his theories of learning, which can best be summarized as "practice, practice, practice, practice." Not only is practice the bedrock of Saxon's program, but the procedures being practised are purely rote and mechanical. What was disturbing was Saxon's claim that his students are doing well on standardized exams used for college admission. Whether or not this is "true" is not the issue. What is important is the way we test students and how these tests are used.

If exams are comprised of items that can be successfully completed by students who have memorized rules and procedures, then a message is being given to them. That message is, you don't need to understand mathematics. The issue that Saxon did not address was whether his students were successful once they entered university. After having spoken to mathematics professors, I have learned that too many students manifest little understanding once they enter university courses even if they obtained high marks in high school mathematics. Many students drop out and/or fail the university calculus courses required for many professions. These failure rates should make us question not only how we are evaluating students but also how we are using the mathematics curricula.

If we say that problem solving and critical thinking are our goals as educators, do we project that message to our students? Are we problem solvers and critical thinkers ourselves, and do we model this behavior in our classrooms? Do we use traditional teaching styles only, demonstrate how-to procedures, have students practise those same procedures, and then wonder why students are unable to think for themselves? Or do we use a variety of teaching styles to get students active in their own learning? Do we demonstrate that we have thought critically about the kinds of mathematical problems we give students or do we merely assign one textbook page after another? In short, do we give the message Do as I say, not as I do by paying lip service to problem solving and critical thinking? Answers to these questions require much more space than is available, but I pose them to help us become aware of the images we are projecting to our students.

Finally, we need to think of ourselves not only as part of the classrooms in which we teach, but also as members of the wider society in which we live. Our values and beliefs about mathematics affect not only the students we teach but they touch the lives of our friends and families. It is important to remember this fact when we recoil from the prevalent image of mathematics as mechanical computational procedures, as being able to balance one's chequebook. We need to remember because we, as members of this culture have helped contribute, often unknowingly, to this attitude.

As teachers, it is difficult to make changes in our classrooms because changes cannot be made in isolation. We are part of a school system. We need the support of our colleagues and the administration. The parents of our students must also understand why it is important to break away from traditional rote drill and practice methods in mathematics.

We need not only to think about ourselves and who we are as mathematics teachers but we also need to speak to other teachers, parents and administrators. We need to reflect and pose questions as I have here. Changing mathematics education begins with changing ourselves, and change begins with the awareness of who we are.

The articles in this issue of *delta-K* can be placed in this context. Ted Aoki's presentation, although not written for the mathematics educator in particular, explores the theme of questioning who we are and how that affects our curriculum. Darlene Hubber's essay about math anxiety makes us question ourselves as teachers in a different way by calling forth an awareness of our influence on our students' self-esteem. Werner Liedtke's article asks us to question our teaching methods, the kinds of questions we ask students and the amount of discussion we promote.

Marlow Ediger poses some issues for us to think about. Although many of the issues he poses may not be "new" for some readers, they are issues worth reflecting on again. James Vance provides us with excerpts from interviews of students' work with rational numbers. These allow us to reflect not only on student concepts of fractions but also on our role as teachers in the creation of these concepts. The article by Yvonne Pothier, Gail Brooks and Daiyo Sawada provides a different twist on this issue's theme. By providing an example of innovative teaching, using mathematics in a dramatic setting, they help expand our repertoire of teaching styles.

Finally, a new feature of *delta-K* is the IDEAS department. In this issue, it contains two articles. I encourage readers to send more manuscripts to this department, which is designed to highlight interesting problems and situations for use in the classroom.

Linda I. Brandau

Wanted

Material for a monograph on problem solving in the high school is now being compiled. Send a brief manuscript, two to five typed pages, outlining a problem and/or situation, and describe how you use it in your classroom, how it relates to the curriculum and the "results" in terms of student learning.

For further information, contact:

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Beyond the Half-Life of Curriculum and Pedagogy

Ted Aoki

This is the text of an address to the ATA Specialist Council Seminar held at Barnett House in September 1989. It is reprinted here with Ted Aoki's permission.

Let me slide into my talk by mentioning three orienting events, all personal.

Event 1: "Education is Being." So announced a banner mounted outside the University of Heidelberg, Germany, where this past summer educators assembled in conference. On seeing this banner, a friend from Montreal excitedly showed it to me and asked, "Could such a banner be possible in Canada?"

Event 2: At the moment I received the above information, I was in the midst of Milan Kundera's *The Art of the Novel.* Kundera is the novelist who wrote the *Unbearable Lightness of Being*, a film version of which was nominated for the Oscar. The latter book portrayed narratively the way in which Czechoslovakians are compelled to live half-lives as humans.

Event 3: Recently, I have been involved with a group of University of Maryland educators who hopefully have a book to be published. Title? *Voices of Educators; Toward Curriculum of Being.* In this book, five educators, critical of their own past half-lives as educators, tell of their experiences trying to understand what authentic teaching is.

Beyond the Half-Life of Excellence in Schools

To call for excellence in schools is a popular call. Those seeking popularity or applause would be wise to call for excellence in schools, for who among us would dare to say no? In first flush, the word excellence conjures forth the superlatives—the best, the top, the cream, the super—reflecting the "super" ethos of our time. Everything excelling is seen to be super—superstore, supermall, superman, supercar. The superlative may be the going understanding of excellence, but I feel it is urgent that we pause to reflect upon what it is to excel. I wish to allow this question to guide me in my quest for a fuller understanding of what it is to excel humanly. But first a detour.

I begin by leading you to Schleiermacher, a great Catholic theologian and hermeneutic scholar who said, "Multifold are the ways a person relates to the universe." To help us understand, let me interpret him in terms of how an architect, a carpenter and a worshiper might relate to a cathedral.

An architect likely experiences the cathedral conceptually and theoretically. Within his intellectual scheme of things, he classifies the cathedral as a special type of church building and, by the shape of it, he can indicate when it was built, the architectural style in which it was built and the materials that entered into its construction. If the architect theoretician is a good scholar, he will have a store of factual and theoretical knowledge about it. In effect, he will have a good intellectual command of the cathedral as an object of study. We will evaluate that person as a good scholar of architecture if he knows much. When you see this scholar, you will likely see his mind at work. For him the cathedral is out there as an object to be subordinated to his intellect.

A carpenter walking into the same cathedral will look for whatever needs making and fixing. He will be bent on making the cathedral serviceable for practical purposes. If the roof leaks, he will fix it. If a window is broken, he will fix that. If an altar is needed, he can make it following a blueprint. His interest is in using his skills to make or fix whatever needs making or fixing. He is a technician and he experiences the cathedral practically. We would evaluate him to be a good carpenter if he has good technical skills and is efficient in making and fixing things. When you see the carpenter, you will probably see his hands at work, and if his head is working, it is likely he is applying the rules of his trade.

When a worshiper enters the cathedral, he or she experiences the cathedral existentially and poetically in a fundamental sense. Understanding the world as a whole with the self included, the person seeks the meaning of what it is to live and to be human. For this person the cathedral is an embodied spiritual dwelling place wherein the fourfold of mortal self, divinity, earth and heaven gather together and shine through as one. Seeking is for oneness, a coming together of the finite and infinite worlds. The meaning of lived experiences is the person's utmost concern. We would evaluate the person to be a good worshiper if the person's quality of being is revealed as deeply human. When you see a true worshiper, you see the whole embodied being in communion with the universe. The language spoken will be that of hopefulness and prayer.

Multifold indeed are the ways in which people relate to the world. Now, if we substitute school for cathedral, we can begin to see three understandings of school. View 1 is a school given primarily to thinking, a school that emphasizes intellectual skills, a school that emphasizes mind building. The curriculum will be a thinking curriculum. It is a school that understands a teacher or student as split into mind and body. Teaching is seen essentially as mind building accomplished by filling empty containers with factual and theoretical knowledge; being a student is like being a blotter, absorbing knowledge, the more the better, the faster the better, as the assessment people get closer. An appropriate metaphor of View 1 is Rodin's sculpture, *The Thinker,* which is symbolic of the Enlightenment and the Renaissance.

View 2 is a school given primarily to "doing," a school that emphasizes practical skills, like the three Rs, a school that nurtures manipulative skills for productive purposes. This school is utility oriented; usefulness in the postschool workplace is often the guide to curriculum building. The school is a preparation place for the marketplace and students are molded into marketable products. If the market needs auto mechanics, an automotive curriculum is built; if the market needs word processors, word processing courses are built; if the market needs computer operators and programmers, computer courses are built. The interest of the market predominates. Adult life is the model, and adolescents are understood as immature adults, yet unskilled.

At the secondary school level, these market-oriented schools are called vocational schools; at the higher education level they are called institutes of technology or professional schools at universities, (schools of engineering, medicine, business, law, education and the like). These are all vocation-oriented schools, vocation understood as jobs.

View 3 is a school given primarily to being and becoming, a school that emphasizes and nurtures the becoming of human beings. Such a school will not neglect doing but asserts the togetherness of doing and being enfolded in becoming. Here it is understood that to do something, one has to be somebody. The teacher or student is seen as being simultaneously an individual and a social being, a being-in-relation-to-others. But as it is a school given to becoming, it emphasizes "reflective seeing again," a reflective reviewing of self and world as well as the taken-for-granted assumptions that make possible our seeing and acting. Teaching is not only a mode of doing but also a mode of being-with-others. Teaching is a relating "with" students in concrete situations guided by the pedagogical good. Teaching is a leading out (from *ex* 'out' and *ducere* 'to lead')—leading students out into a world of possibilities, at the same time being mindful of their finiteness as mortal beings. Whereas the View 1 and 2 schools are grounded in a fragmented view of persons (body *and* mind), the View 3 school sees its origin in an understanding of teachers and students as embodied beings of wholeness. View 3 restores the unity of body and mind, body and soul.

Let us acknowledge that these three views are idealized views and as such they may not exist in reality as clearly separated categories. Nevertheless, holding these views before us begins to allow us to sense what view of school and whose view of school holds sway in certain quarters among educators, administrators, the public and so on. Also, it allows us to acknowledge how a particular view of school assumes a particular view of teaching. And so, when we hear voices telling us what schooling is, we can begin to make sense of what these voices are really saying about our own calling, teaching.

Now we return to the original question. When we speak of excellence in schools, we need to ask, What is it to excel? I suggest that excellence is understood differently depending on the way we are attuned to the world.

Within the theoretical framework, excellence is understood in terms of superiority in thinking power and in the power to acquire knowledge. Academic brilliance would be a paradigm case of this understanding of excellence. A curriculum oriented toward this notion of excellence would likely stress the academic disciplines. The university is an apt model. A belief in the centrality of this understanding of excellence will press for legitimation in schools of theoretical subjects—the disciplines. We saw this curriculum movement peaking in the days of the New Math, of Jerome Bruner's *Process of Education*, the alphabetized curricula such as B.S.C.S., P.S.S.C., CHEM STUDY, MACOS, and a host of others.

Within the practical or utilitarian framework, "excellence" is understood as a high standard in skills. A high level of expertise in the skills of making and fixing will be a manifestation of this understanding of excellence. In teaching, effective and efficient pedagogic skills will be held in high regard à la Madeline Hunter. In the regular classroom, stress will be given to the skills dimension of reading, writing, 'rithmetic and other subjects. Excelling in skills on the students' part will be the main focus of the curriculum.

Within the lived experience framework, the word excellence, as we typically understand it, gives us difficulty. For we find it odd to speak of an excellent worshiper, of excellence in being human or of excellence in becoming more human. We find it difficult to speak of excellence of the beings of teachers or students.

Allow me to venture a thought. When we encounter such difficulty, we might look at ourselves, particularly how we are typically oriented to excellence, that is, to our typical achievement orientation in school curriculum and in life generally. It is likely we find ourselves caught in an orientation that flows from the technological ethos that tends to dominate our thinking and our doing.

To help us in our striving for an appropriate understanding of excellence in the beings and becomings of teachers and students, we might appeal to the etymology of excellence, for there may be in store here an original meaning of what it is to excel. Rooted in the Latin *ex-cellere* (*ex* 'out of' and *cellere* 'to rise or raise oneself'), the source of our word contains, in a deep sense, the notion of one's struggle to surpass who one is, to become, within the world of possibilities, who one is not.

For us, being human beings, to excel can be understood as a coming to a deeper understanding of who we are and a moving beyond, a surpassing of our present being. An excelling person, in this sense, would be a person who is undergoing or has undergone a search for a deep understanding of what it means to be, of what it means to live in this world in such a way that his/her becoming will be guided by that which calls upon the person to be a better human being.

Within this understanding, to understand an educator's true vocation in school life is to understand it as a calling—a true calling that responds to what it really means to be educators, to be administrators, to be teachers, to be curriculum developers, to be appraisers. Within this view, an excellent person is not merely a good intellectual, not merely a good practitioner, but also a good person.

I feel that we have tended to be tuned into the half-life of excellence and thus have become neglectful of the original meaning of excellence. We need to reclaim this fuller meaning that flows from oneness of body and spirit, a oneness that is not forgetful of ethics and morality, a oneness that considers central a proper attunement to life, a oneness that considers that central to the purpose of life, including school life, is to live well, to live surpassingly, to live excellently.

Should we aspire for excellence in schools? Of course, we should. But in aspiring for excellence, let us pause and weigh with care with what understanding of excellence we are calling upon our teachers and students to excel.

Beyond the Half-Life of Parenting, Teaching and Administering

What might human excellence mean in parenting, teaching and administering? Allow me now, through short anecdotes and stories, to tell of the layers of understanding of excellence in parenting, teaching and administering, and through them suggest that at a deep level they converge in pedagogy. As you listen, I ask that you be mindful of what the half-life of parenting, teaching and administering might be like and what it might mean to move beyond such half-lives.

Toward Parenting as Pedagogy

Let's listen to two short stories; the first, concerning father and son, the second concerning mother and child.

"Father and Me"

This is a story told by Bruce Springsteen, from the introductory monologue to "The River."

> "When I was growin' up, me and my dad used to go at it all the time, over almost anything. I used to have really long hair, way down past my shoulders—I was 17 or 18. I used to hate it!
>
> We got to where we were fighting so much that I'd spend a lot of time out of the house. In the summertime it wasn't so bad, 'cause it was warm and your friends were out. But in the winter I remember standing downtown. It would get so cold, and when the wind would blow, I had this phone booth that I used to stand in and I used to call my girl, like for hours at a time, just talkin' to her all night long.
>
> And finally I'd get my nerve up to go home. I'd stand there in the driveway and he'd be waitin' for me in the kitchen. I'd tuck my hair down under my collar and I'd walk in and he'd call me back to sit down with him. The first thing he'd always ask me was what did I think I was doin' with myself—and the worst part about it was I could never explain it to him.
>
> I remember I got in a motorcycle accident once. I was laid up in bed and he had a barber come in and cut my hair. And man, I can remember tellin' him that I hated him and that I would never, ever forget it.
>
> He used to tell me, "I can't wait until the army gets you. When that army gets you they're gonna make a man outta you. They're gonna cut all that hair off and they'll make a man outta you.
>
> This was in, I guess, '68 and there were a lot of guys in the neighborhood goin' to Vietnam. I remember the drummer in my first band comin' over to my house with his Marine uniform on, sayin' that he was goin' and that he didn't know where it was. And a lotta guys went and a lotta guys didn't come back. And a lot that came back weren't the same anymore.
>
> I remember the day I got my draft notice. I hid it from my folks and three days before my physical, me and my friends went out and stayed up all night. We got on the bus to go that morning, and man, we were all so scared! And I went, and I failed.
>
> And I remember comin' home after I'd been gone for three days, walkin' into the kitchen. My mother and father were sittin' there.
>
> My dad said, "Where you been?"
>
> I said, "I went to take my physical."
>
> He said, "What happened?"
>
> I said, "They didn't take me."
>
> And he said "That's good."

In what way does this story speak to pedagogy? What is the meaning of "good" when Bruce's father insists that short hair is good? What is the meaning of "good" when at the end of the story he says, "That's good"? In what way does "that's good" resonate within you? Why?

Women Becoming Mothers

What does it mean for a woman to become a mother? I lean on Vangie Bergum, associate professor in the Faculty of Nursing at the University of Alberta, who in her study conversed with several women who became mothers. In their conversations Vangie asked these mothers-to-be to speak of their lived experiences in an effort to come to an understanding of the meaning of the experiences of becoming a mother. She listened to them with care, turned their tellings into stories, and then she, lingering in the stories, unfolded existential themes of what it means for women to become mothers. You will want to read *Woman to Mother: A Transformation* written by Vangie.

Toward the end of her study, she reflected upon their stories. To get a flavor of these women's transformative experiences, let's read a short statement from her dissertation.

> The transformative experience that is accessible to women who become mothers has been the central focus of this study. The conversations with women have opened ways to explain what it means to become a mother, facing a questioning of the forms of knowledge used by women to understand themselves as mothers. Being a mother is a matter not only of the mother role, not only of caring for the child, not only of caring for a home. It *is* a matter of a changed understanding of who women *are* as mothers. Becoming a mother is a matter not only of maternal tasks, not only of developmental tasks, not only of stressors and satisfactions. It is a realization and acceptance that "I *am* mother."

To open up the domain from the world of becoming mother into the world of teaching, I ask you to reread, with a few changes, what Vangie said about mothers, with the possibility that the passage might say something about teachers. Please return to the above passage for a rereading, substituting "people" for "women" and "teacher" for "mother."

What is it to realize and accept that I am mother, that I am teacher? What is the being of mothers? What is the being of teachers?

Toward Teaching as Pedagogy

To illustrate what being a teacher might mean, let me read a story of teaching from a child's perspective. It is a story so often told by my wife, June, over the past decades.

It was a cloudy day in early April, 1942. I was 13 then, going on 14, in Grade 7 at Fanny Bay School, a two-room school about 40 miles up the island from Nanaimo. It was a bewildering day for many of us. Our Japanese language school had been ordered closed by the Ministry of Education. My father had been sent to a road camp near Blue River in the far-off wilds of the Rockies. We had been hearing rumors that we were to be moved, first to Vancouver, then somewhere to the interior of British Columbia, and possibly beyond. We had been trying not to believe Charlie Tweedie who told my brother, Tim, that all the Japanese were to be herded en masse into Hastings Park, and who had said, teasingly perhaps, "That way only one bomb will do it!"

On this day in April, I went to school solely for the purpose of leaving. As soon as school began, we cleaned out our desks, returned texts that belonged to the school and gathered our books and belongings while our occidental schoolmates silently watched our movements. With our arms full, we left our classroom with feet that seemed to know that they might never return. Cautiously, we moved step-by-step down two flights of stairs and wended our way along the worn path of the school playground bound for home.

The leaving this day was different from our usual taking leave at the end of the school's day. Somehow I felt I was leaving a place to which, like home, I belonged. Why was it that my usually happy feet had no skip to them? Did my feet know that they would never again tread this path whose every bump and bend they had come to know? I guess we were experiencing emptiness in leaving behind what had become so much a part of our everyday lives. As I walked I felt the school's tug. It was like hands that slip away in parting and know not what to say in silent farewell.

I was about to leave the school yard. Something called upon me to turn around for a last look. On the balcony of the school stood my teacher, Mr. McNab, alone, watching as if to keep guard over us in our departure.

I almost felt I did wrong in stealing a look, so without a wave of goodbye I resumed my walk. I wondered, "What is Mr. McNab thinking right now?"

I cannot remember my other teachers in all the years of my schooling which began in Fanny Bay and continued in the Slocan relocation centre and in Picture Butte School in southern Alberta. But Mr. McNab, I remember. He is the one I recall. He was the teacher who urged us to display our Japanese kimonos and to perform some *odori* to Japanese music. He was the one who, on the annual district sports day, insisted he take all the students, the athletic and the not so athletic, breaking the tradition that sports day was for elite athletes. For us the event was something special; we were happy to be loaded in the back of a truck, and to run, jump and throw. It mattered little whether we won or lost. All of us were grateful that Mr. McNab took everyone—swift ones and slow ones, dumpy ones and lean ones, tall ones and short ones.

Last year we returned to the coast, to touch again the earth and water we once knew. Coming home, I wondered if by chance I could make contact with Mr. McNab. I had heard nothing about him for more than four decades.

Through the B.C. Teachers' Federation offices we learned that a Mr. William McNab, a retired teacher, lived in North Vancouver. I felt a stirring in my heart. I phoned him. Most graciously he listened to my story. For him it must have been puzzling, after 44 years, to sort me out from a mountain of memories of hundreds and thousands of students who called him "teacher." But he was my Mr. McNab, my teacher.

He kindly visited us. I experienced a deep inward joy when my hand grasped the hand of him who silently gave watch over us as we left his school that April day 44 years ago. I felt he did not know that over all those years the memory of his watching—watching us leave Fanny Bay School for the last time—stayed vividly with me. For me, that singular moment reflected his being as teacher.

I told Mr. McNab how I had often recalled the image of his watchfulness clothed in care that lived vividly within me. Mustering courage, I asked him if he remembered the moment. There was a moment of silence. Then he simply said, "That was a sad day." That was all he would say. The rest he left unsaid. But I felt in the silence he said much.

I felt blessed to be in the presence of a teacher whose quiet but thoughtful gesture had touched me deeply. Today I feel doubly blessed to be allowed to relive the fullness of this moment in the presence of Mr. McNab, rooted as I am in memories of my teacher of 44 years ago.

A Reflection: Teaching is Watchfulness

What is the voice of teaching that this story speaks of? Could it be merely a student remembering an event? Surely, it is more. Could it be merely that a teacher watched a group of students take leave? Surely, it is more, much more than a recording of a minor historical event in the lives of a teacher and a few students.

How then, shall we understand the voice of Mr. McNab's teaching? Could it be that it is not so much the watching, but the person he was as he watched?

We might see a glimmer of the person he was as teacher if we look with care at his watching. His watching was not so much watching as observing, a looking "at," that is, apart from his self. It was a watching that was watchfulness—a watchfulness filled with a teacher's hope that wherever his students may be, wherever they may wander on this earth away from his presence, they will fare well and no harm will visit them.

We might understand the meaning of watchfulness a little better if we observed with care a mother's watchfulness of her child, a watchfulness that is the voice of the hand in hand of mother and child as they cross a busy street. The watchfulness in the hand in hand is attuned to the care that dwells between mother and child. And it is this logos of care that allows mother to lead from where the child is now to where the child is not yet. For mother, her hand will ever be there, and even in those times when hand does not touch hand, there is a touching that flows from mother's care for the child. And the mother knows that when the child, no longer a child, takes leave, mother's watchful touch in absence will ever be present. Such is the watchfulness of mother with child; such is the watchfulness of teacher with student.

Teachers understand the meaning of the presence of absence growing out of their own experiences of watchfulness. Teachers know that pupils come to them clothed in a bond of entrustment of parents, and parents know that they, in entrusting their children to teachers, can count on the watchful eyes of teachers. So, too, teachers know, that at the end of the year, they and their students will part. The students will advance to the next grade or move to another school. Yet, it is their very leaving that allows them the possibility of return—a turning again to the experiences of the present. And the teachers know that watching their students depart at the end of the year is a watchfulness that is filled with hope that wherever they may be, their students will do well and be well and that no harm will befall them.

Authentic teaching is watchfulness, a mindful watching over, flowing from the good in the situation that the good teacher sees. In this sense, teachers are more than they do; they "are" the teaching. When Mr. McNab watched, he was the teaching. No less, no more.

Administering as Pedagogy

You have heard of Bill 19 in B.C. It effectively separated principals from teachers. I was disturbed because such a separation seemed to stem, at least in part, from an understanding that principals are essentially managers.

In those tumultuous days, I sent to Elsie McMurphy, then president of the British Columbia Teachers' Federation, brief notes on my thoughts. I was pleasantly shocked when the notes appeared as a short article in the *B.C. Teacher.* Entitled "Principals as Managers: An Incomplete Educational View," it reads in part:

- To understand principals as managers is to understand principals within the metaphor of business/industry. The world of education is likened to the world of business, where the prime interests of management and control accompany the goals of effectiveness and efficiency. Education does entail, in part, management, and in that sense education is like a business. Correct. But such a partial understanding is a half-life of what education is. We need to be mindful when metaphors are borrowed; dangers lurk when one thing is likened to another.
- The word principal was at one time understood as "principal teacher"— first or leading teacher. Principal was at one time an adjective. How did it become a noun? What happened when the adjective principal was separated from teacher?
- The separation made it easy for principals to be labelled administrators, usually understood within the business framework as managers. Such an understanding, which might be satisfactory for business, is inappropriate for educational ventures. Business deals with materials and people as resources—as beings that are things (note, dehumanization). Education deals with people—with

beings that are human, making education a venture vastly different from business.

- When we hear "principals are administrators," there is evident forgetfulness of the original meaning of what it is to administer. The original meaning of administer was *ad* 'to' and *minister* 'serve,' to serve. To serve others, to be servants, to minister to the well-being of others was the original meaning of administration. Somewhere along the line, there occurred a reduction through truncation. We need a recovery of the original meaning if we are to speak of educational administration.
- What authorizes a person to be an administrator? In the truest sense, authority does not flow from assignment of position by powered people, nor from receipt of certified pieces of paper. True authority flows from being true to whatever phenomenon claims the person.
- Administrators often talk of leadership. What authorizes a person to be an educational leader? What is it to lead? To lead is to follow the authority of the true. A leader in education must lead as he or she follows the essence, the true, of what education is.
- Principal as manager is correct insofar as education is a business, but not true insofar as education is not a business. Principal as manager, by itself, misunderstands education. As such, it is dangerous.

I have meandered in my musings. Let me gather together Bruce Springsteen's "Father and Me," Vangie Bergum's *Woman To Mother: A Transformation,* June's story of revisiting Mr. McNab and my notes on principals as managers. I claim, at the deep human level, that they converge in pedagogy—pedagogy understood in its original sense.

Pedagogy from the outset meant leading children (from *agogue* 'to lead' and *pedae* 'children'). How fortunate it is that the word education also speaks to leading, for buried in its etymology is *ex* 'out' and *ducere* 'to lead,' a veritable leading out to fresh possibilities.

Allow me now to lead you to a place where leading in terms of human excellence might show itself. I appeal to the Chinese characters for a sage, a wise leader.

耳 —ear, to hear

口 —mouth, to speak

王 —leader who stands tall between heaven and earth

人 —person (it takes at least two to make a person).

Here is my interpretation. A sage is a person whose self is saturated with otherness, that is, with the well-being of others. Standing between heaven and earth, the sage listens with care, not only to what others are saying, but also, perhaps more so, to a calling in the situation that others cannot hear. The sage by heeding the calling is able to speak. In wisdom, the sage leads. In wisdom, parents, teachers and administrators truly lead.

Beyond the Univocity of the Curriculum-As-Plan

Let's consider a course that you teach. If I were to ask you to think curriculum, likely what comes into view is the curriculum as we typically see it in the guide. It speaks to us as something planned—a curriculum-as-plan.

Why this singular view? Why this singular voice? Why is it that the curriculum-as-plan pervades the whole province as if with a single voice? Why is it that it dominates the curriculum world? Under such domination, all else, like curriculum implementation and curriculum assessment/evaluation, flow derivatively from this curriculum-as-plan.

Why so? In a sense, the primacy of the curriculum-as-plan is legitimated by the structure of officialdom in our world of education. In keeping with the venerable *British North America Act,* each provincial authority assumes the responsibility of setting out the curriculum. Within this context, we can understand visibly, the primacy of the curriculum-as-plan with all its institutionalized legitimacy.

But we also know, from having experienced it in our classrooms, that the province-wide voice of the curriculum-as-plan is a globalized voice, necessarily abstract, so abstract at times that its voice begins to dissipate up there in thinner air. What is seriously wrong about the univocal curriculum is that, in its abstractive interest (it cannot help but be abstract), it becomes indifferent to differences among real classroom situations, concerns and interests. In a recent article in *Harper's* magazine, Wendell Berry spoke eloquently of the futility of abstract, globalized thinking.

> All public movements of thought quickly produce a language that works as a code, useless to the extent it is abstract. The heroes of abstraction keep galloping in on their white horses . . . and they keep falling off in front of the grandstand.

What Berry is saying is that abstract language tends to be forgetful of the situational beings of live people. In schools, the classroom teachers whose lived experiences are necessarily situational are the ones who are in the position to know, by the blood, sweat and tears of life in the classroom, that the abstract curriculum-as-plan by itself, no matter how glitzy, is inert, sluggish and at best only a half-life. It could insist on entry into a classroom, but by itself it is a stranger and not at home. In a deep sense the curriculum-as-plan has to await an invitation from the classroom teacher who speaks on behalf of the students. It is only when the teacher, having lived through joys and struggles with the students, receives the curriculum-as-plan and interprets it so it makes sense in the classroom situation, that it can shed its inertness and come alive. By interpreting the curriculum-as-plan, the teacher breathes life into it. The curriculum-as-plan is like a limp violin string; it requires the teacher to give it tautness, hopefully appropriate, such that it can give forth soundings and re-soundings. In its resonance, we can hear the curriculum sing. Indeed, under the teacher's tactful hand, the curriculum-as-plan can leave its abstract form behind and become something else that is alive, our curriculum-as-lived.

What is the point of all this? The point, I hope, is a profound one. It is a challenge to the myth of the univocity of the curriculum-as-plan. It is a challenge to the institutionalized primacy of the curriculum-as-plan. It is a challenge to a claim of wholeness when in reality it is only a half-life.

This is not to negate the curriculum-as-plan. It has its rightful place, but its rightful place is not in its primacy. Its place, I claim, needs to be opened to allow a place for curriculum-as-lived. I call for a twofold curriculum of curriculum-as-plan and curriculum-as-lived.

Within this view, the dwelling place of the teacher is never an easy one, for such a place is the often difficult place of between—a vibrant place between the curriculum-as-plan and curriculum-as-lived. Once we understand this, we can begin to appreciate the toil and struggle of the teacher who stands in the ever-shifting between, in the midst of two curriculum worlds. Then, we can begin to give well-deserved recognition to the gift of hermeneutic creativity that good teachers conscientiously offer. Then, we can begin to give maieutic credit to teachers who allow students to give birth to new unfoldings. Then, we can begin to admire the teachers' interest and concern for the pedagogic good in the teaching situation that they in their wisdom see. Then, we can begin to appreciate the deep sensitivity with which good teachers lead students by listening to the deep calling that the teachers in their wisdom hear. Then, we can begin to appreciate more fully true excellence in our teachers' pedagogic living with students.

Within this twofold curriculum, what then of curriculum implementation? We can begin to see that implementation seen instrumentally as mere "installing" or as mere "delivering" of a given curriculum is only a half-life.

Again within this twofold curriculum, what then of curriculum evaluation/assessment? Here too, we can begin to see that assessment/evaluation understood strictly in terms of fidelity to the voice of the curriculum-as-plan is also a mere half-life.

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It's All Greek to Me: Math Anxiety

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"I'm not a numbers person." "Don't ask me; I never could understand math." "I just don't have a mathematical mind." "What, me do math? I can barely add two and three." It's all Greek to me!"

Sound familiar? If you have heard your students make such proclamations, then you have already had some experience with math anxiety. Tobias defines math anxiety as

> the feeling of panic, helplessness, paralysis and mental disorganization that arises among some people when they are required to solve a mathematical problem. (1980a)

The prevalence of the problem is reported to be as high as 68 percent among students enrolled in math classes (Lindbeck and Dambrot 1986). Some anxiety is desirable. Tension can serve as a motivator, but too much anxiety, as in the case of math anxiety, can inhibit learning.

Several factors contribute to the development of math anxiety (Tobias 1980b; Greenwood 1984; Martinez 1987). The first is the "math as a gift, not as a set of learned and practiced skills" misconception. Math anxious learners think only those born with a mathematical mind can fully comprehend complex numerical operations. They assume that competent math students arrive at solutions instantly, and they have little or no faith in their own ability as a result. Even when they are able to come up with a solution to a problem, they lack confidence in their answers and assume they couldn't possibly have figured it out correctly.

Math anxiety appears to be a uniquely North American phenomena. An examination of the math attitudes of Asians and Americans revealed some interesting discrepancies. The Asians thought math ability was fairly evenly distributed and that skills were developed through study, persistence and hard work. Americans, on the other hand, viewed mathematical ability as a very rare and uncommon talent (Tobias 1987).

Early math experiences can contribute to the development of math anxiety. Many of us painfully recall being summoned to the blackboard to solve a problem, only to make an embarassing blunder in front of a room full of witnesses. Some math anxious students recall negative experiences with particular teachers. They remember the stress and confusion caused by timed tests, the emphasis on one and only one correct answer and on the "right" way to arrive at solutions to problems. They remember pages and pages of drill and practice.

Some researchers believe "the principal cause of math anxiety lies in the teaching method used to convey basic mathematical skills" (Greenwood 1984). They suggest the explain-practice-memorize paradigm isolates facts from the problem solving process of which they are a fundamental part. Students who cannot understand the thought processes that underlie a problem's solution begin to perceive math as an incomprehensible mystery.

Teachers can be math anxious too. Such teachers contribute to the anxiety levels of their students by relying primarily on text explanations and do-the-problems/ correct-the-problems assignments. They often refer authoritatively to the teacher's guide and seldom work out problems with students in front of the class.

Inadequate out-of-class experience with math can contribute to anxiety development as well. Students who are never given an opportunity to solve real-life problems fail to see how the skills they have learned are applicable to situations outside the four walls of the classroom. They lack motivation because the meaningfulness of the skills is not apparent to them.

The language of mathematics can be complicated and ambiguous. For students who never become proficient *readers* of math, this can create confusion and anxiety. Words, such as attitude and root have very different mathematical connotations. One researcher reports interviewing a student who assumed that "least common denominator" meant "most unusual" (Tobias 1978). Based on her understanding of the term, she produced unique, but obviously incorrect responses.

Gender may play a role in the development of math anxiety. Studies show that although girls account for 49 percent of the secondary school population in the United States, they comprise only 20 percent of those taking math beyond geometry (Tobias 1978). This discrepancy is not a result of differing levels of ability so much as it is a reflection of societal perceptions. Parents, teachers and peers are more likely to excuse poor math performance by girls. As a result, girls may perceive math as a primarily masculine domain and lack the confidence and motivation to develop their own skills in this area. Girls may also feel pressure to appear "dumb" in order to conform to these perceptions.

Girls may find math more difficult because they lack math experience. Few of the playthings commonly provided for girls promote the development of mathematical understandings. The toys generally provided for boys however, are of the "take apart and put together" variety that do develop these understandings. In addition, many mathematical concepts are used in sports, such as hockey, football and baseball, which traditionally have much higher levels of male participation.

Role conflict, negative math experiences, inadequate instructional emphasis, lack of understanding of the language of math and misconceptions about the nature of math learning all contribute to the development of math anxiety. Math anxiety leads to math avoidance by students who feel they cannot and will never be able to experience success in the mathematics classroom. It may also contribute to lower achievement levels by otherwise able students who, for whatever reason, panic when asked to perform mathematical computations. Helping these students overcome their anxiety and ideally, preventing it from afflicting the generations to come, is now a critical issue. Math competence has become undeniably necessary for many technological careers. Students without a firm grounding in mathematics, greatly reduce their career options.

Programs have been developed at the college level for the math anxious members of the population. Most programs include counseling and education components. Participants are involved in some form of group or individual counseling where their anxieties are examined. They are also provided with direct instruction in mathematics with an emphasis on meaningful problem solving. Although these programs are beneficial, they treat the symptoms not the cause. Preventing the development of math anxiety is the responsibility of educators.

An anxiety-free learning environment can be created by removing tension and competitiveness; students must not be afraid to ask questions and make mistakes. A nonthreatening learning environment is a necessary component of such a classroom. Allowing students to correct their own work, learn through trial and error, offer answers without fear of humiliation and solve problems as committees helps create this kind of environment and nurture student confidence.

A change in emphasis from the traditional explain-practice-memorize paradigm is also required. Math learning experiences that are creative and encourage active participation by students can contribute to the development of more positive attitudes toward mathematics.

Instruction matched to the cognitive levels of the student results in lower anxiety levels. Concepts must be understood in their concrete forms before more abstract applications are introduced. To help students visualize concepts, manipulatives can be used at all levels.

A lower level of anxiety is also evident in students for whom numbers have real-life significance. Students must be aware of the personal usefulness of math; it must be meaningful from the students' perspective.

Systematic instruction in problem solving helps reduce the levels of anxiety in math classrooms. Understanding a variety of problem solving strategies gives students the tools to attack problems successfully outside the classroom. Students must have experience in looking for solution patterns, substituting simpler numbers and diagramming or sketching problems in order to use these strategies successfully.

Some attention must be paid to the unique reading demands of math. The language of math must be clearly understood for students to succeed. Strategies, such as substituting more easily understood synonyms, underlining and discussing problem words and rereading for clarification, if employed effectively by math students, will reduce anxiety levels.

"Most people leave school as failures at math or at least feeling like failures" (Tobias 1978). They are defeated before they even begin by their lack of confidence in their own abilities. As teachers, we can help reduce anxiety levels by changing the way we teach math. We must demonstrate the importance of math, provide students with a multitude of experimental opportunities to test mathematical concepts, and teach students the strategies necessary for effective problem solving. As our world becomes increasingly technological, we must develop more effective methods of teaching mathematics. Numeracy has become as important as literacy. We must dispell the myth that math is a secret code comprehensible only to an elite few.

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Conceptual Understanding: Promoting Talk in the Mathematics Classroom

Werner Liedtke

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As a visitor from British Columbia, allow me to share with you a few of the changes in our province, recent and contemplated, that have had or may have an impact on teaching and learning mathematics. In 1987, a new *Mathematics Curriculum Guide* was published. The guidelines of this document suggested that conceptual understanding and problem solving should be the focus of mathematics teaching and learning. Recent draft documents published by the Ministry of Education, based on recommendations made by the Sullivan Commission, suggest that at some levels, mathematics could be taught as part of an integrated curriculum via themes or interesting projects. At present, the Ministry of Education has solicited papers on the topic of making thinking the major focus of teaching. A recent publication by the National Council of Teachers of Mathematics, entitled *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) suggests a framework for teaching mathematics and articulates five general goals. According to this document, all students should

1. learn to value mathematics,
2. become confident in their ability to do mathematics,
3. become mathematical problem solvers,
4. learn to communicate mathematically,
5. learn to reason mathematically.

This paper is based on a presentation made at the Mathematics Council conference in Lethbridge in 1989.

These events and these publications indicate that changes in mathematics are in progress. The question that might be posed is, should all of these changes take place?

The reasons for saying yes to most of them are manifold. Let's look at a few possible reasons. Our views and our knowledge about how students learn mathematics and how they transfer knowledge have changed since our parents trained to become teachers. We all come from at least a two-calculator home. Mathematical expectations for new employees in industry have changed and are changing. A summary of these expectations by Pollak is listed in the *Curriculum Standards and Evaluation for School Mathematics,* and this list includes

1. the ability to set up problems,
2. knowledge of a variety of techniques,
3. understanding underlying mathematical features,
4. the ability to work with others,
5. the ability to see the applicability of mathematical ideas
6. preparation for open problem situations
7. belief in the utility and value of mathematics.

The key to attaining the expectations and the five general goals, especially the goal of becoming a problem solver, is the ability to understand the mathematics that is learned or the possession of conceptual understanding (Greenwood and Anderson 1983) or relational as opposed to instrumental, understanding (Skemp 1987). What types of classroom settings and what types of activities are conducive to the acquisition of conceptual understanding?

Greenwood and Anderson (1983), as part of their discussion of the operational and the conceptual domain, present a workable structure that suggests some very effective instructional approaches. They suggest an environment that "promotes, indeed provokes, communication by the student." They also suggest that teaching techniques "require a demonstration of conceptual understanding before expecting computational proficiency."

Post (1988) proposes that "the stereotypical model of students working alone at their desks needs to be expanded to include group discussions, project work and students working cooperatively rather than competitively. Students must talk about mathematical ideas and concepts."

Cobb (1985) agrees with Post and states that

> the most needed curricular innovation is . . . to encourage children to talk with each other and with the teacher about mathematics—their mathematics. Such interactions might sustain the belief that it is acceptable to think about mathematics and that mathematics involves understanding and the gaining of insights rather than finding ways to give the impression that one is behaving "appropriately."

An emphasis on student talk implies that teachers will have to be good listeners. Easley and Zwoyer coined the phrase "teaching by listening," and they conclude that

> If you can both listen to children and accept their answers, not as things to be judged right or wrong but as pieces of information which may reveal what the child is thinking, you will have taken a giant step toward becoming a master teacher rather than merely a disseminator of information. (1975)

Being a good listener means resisting what Kilpatrick calls "teacher lust" (de Groot 1988), or resisting the urge to control what students are doing and thinking. Kilpatrick states that mathematics teachers seem especially likely to be afflicted with teacher lust; after having asked a student to explain something, they often jump in, with scarcely a pause, to provide a clearer explanation themselves.

Are we as parents tempted to succumb to the same affliction as the mathematics teachers described by Kilpatrick? At times, are we tempted to share our wisdom with our children quickly, rather than to sit back and provide the opportunity to teach by listening and questioning?

After arriving home, a colleague was faced by his five-year-old son who, after looking at his digital watch announced, "Dad, it's 4:41. Twenty-one minutes to Scoobie Doo." After a lengthly explanation explanation by the father, which involved pointing out that from 41 to 50 is nine minutes and from 50 to 60 is ten minutes, so you'll have to wait nineteen minutes, the boy responded with, "But Dad, there are two minutes of news first!"

Results from various international studies have shown Japanese superiority in mathematics. I don't think we should ever contemplate copying anther system, but if some of the major reasons for this outcome are known, then perhaps a few of the effective teaching strategies could be kept in mind when we teach mathematics to our students. After observing between one and four mathematics lessons in 16 schools, the Illinois Council of Teacher of Mathematics delegation to Japan (1989) describes a typical mathematics lesson.

1. Students rise and bow	
2. Review previous day's problems or introduce problem solving topic	5 minutes
3. Understanding the problem	5 minutes
4. Problem solving by students, working in pairs or small groups (cooperative learning)	20-25 minutes
5. Comparing and discussing (students put proposed solutions on small or large blackboards)	10 minutes
6. Summing up by teacher	5 minutes
7. Exercises (2 to 4 problems only)	
8. Soft gong sounds to indicate the end of class. Students rise and bow.	

Stigler (1988), in his article about Japanese and American schools, states that the pace of instruction in Japanese classrooms is relaxed, and Japanese teachers constantly stop to discuss and explain. Both of these reports about mathematics teaching indicate the importance of talking in the classroom.

Class discussions are also important at higher levels of education, and my sessions with teachers-to-be are frequently interrupted with periods where students are given the opportunity to discuss points, conjectures or ideas in a cooperative setting. However, there is no truth to the rumor that I am trying to implement the first step of a typical Japanese mathematics lesson!

Allowing students the opportunity or more opportunities to talk is easily arranged, and teachers can use existing materials. Ask yourself what types of questions you could pose using existing computational exercises. What types of questions or instructions could contribute to the conceptual understanding of the ideas on that page of exercises? What types of questions and discussions could you use to increase time spent on developmental tasks and reduce the time students work on their own when little or no new mathematics is learned? What kind of questions could you pose that might lead students to gain new insights into some of the ideas presented?

Teachers in preservice and inservice sessions usually express the hope that by the time the training is completed or the meeting is over, they will have so many ideas and strategies at their disposal that the last thing that would ever come to mind is asking their students to complete more than three or four similar tasks on their own. I hope these teachers will have begun to think about using their ingenuity and creativity to generate ideas and activities for their students that will increase the time spent on developmental activities.

Types of tasks and settings would best be illustrated with excerpts from actual classroom settings. Brief excerpts from videotaped lessons were used during the actual presentation; simulated settings were also used. However, a little imagination will help you think of discussions that groups of two or three students might have after being asked the following questions:

1. Which of the items do you think is easiest (most difficult)?
2. Which of the items has an answer less than (greater than ______)?
3. Which items are similar or which two items do you think are similar?
4. Which items are different?
5. Which items are in some way the same yet will have answers that are different?
6. Which items are different but will yield answers that are similar?
7. The answer is ______ . Which items do not match this answer? Which item(s) could this answer belong to?
8. The estimate is ______ . Which item(s) could this estimate be for? Which items could/should be excluded?
9. Which items would have an "even" number (fractional number) for an answer?
10. Arrange the first five items in order of their estimated answers.

One major advantage of a cooperative setting is that students will have to discuss solutions and reach an agreement before responses are solicited and then compared. As different groups report, it will become obvious that there are different ways of thinking about a question or request and that different ways of looking at a set of items exist. Delaying a report by a group and having the members of the class guess the possible strategies behind the response, is likely to show that there may be different ways of thinking about the same results. Some unique or very creative thinking will surface in any classroom setting. These types of outcomes are valuable because they provide students with the opportunity to think and to think about their thinking. Students will also find out how others think.

Discussion opportunities can also be integrated with assignments or homework. As you picture a set of computational exercises, consider the following questions: What type of thinking skills/strategies are involved in responding to the given instructions or requests that follow? How might you adapt these questions or requests to make them suitable for the grade level you teach? Which tasks or combinations of tasks would you use with your students?

1. You do not have to do all of the items. Just solve those items that have an answer greater than (less than ______ ; greater than ______ and less than ______ .

2. Find the answers to two items that you think are "very easy" and for two items that you think are "not so easy." Give reasons for your choices. How would you explain how to find the answers for those "not so easy" items to someone in a lower grade? Write out your explanations. If possible, use diagrams.

3. Find the answers to two items you think are "very different."

4. Choose two items, and make up word problems that you could find the answers for. Make up a word problem for one item that you could not find the answer for. Why couldn't you find the answer? What would you need to know that would enable you to find the answer?

5. Design a multiple choice question for two items. As part of your choices, include two incorrect answers for each question. Provide your reasons for these choices. Why might someone, who "does not know as much as you do," select these choices?

6. After you have found the answers to four items, think of some mistakes younger students might make if they were asked to solve these tasks. What might they do wrong, and what would you teach them so they wouldn't make these mistakes?

7. Choose five items. Beside each item, list people (professions) who you think use these skills. Make up word problems for each example.

Let's turn our attention to the students and to our goal of having them acquire conceptual understanding of the mathematics they learn. This means that they should be able to talk about what they have learned in their own words. Students should be able to demonstrate their understanding with the assistance of a manipulative. They should also be able to pause and give meaning to their work at any time (Skemp 1987).

If we recognize our terminology, our phrases or our "chants" while listening to our students, we should become concerned. Davis (1986) speaks of "disaster studies" when he reports of the students' inability to talk about mathematics in their own words. He speculates that one reason for this is that teaching mathematics is treated like learning how to sing German lieder without possessing any real command of German as a language.

Some of us may remember the mathematics teacher turned singer/entertainer, Tom Lehrer. He took an algorithmic chant, the procedure for subtraction with regrouping, put it to piano music and wrote a song about the new math. Nightclub patrons loved it; it was mathematics made easy!

If every student is to reach the five general goals suggested by the NCTM standards, classrooms must allow for student discussion, foster self-confidence, lower or eliminate test anxiety, encourage students to take risks and give students an opportunity to think.

I hope the suggestions and ideas presented here will stimulate you to think about such classroom settings.

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Issues In the Mathematics Curriculum

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Some salient issues in the mathematics curriculum must be discussed and resolved. Each teacher and supervisor must take a position on vital issues, but what issues are relevant?

Inductive Versus Deductive Teaching

Pupils may attain significant concepts and generalizations through induction. Thus, with teacher guidance, students use discovery techniques to learn mathematics. Problems and questions are then identified by pupils. Pupils individually and in committees, using a variety of reference sources, secure necessary facts, concepts and generalizations to solve problematic situations. Pupils develop broad ideas or generalizations from specific understandings. The generalizations, supported by facts, are used to answer questions and solve problems.

Deductive teaching procedures are the opposite of inductive teaching strategies. With inductive teaching, the teacher explains a new process to pupils in a meaningful manner. Students then apply the knowledge. Communication exists as a one-way street from teacher to student. However, the pupil must attach meaning to what has been acquired in order to use this knowledge in individual situations.

Active Involvement Versus Passive Recipient

Educators who emphasize active student involvement in lessons and units believe individuals learn by doing. Thus, with teacher guidance, pupils identify and solve lifelike problems in mathematics. To become proficient in problem solving, a student needs to practice specific skills. Subskills in problem selection include, gathering data or information to answer the problem, developing a hypothesis or answer to the problem, testing the hypothesis and revising the hypothesis if necessary. The sequential steps in problem solving are flexible, not rigid.

The passive receiver may secure information (facts, concepts and generalizations) from the sender. The sender is usually the classroom teacher. Thus, content moves from the teacher to the student in explanation/lecture form. Individual differences among students must be taken into account. It is hoped that students will be able to apply what is received from the sender. In contrast, in active pupil involvement, the whole person (intellectual, emotional, social and physical) is involved in projects and activities to solve problems relevant to society. Thus, school and society become integrated entities in the mathematics curriculum.

Advocates of active pupil involvement in learning believe that

1. students are capable and interested in making curricular decisions,
2. students should arrange their own course content rather than follow a logical curriculum offered by adults, and
3. students must be involved in self-appraisal for evaluation techniques to be effective. Otherwise, adult means of appraising learning performance may not affect the student.

Measurably-Stated Versus General Objectives

How precisely should objectives for pupils be stated? The teacher may select learning activities that help students attain chosen objectives. Then, the teacher may measure if a pupil has achieved the stated goal. Successful students may then attempt to attain the next sequential objective. Unsuccessful students may require a new teaching strategy to achieve the previously unattained objective.

Instructional Management Systems (IMS), mastery learning, criterion referenced testing (CRT) and exit objectives are mathematical teaching procedures that are related to measurable objectives. In each of these plans of instruction, precise measurable objectives are used in teaching and learning situations. Advocates of measurable ends believe that teachers should possess a clear intent when teaching. Thus, teacher and pupils have clear and specific ideas about what students will learn.

The teacher can more effectively select learning activities if measurable rather than general objectives are used. Each experience is chosen on the basis of one criterion: Do the activities guide students to specific objectives? If the activity is too complex or not challenging enough, it should be omitted. The teacher may measure personal success in teaching by obtaining objective data to determine if pupils have or have not achieved the desired objectives. Furthermore, student progress may be communicated clearly and precisely to parents. Teachers should also obtain evidence to show that pupils are not achieving measurable objectives.

If pupils are not attaining measurable goals, the teacher receives feedback. The teacher may then need to select a different teaching strategy to help students attain their objectives.

The opposite of measurably stated objectives are

1. broad, general goals that provide some kind of direction in determining the kinds of students a teacher wishes to develop, and
2. evaluation procedures that lack precision in determining if pupils have or have not attained the desired ends.

Student-Centred Versus Society-Centred Curriculum

Should most of the objectives in teaching and learning be set by the pupils themselves, or should attainable goals for pupils be selected on the basis of what society needs and deems to be significant?

How might goals be chosen that reflect the personal interests and purposes of the pupil? First, students can decide which tasks to pursue and which to omit when interacting with learning centres in the school/classroom setting. An adequate number of tasks must be available at learning centres so that pupils may select, as well as omit, sequential experiences. Thus, students might select interesting tasks to pursue. Students may also perceive reasons for participating in ongoing activities.

The following teaching strategies also emphasize the personal interests and purposes of the students:

1. Individualized reading. Students select and read a library book about mathematics. The book must be interesting and suited to the students' reading level. Students may also choose to be evaluated in terms of word recognition techniques and comprehension skills. Thus, students may read a selection orally to the teacher. The teacher might then assist the student in appraising word recognition techniques. To indicate their comprehension, students may develop a mural, diorama, model or creative dramatic presentation to demonstrate what they have learned from the book.
2. The contract system. With teacher guidance, students may specify which mathematics activities they will complete within a particular period of time. The contractual agreement must be reasonable in terms of number of activities students must complete. The contract should also reflect students' enthusiasm and reasons for choosing specific mathematic activities.

To emphasize practical skills in the mathematics curriculum, teachers and supervisors must ascertain what life skills are necessary for students. Teachers might include the following suggestions:

1. Computing the total cost of goods and services purchased in any given situation.
2. Ascertaining the amount of change due in any transaction.
3. Writing cheques and keeping a balanced chequebook.
4. Knowing how to obtain loans to make satisfactory investments.
5. Possessing applicable concepts involving interest rates.
6. Realizing specific abilities involved in ordering materials from mail order companies.
7. Shopping intelligently for necessary goods and services used in the home setting.
8. Buying insurance for property and health in an effective manner.
9. Learning to live within budget requirements.
10. Completing job application forms and gaining knowledge about taxation forms.

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Rational Number Sense: Development and Assessment

James Vance

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The development of number sense is one of the important objectives of mathematics today. Leutzinger and Bertheau define number sense as "sound judgment about the approximate quantity represented by a number" (1989). Children who possess number sense can form a mental image of a number and see relationships between numbers. A strong sense of numbers is needed to be able to perform mental computations, estimate answers and judge the reasonableness of results obtained using an algorithm or a calculator. Having a positive feeling about numbers helps children believe that mathematics is meaningful and makes them confident that they can understand procedures and solve problems.

Rational Number Considerations

Helping children make sense of rational numbers expressed as fractions or decimals is a particularly challenging task for teachers. Current research indicates that many students lack an adequate quantitative basis for thinking about fractions and decimals. They tend to apply rules without understanding why they work, and they have little confidence that the answers they produce will be sensible (Suydam 1984; Hiebert 1987). Rational number concepts and symbols are more difficult to teach and to learn than corresponding whole number ideas and numerals in several respects.

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First, the notation and its referents are more complex. The fraction symbol a/b is used to describe several different situations: a part of a whole, a part of a group, a measurement, a point on the number line and an indicated division. An appreciation of the size of the number represented by a fraction requires students to view the three parts of the symbol as a single entity based on the relationship between the numerator and the denominator (Behr, Post and Wachsmuth 1986). Decimal notation is a way of writing fractions with denominators that are a power of ten and a logical extension to the right of the place value system for whole numbers. Kieren (1984) has pointed out that the latter interpretation does not necessarily provide an easy transition for students because dividing up (going from ones to tenths to hundredths) is different from grouping (going from ones to tens to hundreds).

Second, while whole numbers have a unique standard representation, rational numbers have many names. A rational number can be expressed as a fraction or a decimal, and within each coding system an infinite number of equivalent names can be generated. A nontrivial concept is that the properties and the size of a number remain the same regardless of how it is named.

Third, ordering is a more complicated and difficult process in the rational number system than in the set of whole numbers. Creative thinking, based on visualization and sound understanding, is needed to construct strategies for comparing pairs of fractions and/or decimals in a variety of situations. Standard procedures rely on writing equivalent forms with like denominators or on place value considerations. In addition, rational numbers are dense: between any two rational numbers is another rational numbers.

Unlike whole numbers, there is no number "next to" a given number, but sequences of numbers "closer and closer" to a number can be found.

Number Sense with Fractions and Decimals

Rational number sense has two related components, conceptual understanding and quantitative awareness and involves several abilities. Students display sense with rational numbers when they

1. represent numbers using words, models, diagrams and symbols and make connections among various representations;
2. give other names for numbers within and between the two notational systems and justify the procedures used to generate the equivalent forms;
3. describe the relative magnitude of numbers by comparing them to common benchmarks, giving simple estimates, ordering a set of numbers and finding a number between two numbers.

A Teaching Experiment

As part of a project designed to study the thinking strategies used by children as they construct rational number concepts, a teaching experiment was conducted in the fall of 1988 with six Grade 6 students. Three boys and three girls were taught 21 lessons and interviewed individually four times over a two-month period. The purpose of the instruction was to help the students develop basic concepts of rational numbers and acquire a quantitative feel for numbers expressed in fraction or decimal form.

The lessons included concrete, pictorial and symbolic representations for the numbers, equivalence and order relations and estimation. Operations were not taught, but story problems were used to investigate the possible transfer effects of the instruction. Fraction notation was introduced first; decimal numbers were then taught by building on the concepts, models and language (tenths, hundredths and thousandths) of fractions and by extending place value ideas. The importance of defining the unit and the notion of a variable unit were stressed throughout the experiment.

New ideas were presented as problems, and the students were challenged to use materials or extend their previous knowledge to devise solutions. Alternative methods were encouraged, discussed and evaluated. Students were sometimes asked to explain their answers and thought processes orally as well as in writing. During the interviews, the subjects were instructed to use concrete materials or diagrams to represent numbers or model relationships and to state and justify rules or thinking strategies used to obtain answers or make decisions.

A Number Sense Interview

Tasks and questions that can be used to assess a variety of aspects of conceptual understanding and quantitative awareness of rational numbers follow. The material was taken from transcripts of lessons, interviews and from students' work. Spoken, written and concrete/pictorial responses are included.

Task 1: Represent 6/10

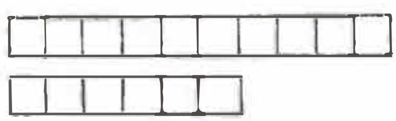
Interviewer: Show 6/10 using base-10 blocks, small cubes, long sticks, flats and cubes.

Student: If the long stick is one, then six small cubes would be 6/10.

Or if the flat is one, then I take six longs. If the cube is one, it would be six flats.

I: Can you show 6/10 using any of these strips? (Set of color-coded strips representing families of various fractions.)

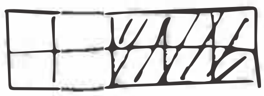
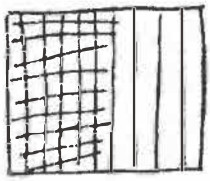
S: First I find the strip divided into 10 equal parts. Then I find a strip that has six parts the same size. It is 6/10 of the longer one.



This one is the same length, so it is 6/10 too, but it is also 3/5.

I: Make a sketch to show 6/10.

S: I divide a figure into 10 equal parts and shade any six.

I: If this is 6/10, sketch one.

S: You divide it into six equal parts and then add four more.

I: Show 6/10 on a number line.

S: You first mark the zero and the one. You divide that section into 10 equal parts, then you count over six from the left.

I: Write 6/10 as a number.

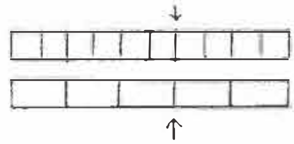
S: You can do it two ways.
6/10 and 0.6.

I: Why do these symbols mean 6/10?

S: For the fraction, the denominator indicates how many equal parts the whole is divided into, and the numerator indicates how many of the parts you take. For the decimal, the decimal points tells you that the zero is in the ones place. The place to the right must be tenths because it takes 10/10 to make one. So the six means 6/10.

I: Can you name 6/10 in another way?

S: 6/10 = 12/20 = 18/30
To get equivalent fractions, you multiply the numerator and denominator by the same number. You can also make it 3/5 by dividing the 6 and the 10 by two. 3/5 is in simplest form. You can see that 3/5 = 6/10 from the diagram. The strips show it too.

You can also make equivalent decimals by adding zeros after the 6.
0.6 = 0.60 = 0.600
6/10 and 60/100 are the same because they just both have 6/10. You can see this with the blocks too. A 10-block has ten-hundredths.

ones	tenths	hundredths
0	6	
0	6	0

0.1 = 0.10

I: Where would 18/30 go on the number line?

S: At the same point as 6/10. They are the same number, just different names.

Task 2: Model Decimals

I: Read this number, 1.3.

S: One and three-tenths.

I: Model it with these materials (long and small cubes only).

I: Can you read it or write it another way?

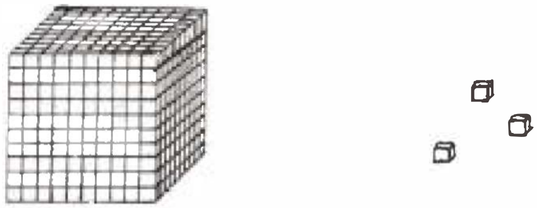
S: 13/10. As a fraction it is 1 3/10 and 13/10.

I: Read and model 1.03.

S: 1 and 3/100. You need the flat for the unit.

I: Read and model 1.003.

S: 1 and 3/1000. The cube is the unit.



I: Read and model 1.0003.

S: 1 and 3/10,000. To model you would need a unit that is ten cubes long. Or you could leave the cube as the unit and cut the small cube into 10 pieces to get ten-thousandths.

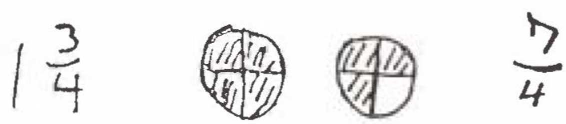
Task 3: Rename 1 3/4

I: Write your answers to the questions on this page.

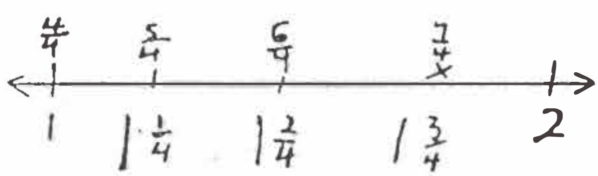
a. Write 1 3/4 as an improper fraction.

7/4

b. Draw a diagram to show that your answer is correct.



c. Mark a number line to show that your answer is correct.



Task 4: Order Numbers

I: Which is the greatest fraction?

1/7, 1/6, 1/8

S: 1/6. When the numerators are the same you go for the smallest denominator because there are less pieces so each piece is bigger.

I: 6/7, 8/9, 7/8

S: 8/9 is the biggest. It has the smallest piece left over.

I: 4/7, 3/8, 1/2

S: 4/7 is biggest because it's greater than a half, and 3/8 is smallest because it's less than half.

I: 2/3, 3/5

S: They are both over half and close together. They have different numerators and denominators. So you make equivalent fractions in fifteenths. 10/15 > 9/15.

I: 0.32, 0.302, 0.3

S: You look at the tenths place and if they're the same, you look at the hundredths place. 32/100 is the greatest and 3/10 is least. Or you can add a zero to 32/100, and add two zeros to 3/10. That one would be 3/100,000, and that one would be 320/1,000, and that would be 302/1,000.

I: Circle the least number, and write down how you decided.

S: 3.6, 3.58, 3.592.

I chose 3.58 for the least because it has 3 units, 5 tenths and 8 hundredths, and all the other ones have more tenths or hundredths.

I: 7/10, 0.68, 4/5.

S: You'd have to change 7/10 to a decimal and that's 68/100, so it would be 70/100, so that's the biggest one so far. 4/5 would be 80/100. 0.70, 0.68, 0.80.

Task 5: Between Two Numbers

I: Write a fraction between 3/5 and 4/5.

S: It could be 3 ½/5, but to name a fraction you make them 6/10 and 8/10, so 7/10 is between. You could also make them 30/50 and 40/50, and there are lots of numbers in between.

I: Write a decimal between 3.7 and 3.71.

S: 3.701. It's thousandths because that's 3 and 7/10 and that's 3 and 71/100. That's only one-hundredth more so you have to go to something smaller.

I: Write a number between 3/8 and 0.6.

S: 0.6 is the larger number because 3/5 is greater than 3/8. 3/7 is also between 3/5 and 3/8.

I: Write a number between 0 and 1/7.

S: 1/8.

I: Could you name another number even closer to zero?

S: 1/10, 1/100. You could go on forever.

Task 6: Estimate Sums

I: Estimate 1/3 + 0.327 + 4/12.

S: 1/3 + 1/3 + 1/3 = 1.

I: Estimate 0.243 + 15/31 + 1.019.

S: 1/4 + 1/2 + 1 = 1 3/4.

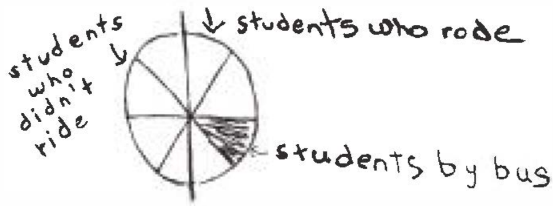
Task 7: Story Problems

I: Show how you arrived at the answers to the following questions.

Jack ate 1/2 of a cake and Mary ate 1/3 of the same cake. What fraction of the cake was eaten?

Answer: 5/6.

One-half of the students rode to school. One-quarter of the students who rode came by the bus. What fraction of all the students came by bus?



Answer: 1/8.

Conclusion

Students can build the mental imagery and understanding needed to make sense of rational numbers, but this awareness develops gradually and progress is often uneven. Obviously, all the students in this teaching experiment did not give all the above responses; answers often revealed lack of understanding or incorrect reasoning. Furthermore, concepts that were apparently well established one day would sometimes appear shaky when revisited at a later time or in a different context. Marked individual differences in the rate of learning, depth of understanding and strategy selection were also noted. Nevertheless, each of the six students made significant progress in acquiring knowledge about rational number size and relationships. Students also displayed increased confidence in their ability to learn and apply mathematics.

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Mathematics for a Magic Theme

Yvonne Pothier, Gail Brooks and Daiyo Sawada

Yvonne Pothier, a professor at Mount St. Vincent University, Halifax, Nova Scotia and Daiyo Sawada, a professor at the University of Alberta, Edmonton, are involved in a collaborative mathematics teaching project at Mount Carmel School in Edmonton, Alberta. Gail Brooks is one of the teachers participating in the project.

Parents and relatives were invited to celebrate Education Week 1989. They came to see class presentations dealing with the magic theme and to view students' creative work.

I, Yvonne Pothier, had been teaching the mathematics problem solving strand to the Grade 4 class as part of a collaborative teaching project. Because 3 by 3 magic squares had recently stimulated a great deal of interest during this unit, the idea of a magic square presentation seemed to be "just the thing." Gail Brooks, the teacher I was working with, agreed and so the short skit presented here was written and eventually performed by an enthusiastic group of students. We offer it to anyone who is interested.

Lesson 1

During the problem solving lessons devoted to magic squares, students were given a large rectangular piece of newsprint paper (50 cm x 30 cm) and were directed to "make the largest square they could by folding the paper." This task proved to be challenging to many students, and the successful ones took on the role of "teacher" for their classmates. When everyone had constructed a large square by making two folds, students were then told to fold their square to make 9 small squares of equal size within the large square. Most students were able to make 4 and then 16 small squares, but some found it impossible to get 9 squares. Again, with the help of classmates, everyone produced the required 3 x 3 square. Some students' 9 squares were scarcely recognizable because so many folds had been made during the process of experimentation; therefore, students were told to draw lines highlighting the "good" folds.

Lesson 2

In lesson two, the students were given a sheet of paper on which a 3 x 3 square was printed. They were reminded of the lessons in which they folded the large square to get 9 small squares, and then the figure on the paper was examined to identify rows, columns and diagonals. Students were then given 9 small squares cut from colored, gummed paper with the numerals from one to nine written on the squares, and asked to arrange them in a 3 x 3 square. The next step was to change the square into a magic square. A magic square was defined, and the students set about moving the colored squares in an attempt to place them in such way that each row, column and diagonal had a sum of 15. Most of the students discovered the solution by the time class ended, and they shared solutions and glued the colored squares onto the sheet of paper to form a magic square (see Figure 1).

Figure 1

A 3 x 3 Magic Square

1	2	3		8	1	6
4	5	6	⟶	3	5	7
7	8	9		4	9	2

The collaborative project has been made possible, in part, by a research grant from Mount St. Vincent University.

Lesson 3

During lesson three, students discussed their solutions, and we asked them what helped them find their solution and which number they tried to "figure out" first. One solution was written on the blackboard, and students discussed why five was the centre number. They also noted the constant difference between the two numbers on either side of the five and directly above and below it. Finally, students noted that the corner numbers were all even. Students discussed the possibility of constructing other magic squares and suggested using different sequences of nine numbers or doubling the first nine numbers. Within minutes, excited cries, such as "Doubling works!" and "Look! I've made one with a sum of 45" were heard.

In addition to presenting a skit, students agreed to make a magic square booklet in which each student constructed a 3 x 3 magic square and explained the number sequence selected. Some number sequences are presented in Figure 2.

Figure 2

Children's Sequences for 3 x 3 Magic Square

a. 2, 4, 6, 8, 10, 12, 14, 16, 18
b. 3, 6, 9, 12, 15, 18, 21, 24, 27
c. 7, 9, 11, 13, 15, 17, 19, 21, 23
d. 11, 12, 13, 14, 15, 16, 17, 18, 19
e. 31, 32, 33, 34, 35, 36, 37, 38, 39
f. 10, 20, 30, 40, 50, 60, 70, 80, 90
g. 8, 16, 24, 32, 40, 48, 56, 64, 72
h. 20, 40, 60, 80, 100, 120, 140, 160, 180

Lesson 4

A final lesson was devoted to "looking for sets of four numbers with a sum of 34 in a 4 x 4 magic square. Some of the students' solutions are presented in Figure 3.

The evening of the school celebration arrived, and the Grade 4s put on a great performance.

Figure 3

A Sample of Four Number Sets With a Sum of 34 in a 4 x 4 Magic Square

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

a. 3, 2, 10, 11
b. 5, 8, 7, 14
c. 3, 10, 13, 8
d. 2, 10, 7, 15
e. 11, 8, 7, 12
f. 5, 9, 8, 12
g. 2, 13, 4, 15
h. 16, 3, 14, 1

It's Mathematics Time—It's Mathemagic Time

A Skit About Magic Squares

Characters: Nine children holding number cards, two narrators, four pupils and one magician. To accommodate all pupils in a larger class, another group of nine children could demonstrate the second magic square, or narrators, pupils and magicians could be added.

Stage Set-Up: Nine children are on stage in a 3 x 3 square formation. Each holds a set of four cards: one card is blank, two have numbers from one to nine on them, and one card is a multiple of 2. The narrators stand on either side of the stage; the pupils and the magician stand at the side of the square.

Magician: With a sweep of the wand, the magician announces, "It's math-a-MAGIC time!"

Narrator 1: Our class wishes to demonstrate some mathematics magic. You see before you a square shape. It's called a 3 x 3 square because there are nine small squares within it.

Narrator 2: We see three rows, three columns and two diagonals. Here's one and here's another. (Magician uses wand to point to each row, column and diagonal in turn.)

N1: To make a magic square, we first number each small square using the numbers one to nine, like this . . . 1, 2, 3, 4, 5, 6, 7, 8, 9! (In sequence, children show cards with numerals on them.)

N2: Now for some magic! We want to move the numbers so that each row, column and diagonal has a sum of 15. (Magician indicates rows, columns and diagonals.)

N1: Let's begin. Which three numbers add up to 15? (Magician moves wand across each row in turn while the children show their third card.)
8 + 1 + 6 = 15
3 + 5 + 7 = 15
4 + 9 + 2 = 15

S1: What about the columns?

N2: Alright, you check them.

S1: 8 + 3 + 4 = 15
1 + 5 + 9 = 15
6 + 7 + 2 = 15

N2: Great! But there's one more thing to check.

S2: The diagonals!

N2: Alright, you do this.

S2: 8 + 5 + 2 = 15
6 + 5 + 4 = 15 Wow!

N1: Bravo! We have made a magic square! The rows add up to 15; the columns add up to 15, and the diagonals add up to 15.

S1: I wonder what would happen if we did something to the numbers.

S2: What do you mean?

S1: Well, we could double each number.

N2: A good idea! Let's try it. We double each number in row 1 (magician points, children change their number cards), double the numbers in row 2 and in row 3.

N1: Now let's check.

S1: 16 + 2 + 12 = 30
6 + 10 + 14 = 30
8 + 18 + 4 = 30 It works so far.

S2: Now the columns. 16 + 2 + 12 = 30
2 + 10 + 18 = 30, 12 + 14 + 4 = 30
Great!

S1: Don't forget the diagonals!

S2: 16 + 10 + 4 = 30, 12 + 10 + 8 = 30

N2: Voilà! Another magic square!

S1: I wonder if we would get another magic square if we multiplied each number by three, by four or by any other number?

S2: I wonder what would happen if we took the numbers in one magic square and added them to the corresponding numbers in another magic square? Would that make a new magic square?

S1: I wonder if we could make a magic square using other numbers like the numbers from 11 to 19, or from 21 to 29 or any other sequence?

S2: I wonder if it's possible to make a 4 x 4 magic square?

All on stage: That's some magic for YOU (pointing to the audience) to try!

Curtain.

The Ups and Downs of Elevator Probability

Sandra Pulver

Sandra Pulver is a professor of mathematics at Pace University, New York, N. Y.

An elevator paradox exists. Whenever I am in a building waiting for the elevator to go up, the elevators are always on their way down. Are the elevators manufactured on the roof and sent down to the basement to be stored? And when I am on an upper floor waiting to go down, the elevators are usually on their way up. Are the elevators being constructed in the basement and carried off the roof by helicopters?

Actually, the intuitive feelings here are correct, and the probabilities are not difficult to compute.

Suppose that an elevator is traveling up and down at constant speed and in continuous cycles in a 20-storey building. If I am on the 4th floor waiting to go up, I will have 3 floors below me and 16 above. Therefore, the probability is 16/19 that the elevator is above me and will be on its way down when it stops. However, if I am on the 17th floor waiting to go down, I will have 3 floors above me and 16 floors below me. Therefore, the probability is 16/19 that the elevator is on some floor below me and will be moving up when it stops!

The solution for two or more elevators is complicated by conditional probability, but Donald E. Knuth, a computer scientist at Stanford University, has made a discovery. As the number of elevators approaches infinity, the probability that the first elevator, going up or down, will stop on any floor, except the top or bottom floors, approaches exactly 1:2.

References

Knuth, Donald E. *Scientific American.* February 1973, 138-39.

Altering Salary Orderings: The Effect of Consecutive Allocations

David R. Duncan and Bonnie H. Litwiller

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A major task in today's schools is to help students acquire number and data sense. This is best done by composing and analyzing meaningful examples. Examples using money are particularly interesting to students and teachers.

Phyllis, an office supervisor, is partially responsible for setting the salaries of 10 subordinates. Phyllis's supervisor, Nan, wishes to play a part in this salary determination; however, Nan indicates that the primary responsibility rests with Phyllis. To accomplish this, Nan assigns $200,000 for Phyllis to divide among the 10 employees. Nan then reserves an additional $100,000 that she will allocate after Phyllis's task is completed.

Since two-thirds of the money is allocated by Phyllis and only one-third by Nan, one might conclude that Phyllis is making the primary determination of salary levels. If Phyllis and Nan are in basic agreement concerning salary levels, it is not important which of them makes the salary allocations. But, what if they disagree?

A matter of great concern to the 10 employees will likely be the ordering of their salaries. Each will be very interested in knowing whether his or her salary ranks near the top or the bottom of the list. What effect can Nan's $100,000 have on the ranking determined by Phyllis's $200,000?

Suppose that Phyllis determines salaries as in Table 1. Although the salaries are fairly close together, there is a clear ranking of the employees.

Now suppose that Nan allocates her $100,000 as in Table 2.

Table 1

Employee	Phyllis's Salary Allocations
A	$22,500
B	$22,000
C	$21,500
D	$21,000
E	$20,500
F	$19,500
G	$19,000
H	$18,500
I	$18,000
J	$17,500

Table 2

Employee	Nan's Salary Allocations
A	$0
B	$2,000
C	$4,000
D	$6,000
E	$8,000
F	$12,000
G	$14,000
H	$16,000
I	$18,000
J	$20,000

Table 3

Employee	Phyllis's Allocation	Nan's Allocation	Total Salary
A	$22,500	$0	$22,500
B	$22,000	$2,000	$24,000
C	$21,500	$4,000	$25,500
D	$21,000	$6,000	$27,000
E	$20,500	$8,000	$28,500
F	$19,500	$12,000	$31,500
G	$19,000	$14,000	$33,000
H	$18,500	$16,000	$34,500
I	$18,000	$18,000	$36,000
J	$17,500	$20,000	$37,500

Clearly, Nan evaluated the employees differently than Phyllis. Nan allocated only one-half the amount that Phyllis did. Will this smaller allocation have a large effect on the final salary?

Table 3 indicates the striking effect of Nan's smaller salary allocations.

Nan's allocations affected the salaries in the following ways:

1. The ordering of the total salaries is completely reversed from Phyllis's original allocations.
2. The difference between consecutive total salaries is actually larger than it was after Phyllis's original allocation. Not only was Nan able to reverse the order that Phyllis preferred but she also dramatically increased the salary "spread" in this reversed order.

Nan's smaller total had a much greater effect than did Phyllis's much larger amount. In practice, Phyllis used her money to establish minimal acceptable salaries with modest variations. Nan had the luxury of allocating money based on "merit" alone; consequently, Nan's judgments are more visible.

Do you know of any organization in which salaries are determined in this way?

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