# Variations on a Theme

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Problem solving is a thinking process, and both critical and creative thinking can be involved. Critical thinking involves analysis of conditions and often depends on the mastery of vocabulary, the technical words and symbols of mathematics. Creative thinking occurs when students develop problems or are given an opportunity to describe conditions.

## Altering a Problem

A problem is altered in at least three different ways:

1. The conditions of the problem are changed.
2. The variable or variables are changed.
3. The information sought is changed.

Consider the following example:

### Basic Problem

1. I'm thinking of a number (a counting number).
2. The number is less than 50 ($<50$).
3. It's also greater than 20 ($>20$). ($20<n<50$)
4. The number is odd ($2n + 1$).
5. The number is a multiple of 5. ($5x = n$) (5 is a factor.)
6. The sum of its two digits is 8. (7 and 9 are other alternatives.)

Too often, once the solution to a particular problem has been found, the problem is forgotten. In reality, the stage for effective problem-solving process has been set. There is a difference between solving a problem and teaching the problem-solving process. An analysis of possible process follows.

## Analyzing the Problem-Solving Process

### Condition I

Grade 3 students will attempt to guess the number the teacher has in mind. Instead of moving to the second condition, explore such questions as:

You guessed 33. Why did you choose that number?

How many numbers are there?

Do you think you could ever guess the number I have chosen?

What other information do you need?

### Condition II ($<50$)

The choice of 50 is completely arbitrary. The condition of the upper limit may be varied.

How many ways may the number 50 be stated? Each poses the upper limit in a different mathematical form.

What does "less than" or its symbol mean to you? Developing synonyms in the students' language broadens and refines the meaning of the word and the ideograph $<$.

How many counting numbers are there that are less than 50? The usual answer is 50.

Could you trap me now?

What would you do to trap me?

### Condition III ($n>20$, $20<n$)

Again, the definition of 20 or any other number the teacher chooses may be varied. Synonyms should be developed and the range defined in words or in symbols ($20<n<50$).

What would you do to find my number?

Where would you start?

Why?

### Condition IV (Odd Number)

What does odd mean?

Can you list?

Before listing, determine how many odd numbers there are between 20 and 50 ($20<\text{odd}<50$)? Some

questions noted in previous conditions may be appropriate. (At this point, allow students to develop final conditions for a number of their choice.)

### Condition V (Multiple of 5)

How many odd numbers that are multiples of 5 are there between 20 and 50?

### Condition VI (Final Clue)

The final clue the teacher chooses will depend on the grade and the mathematics ability of the students in the class. If the number chosen was 35 (the other possible answers are 25 and 45), the following types of clues would be appropriate:

The number is between 34 and 36.

The number is also a multiple of 7.

Seven is a factor of the number.

The number is one less than a perfect square.

The difference between the two digits is 2.

The number is the difference of the squares of two numbers.

The number is the product of 8 and 4 increased by 3.

Condition V may be altered. Consider the following:

Clue 1: The number is prime (e.g., 41).

Clue 2: The number is the sum of two consecutive numbers, and the first is the product of 4 and 5.

Clue 3: The number is the sum of two consecutive counting numbers.

Change Condition VI by stating the numbers.

Ask the students to provide the final clue.

## Summary

Students' creative thinking may be fostered by having students develop similar problems. The problem-solving process is consistent with the Alberta curriculum which focuses on thinking, continuity and integration. Additionally, it is challenging and fun for the students and the teacher.