Logo and Problem Posing

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> Every tool carries with it the spirit by which it has been created.
>
> (Heisenberg 1958, 27)

Educators continue to struggle with Logo. Where does it fit in? I would like to quote from two books about Logo, to set the tone, and then provide examples of problem posing taken from different domains.

> This is a book about exploring mathematics . . .
>
> (Abelson & diSessa 1981, xix)

> You probably won't read the book cover to cover; instead you'll get excited by a particular project and spend a month or two exploring it. . . . You're learning the discipline of serious thinking and of taking pride in your work. . . . You're supposed to be interested enough already to explore on your own. . . . I think it's better to encourage your creativity by letting you invent your own exercises.
>
> (Harvey 1985, x-xiv)

Problem solving was the agenda for action in the 1980s. I hope that problem posing will be on the agenda for the 1990s.

Educators are beginning to realize that a sub-skill approach is relatively ineffective in teaching students to read or write. Mathematics educators have yet to realize that this applies also to mathematics education. A significant proportion of the present mathematics curriculum is predicated on the assumption that practice in computational and algebraic skills will result in an understanding of mathematical ideas, despite substantial evidence that this is incorrect. We have adopted the maxim that if something isn't working, then more of it will. Mathematics has yet to adopt an integrated approach in the sense that the underlying purpose is comprehension and understanding of relationships and ideas.

Even the goal of fostering problem solving has degenerated into rote applicaton of tried-and-true procedures to type problems. The spirit of problem solving has disappeared. How many different ways can you approach a problem? What are the features of each? Where do these approaches come from? The writings of Brown and Walter (1983) and Mason (1985) exemplify such an approach, as do many books on Logo (Abelson & diSessa 1981; Harvey 1985; Clayson 1988).

Playing with "Forward"

FD 50. This may be the first phrase that a novice turtle trainer speaks. The result is straightforward—a line about an inch long. For many, this is also the end of **FD**. They quickly move on to other commands, failing to take advantage of an opportunity to explore. How does one explore a command? How does one explore an idea like addition? Let me leave the latter question, not because I don't have suggestions about it, but because I shouldn't deprive you of the chance to have your own ideas first. However, I would now like to explore the possibilities of **FORWARD**.

We can approach the task from many angles. Let's begin with syntax. A few thoughts come to mind. One pertains to abbreviations. **FD 50** works. Are other abbreviations possible? What about the first two letters? This is okay for California, but not for Montana! We might try **FO 50**. Another possibility is **FWD 50**. I have seen young children try **DF 50**. And what about upper and lower case? **Fd 50** seems promising. And fd **50**? Will **fD 50** generate a line or an error message? Then there is the space between the command and the number. Let's experiment systematically: start with zero spaces, then try one space, two spaces, and so on. Is there an upper limit? In pursuing such questions, the student becomes familiar with the command, its nuances, when it behaves and when it does not. Now, what happens if the number precedes the command, as in **50 FD**?

Now let us focus on the number following the command. What kinds of values work here? **FD 50** works, but what happens when you type **FD 100**? The line is longer. It appears to be about twice as long. How could you be sure? Will **FD 1000** work, or **FD 1000000**? Can you put commas in the number as we do when we write **1,000,000**? Is there a maximum number that is valid for **FD**? Do the answers to any of these questions depend on where the turtle is on the screen? Or on how long the computer has been responding to Logo commands? Besides using a comma, what about a period? More than one period? Thinking in terms of the number line, we have been playing numbers to the right, larger and larger numbers. What happens when we think of the left—negative numbers? Exploring with negative numbers can be interesting in itself. The real number aficionados may have noted that we jumped over another interesting number: zero. **FD 0** should do nothing, shouldn't it? Does it do nothing? There are also zero's near neighbors—the very small numbers. Is there a minimum number? Do you have some other favorites? Some people like pi.

We can extend our questions about the numbers that follow the **FD**. For example, can there be more than one number to the right of the command? Try **FD 2 + 2**, or **FD 2 - 1**. We can explore the use of parentheses. With some experience with Logo, we may use variables or have the expression to the right of the **FD** a procedure that acts as an operation or a command. There is a popular saying to the effect that if we say a word 10 times it is ours. What should we say about Logo primitives?

Playing with Lists

Many people introduced to Logo remain within the subset of primitives associated with turtle geometry. However, Logo, in a fundamental sense, is an example of a list-processing language. Clarification of some of the concepts that are central to list processing can lead to a deeper appreciation of the overall structure of the language. Concepts such as word, sentence and list form the heart of the language. Logo primitives like **FIRST, BUTFIRST, LAST** and **BUTLAST** provide an introduction to the types of operations that are possible. Character strings may consist of letters, numbers, special characters and spaces. In a very real sense, anything goes. The task facing the student is to bring meaning to this situation. Simple receptive learning, where the student is told, or reads, about the distinctions is usually not sufficient. One needs some practice with the possibilities. In order to delimit the problem, at least initially, one may wish to focus on strings that begin with

- a letter
- a "
- a [and end with a]
- a :

as each of these cases represents a special situation in the language.

FIRST TURTLE	
I don't know how to TURTLE	doesn't work
FIRST "TURTLE	try a double-quoted string
You don't say what to do with T	it works, but acts like an operation
PRINT FIRST "TURTLE	
T	okay
PRINT FIRST "TURTLE TRIP	try two words
T	
I don't know how to TRIP	nope
PRINT FIRST "TURTLE "TRIP	try two words again
T	
You don't say what to do with TRIP	still nope
PRINT FIRST "123	try numbers
1	okay

PRINT FIRST 123 1	try again wow!
PRINT BUTFIRST "TURTLE URTLE	try another command okay
PRINT FIRST FIRST "TURTLE T	try combining primitives okay, but I was expecting a U
PRINT FIRST BUTFIRST "TURTLE U	try again now I have my U
PRINT LAST BUTLAST "TURTLE L	can I get an L? yes

One of the problems with an extended protocol like this is that it only reflects the sequence of mental experiments by the author. It represents a two-minute sequence of conjectures. Within half-an-hour or so it should be possible to make substantial progress on how these primitives work together.

Pedagogical questions arise now. For example, (1) is this an efficient way to learn these ideas about list processing, and (2) can students actually do this? Regarding the first question I suggest that a small fact sheet be given to students to help focus the inquiry:

Types of Data

• words	
- unadorned	box
- quoted	"box
- dotted	:box
• numbers	423
• lists	[box]

Logo Primitives

FIRST
BUTFIRST
LAST
BUTLAST

With respect to the second question, I'm not sure. However, I would like to see education move toward a goal where the answer is a clear yes. One difficulty at the moment is that many students have had little experience with such activities—no wonder they feel a little lost at the beginning. The same applies at the teaching and learning levels for most teachers. However, we are now touching on what I see as the real educational value of Logo. It is not the syntactic rules, nor even how to program, but rather the sense to which it focuses debate on the nature of deep understanding and intelligent behavior. In this context, intelligence is viewed as an ability to adapt to new situations, not as a score on an IQ test.

Playing with Time

A number of people have suggested that a slower turtle might be advantageous when young children are playing with Logo. The question can be a springboard for a substantial research agenda. However, I would like to play with a different board. Terms like "slow" and "fast" give rise to other ideas like speed and distance and time. Is it possible to play with Logo-time? What could a term like Logo-time even mean? We seem to have adapted to the notion of a turtle-step. What about a turtle-second?

Here are three ways to move a turtle 200 units.

FD 200

REPEAT 200 [FD 1]

REPEAT 100 [FD 2 WAIT 5]

Each approach is noticeably slower than the preceding one. Now what?

A tentative definition of a turtle-second is the time it takes a turtle to move forward 1 turtle-step. A second approach is to define it in terms of the **WAIT** primitive. Once a particular approach is adopted the questions begin to flow.

How would one create time-conversion procedures so one could move between systems? Analogies to fast-forward and slow-motion come to mind.

If turtles are walking in Europe (metric system) or the United States (imperial system) or Canada (both systems), how can one compare stories of their journeys?

Imagine a turtle walking along the perimeter of a right-angled triangle. Could one prove Pythagoras' theorem using time? Similarly, what about the relation of the radius of a circle to its circumference?

Consider problems of falling bodies. If one threw a barometer off the top of a building, how could one determine the height of the building?

Consider distance problems involving two vehicles. For example, if John (a turtle) leaves A, travelling east at . . . ?

Then there are races, such as those of the tortoise and the hare, Mario Andretti, the America's Cup; hockey—passing a puck from one turtle to another; projectile problems; relativity theory; instant Logo.

This is back where we started.

Playing with Recursion

Books have been written on this topic. Obviously, there is more here than meets the eye. But what? How does one explore the idea(s) of recursion? New branches of mathematics are evolving—witness fractal geometry and chaos theory. Conway's Game of Life and its variants are also relevant (Poundstone, 1985). Harvey (1985) has four chapter fives exploring the idea from different perspectives. Abelson (1982) provided a classic example distinguishing tail recursion from the more general notion. The idea of recursion can be considered within a graphic setting (e.g., fractals), a numeric setting (e.g., Fibonacci numbers), or an alphabetic setting (e.g., palindromes).

Let's begin with a simple example of a program that calls itself.

```
TO BOX
  BACK 50
  LEFT 90
  BOX
END
```

How might we (a) modify, and (b) extend this program? Suppose we have:

```
TO BOX
  BACK 50
  BOX
  LEFT 90
END
```

The first procedure draws an (endless) box, whereas the second does nothing of the kind.

A simple extension is to examine programs that utilize inputs.

```
TO BOX :SIDE
  IF :SIDE > 100 [STOP]
  BACK :SIDE
  LEFT 90
  BOX :SIDE + 2
END
```

Playing with negative numbers can also be interesting:

```
TO BOX :SIDE
  IF :SIDE < -100 [STOP]
  BACK :SIDE
  LEFT 90
  BOX :SIDE - 2
END
```

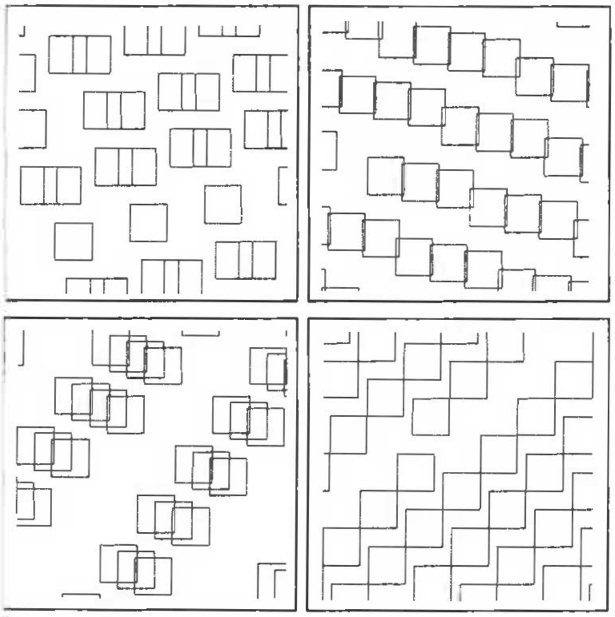
Abelson and diSessa (1981) provide a provoking example right at the beginning of their book when they experiment with different values for spiral procedures (pp. 17-20). A simple three-line procedure can provide hours of thoughtful exploration. But who is to say how to explore?

Playing with Patterns

One can make a career of playing with patterns. Mathematicians tend to think of patterns in terms of the shapes that are to be combined. Topics like tessellation and symmetry arise. Artists tend to focus more on the space between the shapes. People such as M.C. Escher bridge both approaches.

I would like to begin with the simple square. It is a relatively easy task to write a Logo procedure that tessellates these squares to form a perfect checkerboard. While tessellations are pleasing in the sense that they come out right, they tend to bore most people. Clayson (1988) has shown that Logo can be used as a vehicle for visual thinking. He approaches squares in a less inhibited manner, splashing them on the screen to see if he can produce some mathematically messy but aesthetically pleasing patterns. He admonishes us to "get in the habit of tinkering" (p. 5). See the page opposite for four examples.

It is tempting to imagine what could be done next with a powerful color facility, on the screen and on paper. We have just begun to play. After squares come other shapes—I suspect triangles would be particularly interesting. Burnett (1985) played with Islamic art patterns, which turned out to be a fascinating trip to other regions of the mind. One version of Logo permits the drawing of three-dimensional



figures. If this could be combined with animation and color, the resulting product would be dazzling to contemplate. And some people think Logo is only for kids! Others complain, "What do you do after you have drawn a house?"

Perhaps the difficulties lie with our conceptions of the curriculum (Burnett 1988). A curriculum that has trouble finding a place for imagination and extensive thought is a curriculum in trouble.

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