Problem Solving: Some Points To Consider

Irvin K. Burbank

Irvin Burbank is a professor of elementary education at the University of Victoria, B.C., and senior author of Houghton Mifflin Mathematics (1–8).

Generally, educators agree that a major purpose (if not the goal) of school mathematics is to teach and encourage students to solve problems. Achieving success in problem solving continues to be a major challenge for teachers and students at all grade levels. To meet this challenge, one first needs to find satisfying answers to the following questions:

- What is problem solving?
- What is a problem?

George Poyla, a math educator who has devoted a lifetime to the study of problem solving, has provided its simplest description. He has defined it as finding a way out of difficulty, where no way is known offhand; or as finding a way around an obstacle; or as a way of attaining a desired end that is not attainable by obvious means. Because answers do not always come easily or quickly, problem solving can be frustrating for students, who then become impatient and try to avoid the activity.

In answering the first question, Poyla hints at the answer to the question: What is a problem? A problem must be defined in terms of the student. For a situation to constitute a problem, it must meet the following criteria:

1. The student must have a desire to solve the problem. If a student has no interest in finding the prime factors of 60, the situation is not a problem for that student. In addition to wanting to solve the problem, the student needs to feel that the problem is within his/her capability and be willing to attack the problem.
2. The situation must be new or unfamiliar, with no immediate, obvious or apparent solution. If a student has encountered this kind of situation before and resolved it, it may not present a problem now.
3. The situation must involve an initial state and a goal state with obstacles in between that present a challenge to be overcome.

Objective: To logically bridge the gap from point A to point B

The following diagrams illustrate what often occurs in the process of solving problems.

Diagram 1 shows the student starting at point A (given state) and endeavoring to get to point B (goal state). This is a new situation the student has not encountered before. He/she gathers his/her resources and ventures out in an attempt to reach point B. The path followed may lead in a direction away from point B (path I).

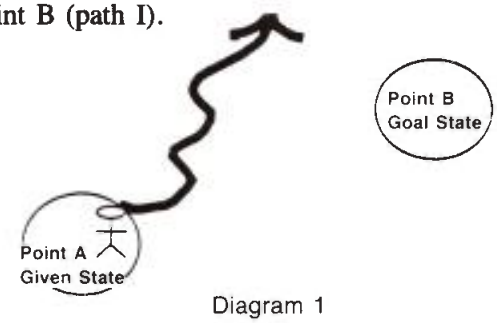


Diagram 1

Realizing he/she is not progressing, the student backtracks to point A, regroups, gains more information and starts off on another path in the attempt to reach point B. However, this time the selected paths may prove to be dead ends, as illustrated by paths II and III in Diagram 2.

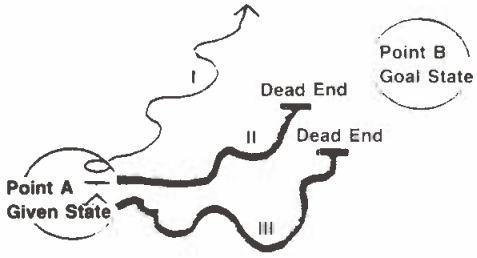


Diagram 2

Once again the student backtracks, regroups, redefines and gets more information before starting off on another path. It may take a long time, but he/she eventually reaches the goal, as illustrated by the winding path IV, in Diagram 3.

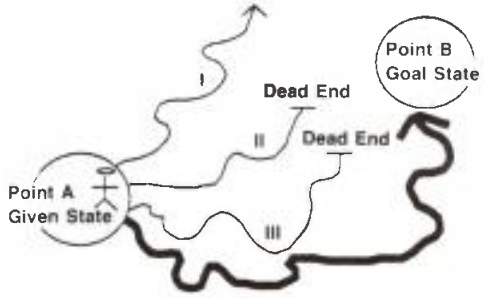


Diagram 3

Once a student arrives at point B, it is often possible to look back and see bridges and shortcuts. For example, he/she may see how paths III and IV can be bridged, and likewise where to build a bridge between II and III, as illustrated in Diagram 4.

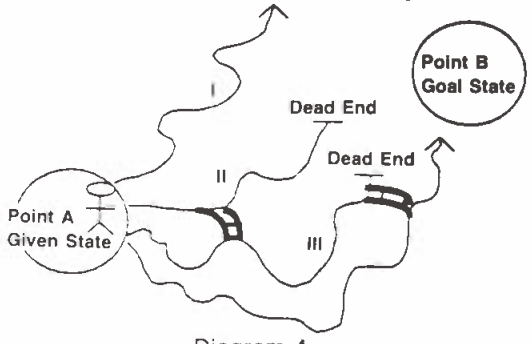


Diagram 4

The next time the student is faced with this type of problem, he/she will start on path II, cross over to path III, cross the bridge to path IV, and then proceed to point B, instead of taking the original long path IV, as shown in Diagram 5.

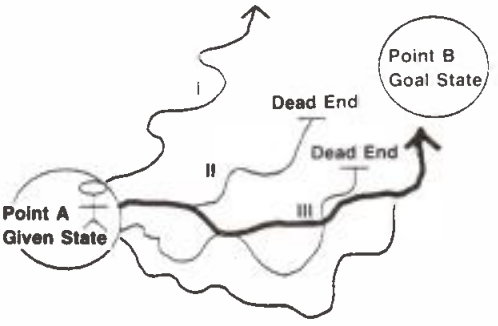


Diagram 5

The student uses the new, shorter path over and over again when solving similar problems. Interestingly, the task is no longer a problem: it has become an exercise.

So much for attempts to answer the questions: What is problem solving? and What is a problem? However, other factors also are involved in the development of problem solving skills. In the rest of this article I will discuss some of them.

Factor 1

Often teachers do not have a clear understanding of the nature of problem solving, and thus may give the students a false impression of what it entails. For example, a student is asked to go to the chalkboard to demonstrate his/her solution to a problem. The student fills up two boards with diagrams, steps and calculations before arriving at the solution. The teacher steps forward, impatiently takes the chalk and quickly solves the problem in four short steps. The student replies, "But sir, you asked to see *my* solution to the problem, you did not ask me to show your solution to the problem. If I could've solved it in four steps, I would've solved it in four steps, but this is the first time I have solved this problem."

In this example, the student was correct. He/she was engaged in problem solving: the teacher was doing a four-step exercise. If the truth be known, it is highly likely the teacher did not solve the problem in four steps the first time he/she solved it. Thus we can conclude that true problem solving takes time and that good problem solvers must put up with frustration as they strive to move from the given state (point A) to the goal state (point B). If the teacher conveys to the student that problem solving is a quick,

direct progression from point A to point B, he/she misleads the student about the nature of problem solving. For example, the teacher has the following routine:

1. Reads the question
2. Writes the formula or equation
3. Gets the answer

The student is led to believe this is the way problems are solved. He/she tries this routine:

1. Reads the question
2. No formula pops into his/her head
3. Reads the question
4. No equation comes to mind
5. Reads the question
6. . . .

The student concludes, "I can't solve problems!" and is correct, because no one solves problems this way: these are the steps of an exercise, not strategies for solving a problem.

Factor 2

Intimidation is a major factor hindering the development of problem solving skills in the classroom. Unfortunately, it is often quite unintentional. For example, the teacher who solves a problem in 4 steps and 10 seconds flat often does not realize how this intimidates a child who has just struggled through many steps to get the solution.

Suppose a teacher prefaces the problem solving with a statement such as, "Here is an easy problem for you, Walter." This is a devastating remark to make to Walter because it implies that he could not possibly solve an ordinary problem. If Walter can solve the "easy" problem then he gains no satisfaction from it. If he cannot solve it, then that confirms for him that he is a dunce. It would have been better to have said, "Here is a tough (or challenging) problem for you." If Walter can then solve it, he will feel a lot of satisfaction; if he cannot, he can conclude, without loss of confidence, that he could not solve it because it was too tough, but that he is still learning, and one day he may be able to solve it.

Factor 3

Students often witness the teacher and other students doing mathematical exercises, but they seldom see the teacher in the process of solving problems. They do not witness what the teacher does when the attempt does not seem to be leading to a solution, or when the attempt reaches a dead end. A child cannot learn to kick, catch or throw a ball by engaging in worksheet activity, only by observing others practising these skills. Hence it is not surprising that students have difficulty solving problems if they seldom see others doing it and are only exposed to problem solving through worksheets.

Factor 4

If students' exposure to problem solving comes only in the form of a worksheet, then they are not involved in the critical problem solving activities of asking questions and vocalizing the terms related to the problem. If students ask: Why? How come? How much? How many? then they are at the first and crucial stage in problem solving. The importance of asking questions and vocalizing is evidenced by the fact that often by the time a student has said the last word, the answer pops into his/her mind. The reason may be that this was the first time the student had verbalized the math terms or phrases.

Factor 5

If the teacher is not keen on problem solving, he/she will probably find it hard to motivate the students to develop the skills and strategies necessary for solving problems. Unfortunately, many teachers share their students' fear and dislike of problem solving.

Factor 6

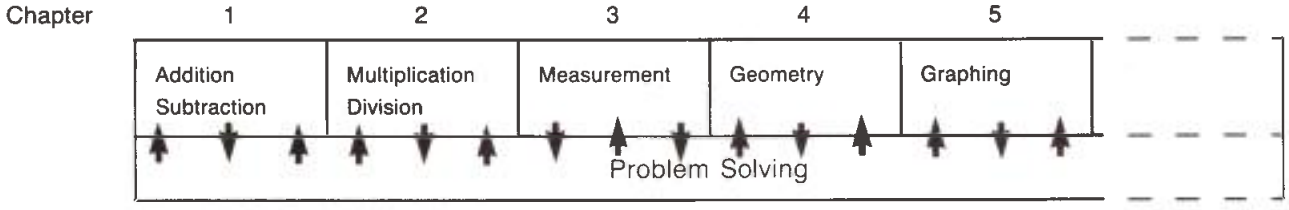
This last factor has to do with how problem solving fits into the elementary math curriculum. Some of the old math programs placed problem solving in the last chapter of the textbook. For example:

Chapter	1	2	3	4	5	6
	Addition Subtraction	Multiplication Division	Measurement	Geometry	Graphing	Problem Solving

Placing problem solving in the last chapter often meant the class never studied it. Other programs placed problem solving at the end of each chapter, a slight improvement over the first format:

Chapter 1		2		3		4		5	
Addition Subtraction	Problem Solving	Multiplica-ment Division	Problem Solving	Measure-ment	Problem Solving	Geometry	Problem Solving	Graphing	Problem Solving

However, it is generally agreed that problem solving should be interwoven throughout the math program, as follows:



In this type of program the introduction to a lesson involves problem solving; teaching the lesson involves problem solving; and practice, reinforcement and enrichment activities all use various elements of problem solving.

After studying problem solving and some of the factors to be considered in promoting it, teachers will, I hope, conclude that problem solving has a dual personality: it is the goal of mathematics education and the means of achieving the goal. It is the product and the process. Moreover, carrying out problem solving is the best method for teaching problem solving.