# Motion Geometry with the Apple IIe

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The computer can be used effectively as an environment in which mathematics may be discussed and learned; however, the key to such a use of the computer is the teacher's or programmer's ability to pose problems that challenge students' conceptualizations of mathematical ideas. The teacher-made program presented below is an attempt to construct a computer-learning environment to facilitate the instruction of transformations.

In this application of the computer, transformations are represented as the manipulation of a two-dimensional shape within a Cartesian coordinate system. The program functions by accepting as input the vertices of a closed figure, and these vertices are translated, reflected and rotated. For example, figure ABCD can be translated upward 12 units simply by adding 12 to the ordinate of each vertex of the figure (Figure 1). Thus point A (-5,-5) would translate to point A' (-5,7). Likewise, this constant value (12) would be added to each of the ordinates of the points B, C and D.

It may be worth pausing at this point to consider the following question. What manipulation(s) should be performed on either the abscissa or ordinate (or both) to achieve the following translations:

- Translate figure ABCD downward 2 units *(subtract 2 from ordinates)?*

- Translate figure ABCD 6 units to the right *(add 6 to abscissas)?*

- Translate figure ABCD 1 unit left and 2 units up *(decrease abscissas by 1, increase ordinates by 2)?*

- Reflect figure ABCD about the x-axis *(multiply ordinates by -1)?*

- Reflect figure ABCD about the y-axis *(multiply abscissas by -1)?*

Figure 1. A Basic Figure Translated Upward 12 Units
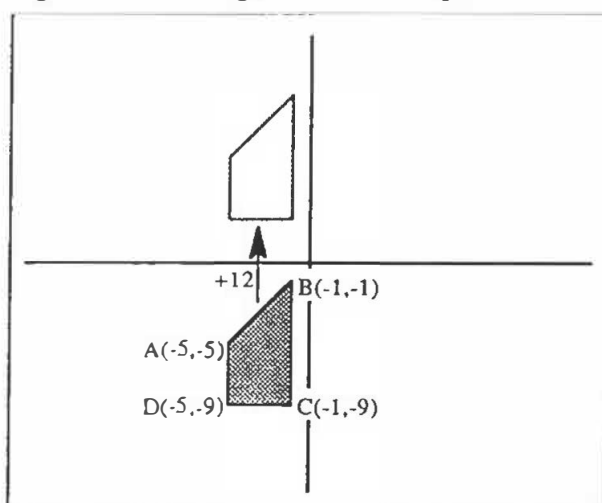
It is important that questions like these are presented to, and discussed with, students at some point during this exercise. Such discussions encourage generalization of the mathematical processes carried out by the computer.

Getting Started

Motion Geometry is written in Apple BASIC and is intended for the familiar Apple IIe computer. The program is listed in Figure 2 and must be typed in to begin the activity. Once the program is entered into the computer, be sure to save it so that you will not have to type it again the next time you want to use it.

To begin the program, you simply type RUN and press return. This will clear the screen and prepare you to input your vertices. The x- and y-axes of a Cartesian coordinate system are drawn at the top of the screen, and instructions are given at the bottom of the screen.

Figure 2. Motion Geometry Program

```
10   REM   WRITTEN BY A. CRAIG LOEWEN
20   REM   NOVEMBER 17, 1986
30   REM   MOTION GEOMETRY
40   DIM A(20,2),B(20,2)
50   HGR
60   HCOLOR= 2
70   HOME
80   HPLOT 140,0 TO 140,160
90   HPLOT 0,80 TO 279,80
100  HCOLOR= 6
105  FOR X = 1 TO 20:A(X,1) = 0:A(X,2) = 0:B(X,1) = 0:B(X,2) = 0: NEXT
110  VTAB 21
120  PRINT "INPUT CORNER POINTS IN CLOCKWISE ORDER":K = 2
130  INPUT "X,Y: ":R,S
140 A(1,1) = R:A(1,2) = S
145  HPLOT R * 6 + 140,S * ( - 4) + 80
150  INPUT "X,Y: ":A(K,1),A(K,2)
160  HPLOT  TO A(K,1) * 6 + 140,A(K,2) * ( - 4) + 80
170  IF A(K,1) = R AND A(K,2) = S THEN 200
180  K = K + 1: IF K > 20 THEN K = 20
190  GOTO 150
200  FOR X = 1 TO K:B(X,1) = A(X,1):B(X,2) = A(X,2): NEXT X
205  HOME
210  VTAB 21: PRINT "OPTIONS: ";: INVERSE : PRINT "Q";: NORMAL : PRINT "
     UIT ":
211  INVERSE : PRINT "N";: NORMAL : PRINT "EW ";
212  INVERSE : PRINT "C";: NORMAL : PRINT "ONTINUE"
220  GET A$: IF A$ = "" THEN 220
230  IF A$ = "Q" THEN  TEXT : HOME : END
240  IF A$ = "N" THEN  RUN
250  IF A$ <  > "C" THEN  PRINT  CHR$ (7);: GOTO 220
270  VTAB 21: PRINT "OPTIONS: ADD";: INVERSE : PRINT "X";: NORMAL : PRINT
     " ADD":
271  INVERSE : PRINT "Y";: NORMAL : PRINT " S";
272  INVERSE : PRINT "I";: NORMAL : PRINT "GNX SI":
273  INVERSE : PRINT "G";: NORMAL : PRINT "NY ";
274  INVERSE : PRINT "S";: NORMAL : PRINT "WITCH"
280  PRINT "             ";: INVERSE : PRINT "D";: NORMAL : PRINT "RAW ":
281  INVERSE : PRINT "Q";: NORMAL : PRINT "UIT ";
282  INVERSE : PRINT "C";: NORMAL : PRINT "LEAR ";
283  INVERSE : PRINT "N";: NORMAL : PRINT "EW"
285  GET A$: IF A$ = "" THEN 285
290  IF A$ <  > "X" THEN 340
300  VTAB 23: HTAB 1
310  INPUT "CONSTANT VALUE: ":V
315  VTAB 23: HTAB 1: PRINT "                              ";: VTAB
     23: HTAB 1
```

```
320   FOR X = 1 TO K:B(X,1) = B(X,1) + V: NEXT X
330   GOTO 285
340   IF A$ < > "Y" THEN 390
350   VTAB 23: HTAB 1
360   INPUT "CONSTANT VALUE: ";V
370   FOR X = 1 TO K:B(X,2) = B(X,2) + V: NEXT X
375   VTAB 23: HTAB 1: PRINT "                                    ":: VTAB
      23: HTAB 1
380   GOTO 285
390   IF A$ < > "C" THEN 480
400   HGR : HCOLOR= 2
410   HPLOT 140,0 TO 140,160
420   HPLOT 0,80 TO 279,80
430   HCOLOR= 6
440   HPLOT A(1,1) * 6 + 140,A(1,2) * ( - 4) + 80
450   FOR X = 2 TO K
460   HPLOT  TO A(X,1) * 6 + 140,A(X,2) * ( - 4) + 80: NEXT X
465   FOR X = 1 TO K:B(X,1) = A(X,1):B(X,2) = A(X,2): NEXT X
470   GOTO 285
480   IF A$ < > "D" THEN 530
490   HPLOT B(1,1) * 6 + 140,B(1,2) * ( - 4) + 80
500   FOR X = 2 TO K
510   HPLOT  TO B(X,1) * 6 + 140,B(X,2) * ( - 4) + 80: NEXT X
520   GOTO 285
530   IF A$ < > "N" THEN 560
540   RUN
560   IF A$ = "Q" THEN  TEXT : HOME : END
570   IF A$ < > "I" THEN 620
580   FOR X = 1 TO K
590 B(X,1) = B(X,1) * ( - 1)
600   NEXT X
610   GOTO 285
620   IF A$ < > "G" THEN 670
630   FOR X = 1 TO K
640 B(X,2) = B(X,2) * ( - 1)
650   NEXT X
660   GOTO 285
670   IF A$ < > "S" THEN  PRINT  CHR$ (7);: GOTO 260
680   FOR X = 1 TO K
690 T = B(X,1):B(X,1) = B(X,2):B(X,2) = T
700   NEXT X
710   GOTO 285
```

The program is now waiting for you to input the vertices of your two-dimensional figure. These vertices should be entered in a clockwise order. Vertices are entered simply by typing the value of the abscissa and then of the ordinate, separated by a comma. Be sure you hit return only after entering the value of the ordinate. You are allowed to enter a maximum of 20 points, but the last point you enter should close the figure. This is done by ending with the first vertex you input. To enter figure ABCD (Figure 1), you would type:

-5,-5 *(hit the return key)*
-1,-1 *(return)*
-1,-9 *(return)*
-5,-9 *(return)*
-5,-5 *(return)*

Notice that the first and last entry are the same; this closes the figure. The computer will draw each edge of the figure on the top part of the screen as the vertices are entered.

When you have entered a closed figure, you will be given three options to choose from: QUIT, NEW or CONTINUE. You select your option simply by pressing the key highlighted on the screen (Q for quit, N for new and C for continue). If you choose to QUIT, the screen will be cleared and the program ended. To restart the program, you must type RUN and begin again from the start. If you select NEW, the screen will be cleared, and you will be asked to enter new vertices for your figure (you would select NEW if you made an error in entering the vertices). If you select CONTINUE, you will be admitted to the second part of the program where you will manipulate your figure through the various transformations.

Transforming Your Figure

At this point in the program, you will be asked to select commands that will complete the various transformations on your figure. Two sets of commands are needed. The first set of commands facilitates the transformations, and include ADDX, ADDY, SIGNX, SIGNY and SWITCH. The second set is used to control the screen and exit the program and includes DRAW, NEW, CLEAR and QUIT. To select a command, simply type the highlighted character. Each command is explained below.

ADDX: This command is used to add a constant to the abscissa of each vertex of your figure. After selecting this option, you will be asked to input the

integer value of the constant. You may wish to experiment and determine the domain of the coordinate system displayed on your screen.

ADDY: This command is used to add a constant to the ordinate of each vertex in your figure. Again, you will be asked to input the value of the constant. More experimentation may be necessary to determine the range of the system shown on your screen.

SIGNX: This command simply multiplies the value of the abscissa of each vertex by -1. The effect of the SIGNX command is to complete a reflection about the y-axis. For example, point B (-1,-1) becomes B' (1,-1).

SIGNY: This command multiplies the value of the ordinate of each vertex by -1. The effect of SIGNY is a reflection about the x-axis. For example, point C (-1,-9) becomes C' (-1,9).

SWITCH: This command switches the abscissa and ordinate in each vertex. For example, point D (-5,-9) becomes D' (-9,-5). The effect of this move is left for the reader to determine by experimentation.

The reader may have noticed that nothing has yet changed on the screen regardless of what command has been tried. This is deliberate. To view the transformed figure, the DRAW option must be selected. This delay has several advantages: (1) it allows the joining of a series of manipulations before the image is shown; (2) it challenges the predictive capabilities of the students; and (3) it facilitates the development of visualization skills. The remaining commands are discussed below.

DRAW: This command simply sketches the image of the figure according to the transformations selected. None of the other figures previously drawn are erased; the image is simply added to those already shown on the screen.

NEW: When selected, this command clears the screen and takes the student back to the first part of the program where the vertices of the figure were entered. It is used when the student wishes to start over with a new figure.

CLEAR: This command simply clears the screen and redraws the original figure and is ready to accept new transformations. It would be used when the screen becomes cluttered.

QUIT: When selected, this command clears the screen and exits the program. To restart, the student types RUN.

An Example

Problem: Using the basic shape and coordinates found in Figure 1, create the diagram shown in Figure 3.

Figure 3. A Design Built from ABCD in Figure 1
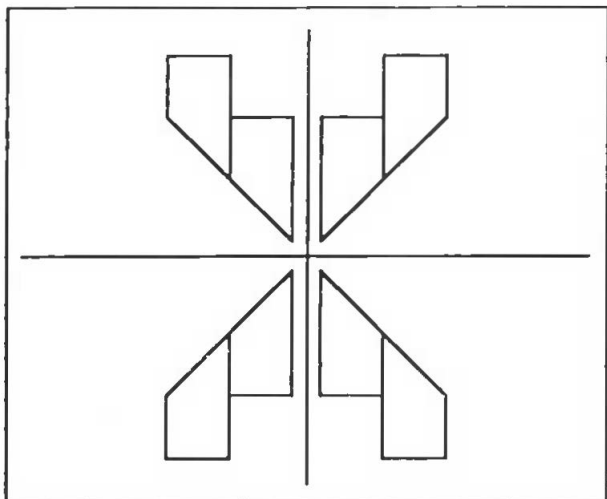
The first series of moves will be a series of reflections. If we reflect the figure about the y-axis, we can get the shape in the fourth quadrant closest to the origin. This reflection is achieved by selecting SIGNX. Remember to select DRAW to view the translated shape. Now, we need to reflect the new shape about the x-axis. This can be achieved by selecting SIGNY followed by DRAW. To get the fourth shape, we complete another reflection about the y-axis and so select SIGNX and DRAW. We now have all four of the shapes closest to the origin.

The last shape we drew was in quadrant two. We must now translate this shape left four units and up four units. This move may be accomplished by selecting ADDX with a constant value of -4. We do not draw the figure yet, as it is not fully positioned. Next, we ADDY with a constant value of 4. We are now ready to DRAW.

To finish our design, we need to make one final set of reflections. First, we reflect the figure about the y-axis using SIGNX and DRAW. Next, we reflect about the x-axis using SIGNY and DRAW, and finally, we reflect about the y-axis, again using SIGNX and DRAW. Our design is now complete.

This simple example can be easily extended to create a richer problem-solving environment by asking questions such as: Is there an easier or faster way? Can this task be completed in fewer steps? Does the order of the steps matter? The ability to answer these questions requires a genuine understanding of the processes involved in the translation of figures about a Cartesian coordinate system.

Problems and Questions

The first set of problems is found in Figure 4. The coordinates for the shaded figure are provided. Using only translations, reflections and rotations, create each of the patterns.
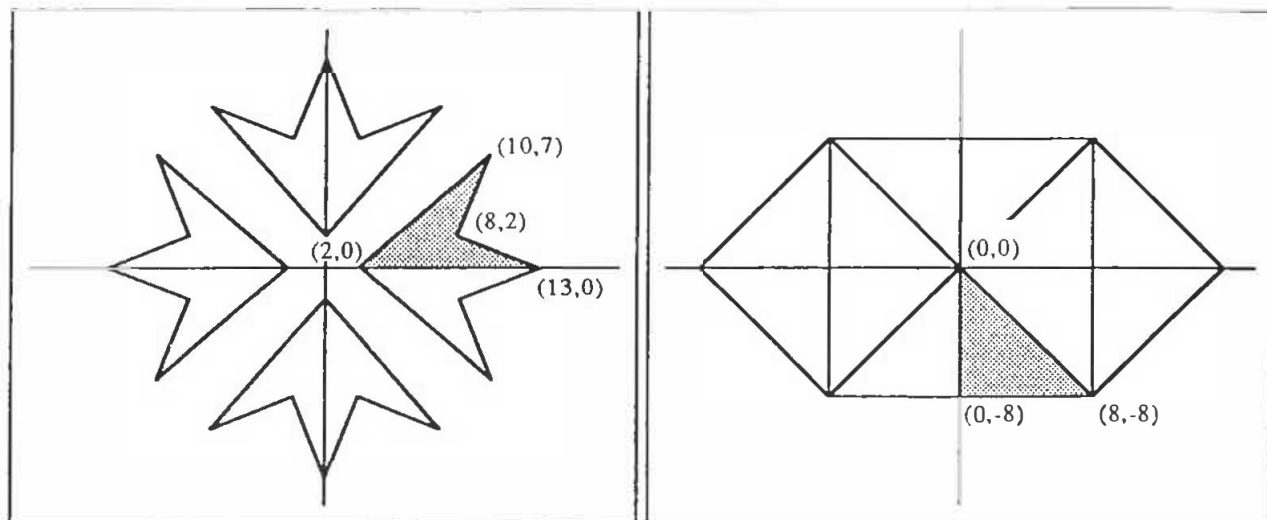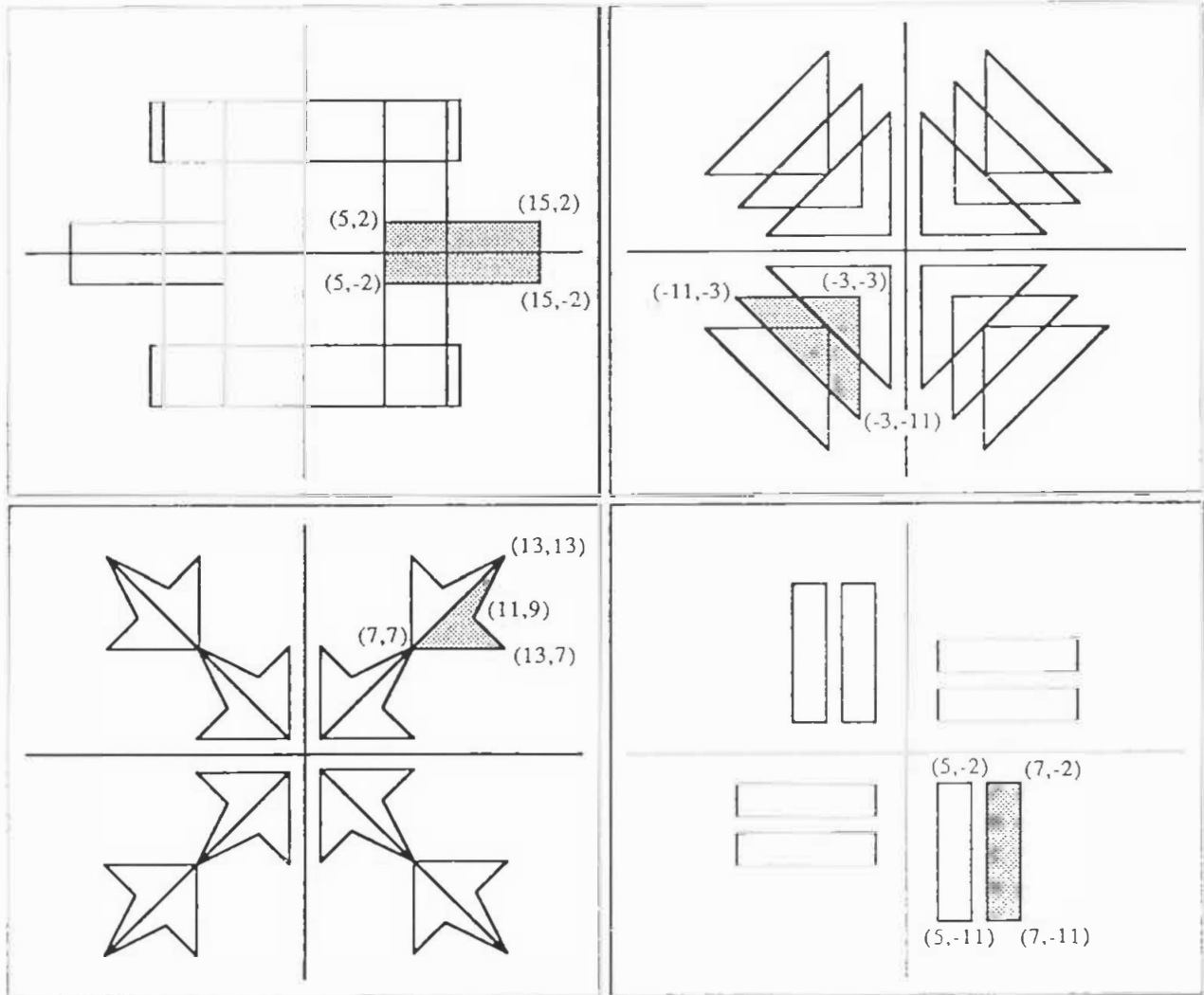
Figure 4. Motion Geometry Problems

Figure 4. (continued)

Once you are familiar with the manner in which shapes and figures are manipulated, see if you can answer the questions below:

- How is a quarter turn clockwise different from a quarter turn counterclockwise? What is the fastest way to get a half turn?
- How can you reflect a figure about a horizontal or vertical line that is not an axis? For example, try reflecting figure ABCD shown in Figure 1 about the line x = 5.
- How can you rotate a figure one-quarter turn counterclockwise about a point that is not the origin? For example, try rotating figure ABCD in Figure 1 about the point (4,2).

A Little Hint

Beginning with triangle ABC with points A (2,1), B (10,1) and C (10,5), what is the effect of the following commands used together in order: SWITCH SIGNX DRAW?